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2進ジャンピング回路網のスパナについて

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あらまし

非同期式分散処理回路網上でシンクロナイザを実行することは重要であり，シンクロナイザとスパナの構成には密接な関係がある．本稿では，無向2進ジャンピング回路網(UBJN)のスパナの構成方法を3タイプ紹介する．これらのタイプのスパナの伸び率は，UBJNのノード数をNとすると，高々$\lceil \log_2 N \rceil$，任意の$1 \leq k \leq \lceil \log_2 N \rceil$に対し$2k-1$，任意の$0 \leq k \leq \lceil \log_2 N \rceil - 1$に対し$2k-1$である．また，これらのタイプのスパナを構成する辺の数は，$N-1$，高々$N\lceil (\log_2 N)/k \rceil$，高々$N(\log_2 N - k)/2k + Nk$である．最後に，UBJNの$\gamma$シンクロナイザに関する伸び率と辺の数の表を示す．

Spanners of Binary Jumping Networks

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Abstract

Synchronizers play an important role in an asynchronous distributed computing network system, and there is a close connection between synchronizers and the structure of spanners on the network. In this paper we show three types of spanners of undirected binary jumping networks (UBJN's for short). The stretch factors of these types of spanners of an UBJN with N nodes are at most $\lceil \log_2 N \rceil$, $2k-1$ for any $1 \leq k \leq \lceil \log_2 N \rceil$, and $2k-1$ for any $0 \leq k \leq \lceil \log_2 N \rceil - 1$. The number of edges of these types of spanners are $N-1$, at most $N\lceil (\log_2 N)/k \rceil$ and at most $N(\log_2 N - k)/2k + Nk$, respectively. Some of these spanners are superior in both stretch factors and numbers of edges to corresponding spanners for synchronizer γ of UBJN's.

1 Introduction

A distributed computing network system consists of processors and links. Each processor communicates with a neighbor processor by message transmission through a link. If the system has a global clock, it is called a synchronous system, and otherwise an asynchronous system. A distributed computing network system is essentially asynchronous. However, asynchronous algorithms are in many cases substantially inferior in terms of their time and communication complexity to corresponding synchronous algorithms [4][14]. It is frequently required for the processors to obtain some common notion of time or some simulation mechanism of synchronous behavior in the asynchronous system.

Awerbuch introduced a notion of synchronizers so that the user will be allowed to write an algorithm as if it were run in a synchronous system [4]. He proposed the construction of three types of synchronizers α, β, and γ [AW]. Since then the construction and properties of various synchronizers have been much studied [3][5][6][10][13]~[16].

Let $T(\nu)$ and $C(\nu)$ be the time and communication requirements added by a synchronizer ν for each time step of the synchronous algorithm. Awebuch showed that for synchronizers α, β and γ in network $G = (V, E)$, $T(\alpha) = O(1)$, $C(\alpha) = O(|E|)$, $T(\beta) = O(|v|)$, $C(\beta) = O(|E|)$, $T(\gamma) = O(\log_k |v|)$ and $C(\gamma) = O(k|V|)$, where k is a parameter in the range $2 \leq k < |V|$ [4]. Given a connected simple graph $G = (V, E)$, subgraph $G' = (V, E')$ is called a t-spanner of G if for every u, and v in V,

$$\frac{dist(u,v,G')}{dist(u,v,G)} \leq t,$$

where $dist(u, v, H)$ denotes the distance from u to v in H. We refer to t as the stretch factor of G. There is a close connection between synchronizers and the structure of t-spanners on a network [13]. They gave the next theorem.

Theorem 1 [13]

(1) If the network G has no t-spanners with at most m edges, then every synchronizer ν for G requires either $T(\nu) \geq t + 1$ or $C(\nu) \geq m + 1$.

(2) If the network G has a t-spanner with m edges, then it has a synchronizer δ with $T(\delta) = O(t)$ and $C(\delta) = O(tm)$.

From the avobe theorem, it is important for us to know whether we can show the existence of a family of spanners with small stretch factors and small numbers of edges for a given network family. Awerbuch proved that for some network families the best possible improvements concerning both stretch factors and numbers of edges are within constant factors from synchronizer γ [4]. However, this is not true for some networks. In fact, Peleg and Ulllman showed that the existence of a 3-spanner with a linear number of edges for the hypercube of dimenssion n (i.e., with $O(2^n)$ edges for the n-cube). The 3-spanner is supperior to synchronizer γ of the hypercube. Another example is the class of bounded-degree networks. For any bounded-degree network, synchronizer α is optima in both time and communication complexity [13].

