アルゴリズム 42−13
（1994. 11. 18）

最小最大化部分集合問題

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ある組合せ最小化構造を考えた時、与えられた重みつきグラフのＫ個の頂点集合で、最小化構造が最大になるものを探す問題を遠隔Ｋ集合問題と呼ぶ。代表的な最小化構造には最小木があり、この時は最小木遠隔Ｋ集合問題と呼ばれる。同様にスタイナー木遠隔Ｋ集合問題や巡回路遠隔Ｋ集合問題も考える。この論文では（１） 遠隔Ｋ集合問題に対してNP完全性と近似アルゴリズムの効率の理論的下界（２）三角不等式がなり立つ重みつきグラフでの最小木遠隔Ｋ集合問題に対する貪欲算法の最適近似比４の導出（３）平面距離に対応する時の最小木遠隔Ｋ集合問題の近似比2.25のアルゴリズムの設計（４）２と３に対応するスタイナー木及び巡回路遠隔Ｋ集合問題についての結果を与える。

Finding subsets maximizing minimum structures

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We consider the following problem: "Given a graph $G = (V, E)$ and a number K, compute the *subset* of V, P, of cardinality K such that the total weight of a *minimum structure* is *maximized*."

If the structure is "spanning tree", the problem is named the *MST-remote K-set problem.* If we maximize the minimum Steiner tree of P instead of the minimum spanning tree, it is named the *Steiner-remote K-set problem.* Also, we consider *TSP-remote K-set* maximizing the TSP tour of the subset.

We give: (1) NP-hardness and lower bounds of approximation ratios of the above remote K-set problems. (2) tight analysis of the furthest-point greedy approximation algorithm with an approximation ratio 4 for the MST-remote K-set problem when the graph is metric (that is, when the edge weights satisfy the triangle inequality) (3) an approximation algorithm computing MST-remote K-set with a performance ratio 2.25 for a graph induced from Euclidean distance of a set of points in a plane, and (4) corresponding results for Steiner-remote and TSP-remote K-set problems.

1 Introduction

Let $G = (V, E)$ be a complete weighted undirected graph such that each weight is non-negative (possibly infinity). Let P be a subset of V. A spanning tree of P is a subtree of G whose node set is P. A Steiner tree of P is defined as a spanning tree of a *superset* of P. A subcircuit of G (regarding an undirected edge as a pair of oppositely directed edges) whose node set is a superset of P is called a tour of P. The minimum weight tour of P is often called a TSP tour of P.

Minimum weight spanning tree (in short, minimum spanning tree or MST), minimum Steiner tree and TSP tour are famous combinatorial structures, which are not only useful in applications but also rich source of research on exact and approximation algorithms.

We denote minimum spanning tree, minimum Steiner tree, and TSP tour of P by $MST(P)$, $St(P)$, and $TSP(P)$ respectively.

The total weight of a tree (or a tour) T is denoted by $l(T)$. We consider the following problems:

MST-remote K-set problem: Find a subset P of cardinality K of V such that $l(MST(P))$ is maximized. The set P is called the *MST-remote K-set* of G.

Steiner-remote K-set problem: Find a subset P of cardinality K of V such that $l(St(P))$ is maximized. The set P is called the *Steiner-remote K-set* of G.

TSP-remote K-set problem: Find a subset P of cardinality K of V such that $l(TSP(P))$ is maximized. The set P is called the *TSP-remote K-set* of G.

All of these problems find subsets (of the node-set of a weighted graph) *maximizing* the weight of minimum combinatorial structures constructed from the subsets.

If we consider a complete graph induced from a set of points in an Euclidean space such that the edge weights are given as the Euclidean distances, the problems can be regarded as computational geometry problems. In computational geometry, the problem of finding a subset with cardinarity K of a planar point set maximizing the perimeter or area of convex hull (minimum perimeter enclosing polygon) of the subset was studied in literature [2, 3, 5]. However, the authors know no previous results on problem of computing subsets maximizing other minimum structures.

The problem of finding subsets *minimizing* the minimum weight of a combinatoral strucure is probably more popular[1, 7, 15, 10]. In particular, the problem of finding the K-set minimizing the weight of the minimum MST was recently studied by Ravi et al.[15] who proved NP-hardness, and gave an $O(K^{1/4})$ approximation algorithm for a graph induced from a set of points in a plane. Garg and Hochbaum[10] improved the approximation ratio to $O(\log K)$.

