完全な透明マント（不可視装置）の数理

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あらまし　不可視装置（透明マント）とは、光をうまく迂回させて、中にある物体を見えなくする技術であり、いわゆるハリーポッターの透明マントのことである。今回、左手系メタマテリアル（負の屈折率をもつマテリアル）を用いて、等方性媒質において完全な不可視装置（透明マント）を理論的に設計した。この設計においては、完全な不可視性を実現でき、時間遅れと反射の両方をゼロにすることができる。

キーワード　透明マント、不可視装置、左手系メタマテリアル

Mathematical model of perfect invisibility devices

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Abstract　The aim of an invisibility device is to guide light around any object put inside, being able to hide objects from sight. In this work, we propose a novel design of dielectric invisibility media based on negative refraction that creates perfect invisibility. In this device, both the time delay and the reflection are zero. These findings strongly indicate that perfect invisibility with optically isotropic materials is possible.

Keyword　Invisibility device, Left-handed metamaterial, Isotropic media

1. Introduction

The aim of an invisibility device is to guide light around any object put inside, being able to hide objects from sight. This device is also known as Harry Potter's 'Invisibility Cloak', which, of course, has not been realized yet. However, recent developments on invisibility devices indicate that they could be realized in near future. Recently, a few models have been proposed to create a perfect illusion of invisibility. The main idea is to design a dielectric medium composed of a specific refractive index that literally bends and guides the light around a desired object. As the device itself should be invisible, an external observer would not see the object. The optical effect will be equivalent to observe the light rays propagating across empty space, because objects in the background will be visible. Until now, a perfect invisibility effect based on isotropic media has not been achieved and the ultimate wave nature of light has been argued as the main reason. This issue has been extensively debated in several works[1)2)6)7)]. However, using highly anisotropic media these distortions could be reduced to zero. Furthermore, a recent experimental demonstration of a cloaking device based on artificially structured metamaterials[4)] has succeed in decreasing scattered waves emitted from the hidden object. The object remains invisible over a narrow band of microwave frequencies.

In this work[3)], we extend the original idea of one of us [1)2)] based on optimal conformal mapping to create dielectric invisibility devices. In this context, we propose a new design for a cloaking device made of a metamaterial with a partially negatively refracting index. Our scheme reduces both the time delay and reflection caused by the device to zero and relaxes the constraints imposed by the requirement for bounded orbits in the Riemann sheet. Interestingly, we have designed it using isotropic media, which seems easier to construct if compared to the highly anisotropic and complex structures suggested in [4)5)]

2. Methods

By following[1)2)], a dielectric medium conformally maps a physical space z onto Riemann sheets given by an analytic function w(z). As a feature of conformal maps, the angles between coordinate lines are conserved. By using, for example, the well-known Joukowski transformation, the papers [1)2)] exploit a mapping between the physical space described by the complex field z and the analytic function w(z) that represents the mathematical space composed of two Riemann sheets and a branch cut which connects both sheets. As a result any object located at the center of physical space cannot be detected by an external observer. In other words, it seems hidden inside the Riemann sheets.

Our device is composed of five artificial dielectric materials with different values of refractive indices *n* and different index profiles, which are combined with one another like building blocks. The cloaking device is then composed of two Riemann sheets. While the first Riemann sheet the material has refractive index $n=1$, the second Riemann sheet is composed of four quadrants with alternative values negative and positive refractive index profile *n* (see Fig.1).

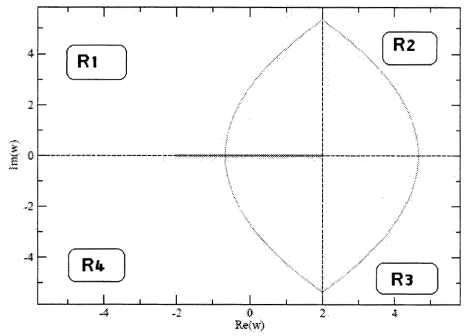

Fig.1 Trajectory of light in the second Riemann sheet

The light rays are bent and form bound orbits defined by the refractive index profiles (acting like *potentials* in classical fields) on the interior sheet. The existence of quadrants with different refractive index makes it possible to generate bound orbits with more relaxing constrains than suggested previously[1)2)]. By using the negative refractive index, the trajectory around branch cut points in the second Riemann sheet can be seen as a mirror-like reflection of light.

But more importantly, the proposed device based on negatively refracting material represents an example where the time delay in a dielectric invisibility device is zero. Furthermore, due to impedance matching of negatively refracting materials the reflection should be zero. These findings strongly indicate that perfect invisibility with optically isotropic materials is possible.

We show the light rays in physical space in Fig 2 and 3. In these figures, the central area of the cloak is impenetrable to light coming from any angle. Thus, anything placed in that area becomes invisible when seen from a distance (see Figs.2-3)

3. Conclusions

While previous cloaking devices were proposed using highly anisotropic materials with complex structures [4,5], here we have designed a device using isotropic media, which seems easier to construct. From the theoretical side, our approach has also shown that it is possible in isotropic media to develop new methods to get around Nachman's theorem [6]. In summary, our findings strongly indicate that perfect invisibility with isotropic materials is possible and opens new possibilities on the way to invisibility in the visible range of the spectrum.

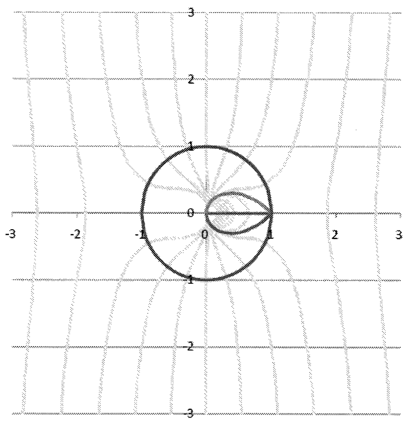

Fig. 2 Trajectory of light in the physical space

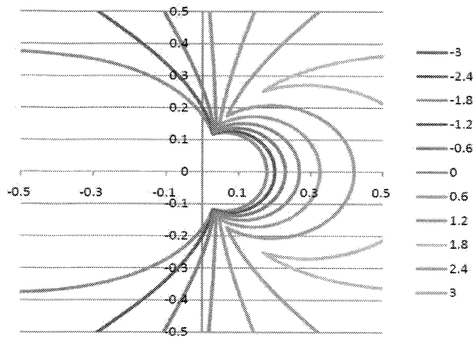

Fig.3 Enlarged view of the central area shown in Fig.2. The central portion impenetrable to light is where an object can be hidden.

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