Binary jumping networks have been proposed for interconnecting processors [1][8]. These networks can be considered to be similar to hypercubes in a sense. However, for any N there exists a binary jumping network with N node, whereas there exists a hypercube with N nodes only if N is a power of 2. An information disseminating scheme in binary jumping networks and its fault tolerance have been studied in [8][9]. In this paper we study the construction of various spanners of binary jumping networks, and discuss the time and communication complexity when we use them as synchronizers (i.e., the stretch factor and the number of edges). We also implement the program given by Awebuch [4] for constructing synchronizer γ of binary jumping networks for various values for factor k and various network sizes N.

2 Sparse Spanners

If a spanner of $G = (V, E)$ has $O(|V|)$ edges, the spanner is called sparse spanner. Any spanning tree is a sparse spanner, since it has exactly $|V| - 1$ edges. The number of nodes in the network is denoted by N. We denote m modulo r by $[m]_r$. A graph is called an undirected binary jumping network with N nodes (UBJN with N nodes or N-UBJN for short) if $V = \{0, 1, \cdots, N-1\}$ and $E = \{\{u,v\} | u, v \in V$ and either (or both) $[v-u]_N$ or $[u-v]_N$ is 2^k for some $0 \leq k \leq \lceil \log_2 N \rceil - 1\}$. Let us conside the breadth-first-search tree (BFST for short) of a UBJN. Then we have the next thorem.

Lemma 1 *The depth of the BFST of N-UBJN is at most* $\lceil \frac{\log_2 N}{2} \rceil$.

Proof. For $N \leq 8$ the assertion of the lemma obviously holds. Let $n = \lceil \log_2 N \rceil$, and let $D(N)$ denote the depth of the BFST of N-UBJN. Since the depth of any node between 2^{n-2} and $3 \times 2^{n-2}$ is at most $D(2^{n-2}) + 1$, we have the following recurrence inequality:

$$D(N) \leq D(2^n) \leq D(2^{n-2}) + 1,$$

$$D(2^2) = 1 \text{ and } D(2^3) = 2.$$

From the above inequality and the values of $D(2^2)$ and $D(2^3)$ we have $D(N) \leq D(2^n) \leq \lceil n/2 \rceil$. □

Theorem 2 *The stretch factor of N-UBJN is at most* $\lceil \log_2 N \rceil$.

Proof. Let $n = \lceil \log_2 N \rceil$. If n is even, then from Lemma 1 its stretch factor is at most n. Suppose that n is odd. Consider 2^n-UBJN and two subgraphs A_n and B_n of the 2^n-UBJN, where A_n consists of all even numbered nodes and their induced edges, and B_n consists of all odd numbered nodes and their induced edges. Then from Lemma 1 the distance between any pair of nodes in A_n or any pair of nodes in B_n is at most $n-1$. The distance between any node in A_n and any node of B_n is at most $2(n-1)/2 + 1 = n$. Since $D(N) \leq D(2^n)$, the theorem holds. □

The BFST of 12-UBJN is shown in Figure 1. The number of edges of the BFST is 11.

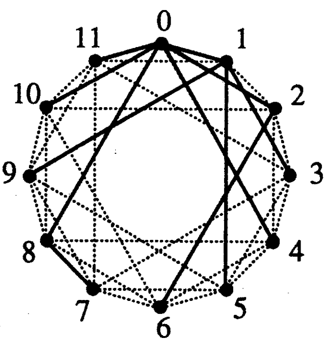


Figure 1: The BFST of 12-UBJN

The stretch factors of the BFST's of N-UBJN's up to $N = 130$ are shown in Table 1. We note that for many values of N the bound given in Theorem 2 is tight and that for some values of N the stretch factors are less than $\lceil \log_2 N \rceil$ by 2. For example, the stretch factor of the BFST of 43-UBJN is $\lceil \log_2 43 \rceil - 2 = 4$.

a \ b	1	2	3	4	5	6	7	8	9	10
0	1	1	2	2	2	3	2	3	3	3
1	4	4	4	3	4	4	4	4	4	4
2	4	5	4	5	4	5	4	4	4	5
3	4	5	5	5	4	5	4	5	5	5
4	5	6	4	6	6	6	6	6	6	6
5	6	6	6	6	6	5	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6
7	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	6	7	6	7
9	6	6	6	7	6	7	6	7	6	6
10	6	7	6	7	6	7	6	6	6	6
11	6	6	6	6	6	6	6	7	6	7
12	6	7	6	6	6	7	6	7	7	7