From a practical point of view, the MST-remote (or Steiner-remote) K-set of a network can be considered as the set of K nodes among which it is most expensive to communicate information. Thus, the remote subsets can be applied to the evaluation of the communication performance of networks.

They can also be applied to clustering problems. In particular, when G is a graph induced from distances among a set of points in a plane (Figure 1), the problems are strongly related to geometric clustering problems. Indeed, we originally faced these problems when we tried to obtain a good "starting tour" of a large TSP instance (it was a circuit board drilling problem [12] which occured at a manufacturing plant) with more than 10,000 non-uniformly distributed cities. For obtaining a short approximate TSP tour by using a construction heuristics, it is effective to start with a subtour (starting tour) consisting of relatively small number of sample cities, which effectively captures a global structure of the point distribution [13]. For the purpose, random sampling is not suitable, since it may miss some critical cities, and an approximate TSP tour constructed from the associated starting tour is often a bad instance as an input of a local heuristics algorithm. The MST-remote K-set and/or the TSP-remote K-set (or those approximate solutions) seem to give better starting tours.

If we remove the cardinality condition from MST-remote K-set problem, we have the following

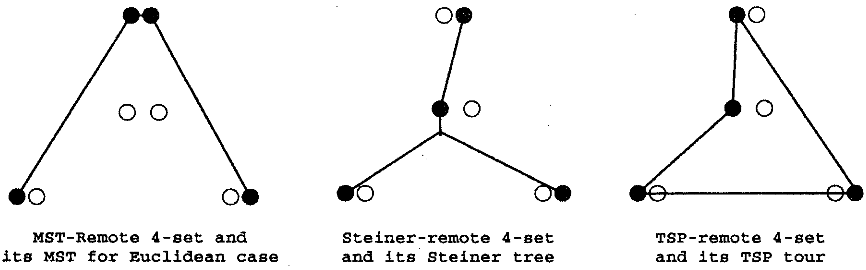

Figure 1: remote K-sets

problem:

MST-remote subset problem: Find a subset Q of V such that $l(MST(Q))$ is maximized.

The MST-remote subset problem can be considered as a *inverse problem* of the Steiner problem. Whereas the Steiner problem find a superset Q' of P minimizing $MST(Q')$, the MST-remote subset problem find a subset Q of V maximizing $MST(Q)$.

We first show that remote K-set problems are NP-hard for general graphs. Moreover, it is NP-hard to obtain a constant-ratio approximation algorithm for the MST-remote K-set problem. Here, the approximation ratio is defined by the ratio of the weight of the optimal MST (resp. Steiner tree, TSP) to that of the output MST (resp. Steiner tree, TSP).

However, in practical cases, it is possible to obtain constant-ratio approximation algorithms. Indeed, we can usually assume the metric condition when we consider applications of MST. The graph G is called *metric* if the weight of G satisfies the triangle inequality. That is, for any three nodes u, v, w, $d(u,v) + d(v,w) \geq d(u,w)$. Here, $d(u,v)$ is the weight of the edge $e(u,v)$ connecting u and v, and often called *distance* when the graph is metric.

An example of a metric graph can be constructed as follows: Given a weighted graph G, we define a complete graph $D(G)$ with the same node set such that the edge weight of $e(u,v)$ is defined to be the weight of the minimum weight path between u and v of G. The graph $D(G)$, which we called the *shortest-path distance graph*, is metric. Graph G is sometimes called the seed of $D(G)$. Another example of a metric graph is a graph induced from distances of set of points in a metric space.

If the graph G is metric, a constant-ratio approximation for the MST-remote K-set is possible. We shall give an approximation algorithm with the approximation ratio 4 for metric cases, and show that this ratio is tight for the algorithm. Furthermore, if the graph is induced from Euclidean distances of a set of points in a plane, we give an approximation algorithm with the approximation ratio 2.25.

Both algorithms are based on the greedy furthest-point algorithm, which constructs "K-central-circle covering", that was originally introduced by Feder and Greene [8] to obtain a 2-approximation algorithm for the Euclidean K-clustering problem.