The value in entry (a, b) denotes the stretch factor of the BFST of $(10a+b)$-UBJN (i.e., $N = 10a + b$)

Table 1: Stretch factors of BFST's of N-UBJN's

3 2^{k-1}-Spanners

Let N-UBJN(k) be a spanner of N-UBJN defined as follows: N-UBJN(k)= (V, E), where $V = \{0, 1, \cdots, N-1\}$ and $E = \{\{u, v\} | [u-v]_N$ or $[v-u]_N$ is 2^i for some i such that i is 0 modulo k, $0 \leq i \leq \lceil \log_2 N \rceil - 1\}$. Then the stretch factor of N-UBJN(k) is obviously at most 2^{k-1}, and the number of edges of this spanner is at most $N\lceil(\log_2 N)/k\rceil$.

For $k = 1$, this spanner is N-UBJN itself and the spanner for α-synchronizer [4]. For $k = 2$, it is a 2-spanner with at most $N\lceil(\log_2 N)/2\rceil$ edges. We show the 2-spanner and the 4-spanner of 12-UBJN constructed in this way in Figure 2. At present we do not know whether there exists a 2-spanner of N-UBJN with a smaller number of edges than the number of edges of N-UBJN(2).

4 Spanner Construction by Partition

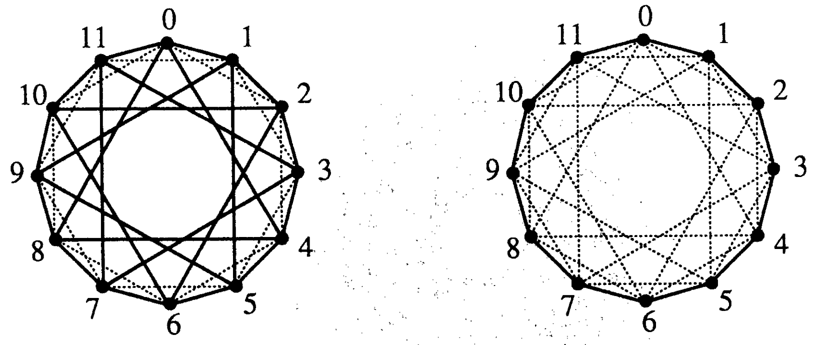
Let N-UBJN $= (V, E)$, and let $K = 2^k, 0 \leq k \leq \lceil \log_2 N \rceil - 1$. In this section we assume that N is a multiple of K. For each $i (0 \leq i \leq K-1)$ let $G_i = (V_i, E_i)$, where $V_i = \{v | 0 \leq v \leq N-1$ and $v = i$ modulo $K\}$ and E_i is the set of all induced edges by V_i in E of N-UBJN. Then G_i is isomorphic to N/K-UBJN, where each node u in N/K-UBJN corresponds to node $uK + i$ in G_i. We show 4 partitioned subgraphs $G_i (0 \leq i \leq 3)$ of 20-UBJN in Figure 4.

Let $E' = E - (E_1 \cup E_2 \cup \cdots \cup E_{K-1})$, and let $G' = (V, E')$. Then G' is a spanner of N-UBJN. For any edge $\{u, v\} \in E_1 \cup E_2 \cup \cdots \cup E_{k-1}$, $dist(u, v, G') \leq 2(k-1) + 1 = 2k - 1$. Therefore, the stretch factor of G' is at most $2k - 1$ and the number of edges of G' is at most $N(\log_2 N - k)/2^k + Nk$.

The product of t and m of a t-spanner with m edges is an interesting measure of the spanner. It was an interesting question whether for N-UBJN there exists of a t-spanner with m edges such that $mt = o(N \log N)$. If we choose $k = \log_2 \log_2 N$, for the spanner constructed in this section the product of the stretch factor and the number of edges is $O(N \log \log N)$. Hence, we can affirmatively answer to the above question.

5 Spanners as Synchronizer γ

As described in [4], synchronizer γ is a combination of two synchronizers α and β. The spanners for synchronizer α and β of a network are the network itself and the BFST of the network, respectively. Awerbuch demonstrated that spanners for synchronizer γ is optimal for some networks in the sense of time and commu-



(a) a 2-spanner (b) a 4-sppaner

Figure 2: Spanners of 12-UBJN

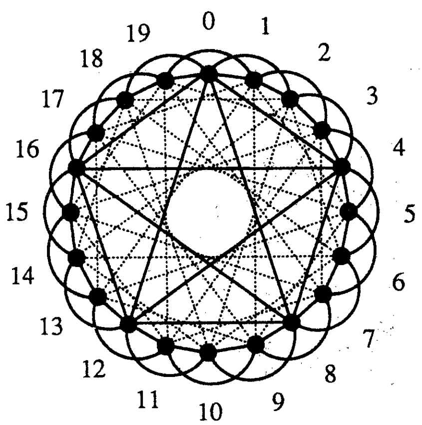


Figure 3: A 3-spanner of 20-UBJN

nication complexity within constant factors [4]. However, as shown in [13] for some networks there exist better spanners than spanners for synchronizer γ. We implemented the Awerbuch's algorithm for UBJN's to construct spanners. The result of this implementation is shown in Table 2. Notice that many spanners constructed in the ways in Sections 2, 3 and 4 are superior to spanners of UBJN's for synchronizer γ.

6 Concluding Remarks

We have shown various spanners of UBJN's. We do not know at present whether spanners given in Section 4 can be optimal within constant factors in the sense of the smallest value for the product of the stretch factor and the number of edges if we choose an appropriate value for k. We therefore are interested in deriving a nontrivial lower bound on the product of the stretch factor and the number of edges. The following problems are also worth of further studies:

(1) Construct various spanners of directed binary jumping networks.

(2) Find efficient methods for constructing superior spanners of more general families of networks.

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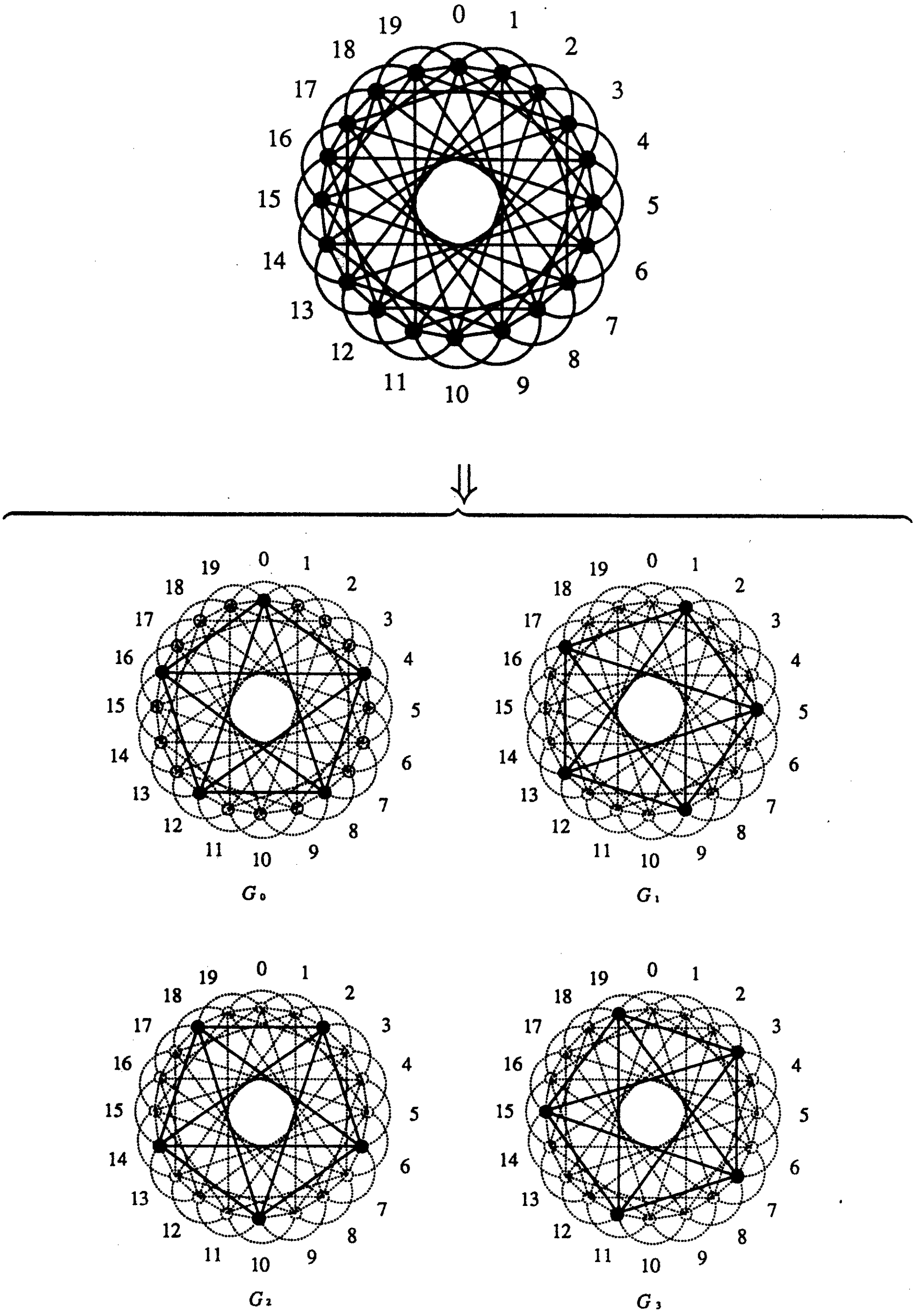