The approximation ratios of both algorithms are analyzed by using a nice property of the K-central-circle cover, in conjunction with well-known results concerning the Steiner ratios of metric and Euclidean Steiner trees.

Steiner-remote K-set problem is NP-hard, and TSP-remote K-set problem is hard to approximate within a factor of 4/3. We can give 3-approximation results for both problems.

We also give a linear time algorithm for computing Steiner-remote K-set if the set of edges with non-infinity weights forms a tree in G.

2 Hardness results

The decision version of the MST-remote K-set problem (decide whether there exists a set of size K whose MST has a weight more than a given threshold) is obviously in NP. Instead of showing NP-hardness, we show the approximation-hardness. The approximation ratio of an algorithm for computing a MST-remote K-set is the ratio of the weight of the MST for the optimal MST-remote K-set to that for the output approximate MST-remote K-set.

Proposition 2.1 *It is NP-hard to obtain an approximate solution of the MST-remote K-set problem within an approximation factor n^{ϵ} for some $\epsilon > 0$.*

Proof We consider a graph $H = (V, E_1)$ and its cograph $H^c = (V, E_2)$ in the complete graph. We give a weight 1 to each edge of E_1, and give a sufficiently large weight $\alpha > n$ to each edge of E_2. Suppose that H is the graph of Arora et al.[4] such that it is hard to approximate an independent set within a factor of $\gamma = n^{\epsilon}$ for some ϵ unless $P = NP$. Let us consider $G = (V, E_1 \cup E_2)$. Suppose K_{opt} is the size of the maximum independent set of H. If we have an approximate solution for the remote K-set whose MST has a weight $(s_K - 1)\alpha + (K - s_K)$, we can compute an independent set of H with size s_K. Indeed, if we erase all edges in E_2 from the MST, and select a node from each connected component, it forms an independent set. Let s_{max} be the maximum of s_K over all $K = 1, 2, .., n$. Thus, we can construct an approximation algorithm for the independent set within the factor of K_{opt}/s_{max}, which is not larger than $K_{opt}/s_{K_{opt}}$.

Thus, unless $P = NP$, $S_{K_{opt}}$ may be smaller than $\gamma^{-1}K_{opt}$. Hence, the ratio of $(s_{K_{opt}} - 1)\alpha + (K - s_{K_{opt}})$ to the weight of MST of the optimal MST-remote $(K_{opt} - 1)$-set may become smaller than $\gamma^{-1} + \alpha^{-1} < \gamma^{-1} + n^{-1}$. Thus, it is hard to obtain an approximation ratio better than γ. □

Similarly, it is NP-hard to approximate the MST-remote subset problem within a constant factor.

If the graph is metric, we can only have a constant lower bound for the MST-remote K-set problem.

Proposition 2.2 *It is NP-hard to obtain an approximate solution of the MST-remote K-set problem of a metric graph within an asymptotic factor less than 2.*

Proof The proposition is similarly proved as Proposition2.1, setting $\alpha = 2$. □

For Steiner-remote K-set problem and TSP-remote K-set problems, one can always assume that graph G is metric, since TSP and minimum Steiner tree of a node set P in G can be realized as those of P in the shortest-path distance graph $D(G)$. Hence, these problems look like easier to approximate than MST-remote K-set problem.

Proposition 2.3 *It is NP-hard to obtain an approximation solution of the Steiner-remote K-set problem within an asymptotic factor less than 4/3.*

Proof Given graph G, we construct a graph H' as follows. Replace each edge of G by a path with two-edges, and connect the middle vertices of the paths into a clique. The input to Steiner-remote K-set problem is the distance graph $D(H')$ of H'. If we consider two vertices in G, they will be of distance 2 in H' if they are adjacent in G, and of distance 3 in H' if they are non-adjacent in G.

An independent set in G corresponds to a set of vertices in H' that have no neighbors in common. Hence, the cost of the minimum Steiner tree of that set in $D(H')$ is $2(K - 1)$. On the other hand, given a K-set, we can easily find a Steiner tree that is minimal in the following sense: the Steiner vertices adjacent to only one terminal each can not be reduced in that no two of the corresponding terminals share a common neighbor. These terminals are all of pairwise distance 3 in H', and thus form an independent set in G. Let P be the number of those terminals. The cost of the Steiner tree constructed will be at most $3/2(K-P-1)+2P = 3/2(K-1)+1/2P$, and that upper bound holds naturally for the minimum Steiner tree as well.