Figure 4: 4 partitioned subgraphs of 20-UBJN

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k	2		3		4		5		6		7		8		9		10		11		12		13	
N	t	m	t	m	t	m	t	m	t	m	t	m	t	m	t	m	t	m	t	m	t	m	t	m
3	2	2	1	3	1	3	1	3																
4	2	3	2	3	1	6	1	6																
5	2	4	2	4	2	4	1	10	1	10	1	10												
6	3	5	3	5	3	5	1	15	1	15	1	15												
7	2	6	2	6	2	6	2	6	2	6	1	21												
8	3	8	3	8	3	8	3	8	1	28	1	28												
9	3	9	3	9	3	9	3	9	3	9	1	36	1	36	1	36								
10	4	9	3	12	3	12	3	12	3	12	1	45	1	45	1	45								
11	3	11	3	11	3	11	3	11	3	11	3	11	3	11	1	55								
12	4	14	4	12	3	21	3	21	3	21	1	66	1	66	1	66								
13	3	18	3	18	3	18	3	18	3	18	3	18	3	18	1	78								
14	4	13	4	13	4	13	3	23	3	23	3	23	3	23	1	91								
15	4	20	4	17	3	29	3	29	3	29	3	29	3	29	1	105								
16	4	25	4	25	4	21	3	43	3	43	3	43	1	120	1	120								
17	5	17	4	22	4	22	3	44	3	44	3	44	3	44	1	136	1	136	1	136				
18	4	27	4	27	4	27	4	23	3	53	3	53	3	53	1	153	1	153	1	153				
19	5	19	4	24	4	24	3	46	3	46	3	46	3	46	3	46	3	46	1	171				
20	4	22	4	40	4	40	4	34	3	74	3	74	3	74	1	190	1	190	1	190				
21	4	35	5	30	5	30	5	30	5	26	3	65	3	65	3	65	3	65	1	210				
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23	4	37	5	25	5	25	4	43	5	37	3	88	3	88	3	88	3	88	1	253				
24	4	44	5	29	5	44	4	68	3	128	3	128	3	128	1	276	1	276	1	276				
25	5	30	5	30	5	30	4	52	4	52	3	115	3	115	3	115	3	115	1	300				
26	5	46	4	61	4	61	4	61	4	61	4	53	3	130	3	130	3	130	1	325				
27	5	27	5	36	5	54	4	71	4	71	3	146	3	146	3	146	3	146	1	351				
28	4	30	4	30	4	30	4	82	4	82	4	72	3	163	3	163	3	163	1	378				
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52	6	61	5	129	4	222	4	222	4	222	5	171	5	304	4	486	3	792	3	792	3	792	1	1326
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59	6	64	5	73	5	149	5	268	4	383	5	311	5	493	3	1093	3	1093	3	1093	3	1093	1	1711
60	6	74	5	80	5	212	5	195	4	410	5	335	5	312	4	762	3	1140	3	1140	3	1140	1	1770
61	6	75	5	81	6	115	5	138	5	411	5	588	4	525	4	801	3	1188	3	1188	3	1188	1	1830
62	6	71	6	82	5	181	5	337	5	314	5	232	5	361	4	841	3	1237	3	1237	3	1237	1	1891
63	6	83	6	77	5	198	5	198	5	198	5	413	5	387	4	882	3	1287	3	1287	3	1287	1	1953
64	7	73	5	108	5	183	5	141	5	388	5	528	4	1009	3	1389	3	1389	3	1389	1	2016	1	2016

Table 2: Spanners for synchronizer γ