If, now, we could guarantee finding a K-set where the minimum Steiner tree is at least $3/2K + 1/2K^{\epsilon}$, it follows that P, the size of the independent set found in G, will be at least K^{ϵ}. From [4], this is NP-complete. □

Proposition 2.4 *It is NP-hard to obtain an approximate solution of the TSP-remote K-set within an asymptotic factor less than 2.*

Proof Given an unweighted graph G, we construct a complete graph H, with the same vertex set, and weight of an edge equal to 1 if the edge is in G and 2 otherwise. Clearly, this graph satisfies the triangular inequality.

If G contains an independent set of size K, then any tour of that K-set in H will have cost $2K$. On the other hand, suppose the algorithm finds a K-set S in H for which $TSP(S) \geq K + K^{\epsilon}$. Grow a collection of disjoint paths in S using only edges of cost 1, until no endpoints of two paths can be joined with a cost 1 edge. Let the number of paths be P, and thus the cost of any tour formed by arbitrarily connecting the paths into a circuit is $K + P - 1$. Select one endpoint from each path; any pair will be an edge of cost 2, hence they form an independent set in G of size P. The number of paths P equals the cost of our tour less $K - 1$, which is at least $TSP(S) - K + 1$, which is by assumption at least K^{ϵ}. Hence, the algorithm has produced an approximation of Independent Set proved to be NP-complete in [4]. □

The Steiner-remote K-set problem and TSP-remote K-set problem are in Σ^2. We do not know whether these problems are in NP or not. On the other hand, as is shown in the next section, there are polynomial time constant-ratio approximate algorithms for these problems

3 Aproximation algorithms for Metric case

In this section, we assume that $G = (V, E)$ is metric unless it is explicitly declared to be general. We give an approximation algorithm for the MST-remote K-set problem.

K-central-circle covering: We find a set P of K nodes $p_1, \ldots, p_K$ such that there exists a real number r (called the radius of the circle covering) satisfying $d(p_i, p_j) \geq r$ for $i \neq j$ and $Min_i\{d(v, p_i)\} \leq r$ for any node $v \in V$. Such a node set P is called the set of *centers* of a K-central-circle covering, and can be constructed by the greedy furthest-point algorithm.

Greedy furthest-point algorithm: We choose an arbitrary node (starting node) v of V, and initially set $P = \{v\}$. The distance between a node u and the set P is the shortest distance between u and nodes in P. We choose the furthest node q in $V - P$ from P, and insert q into P until the cardinality of P becomes K. Then, it is easy to see that the node set is a set of centers of a K-central-circle covering. The radius r of the circle covering is half of the distance between the last chosen node q and $P - \{q\}$.

Theorem 3.1 *The set P of centers of a K-central-circle covering is a 4-approximation of the optimal MST-remote K-set. Similarly, P is a 3-approximation of the optimal Steiner-remote K-set.*

Proof Obviously, $l(MST(P))$ is at least $(K-1)r$. Suppose Q is the optimal MST-remote K-set. Let us consider a tree $T(P \cup Q)$ formed by joining each point of Q to the nearest node of $MST(P)$. Since the length of each newly created edge is at most r, $l(T(P \cup Q))$ is at most $l(MST(P)) + Kr$. This tree $T(P \cup Q)$ is a (not necessarily minimum) Steiner tree of Q. Since the approximation ratio (Steiner ratio) of a minimum Steiner tree to that of the MST is known to be $2(K-1)/K$, $l(MST(Q)) \leq 2(K-1)l(T(P \cup Q))/K \leq 2(K-1)(l(MST(P)) + Kr)/K$. The ratio $l(MST(Q))/l(MST(P)) \leq$

$2(K-1)(l(MST(P))+Kr)/Kl(MST(P)) \leq (4K-2)/K$. Thus, the approximation ratio is at most 4. Similarly, $l(St(P)) \geq Kr/2$ because of the Steiner ratio, and $l(St(Q)) \leq l(St(P \cup Q)) \leq l(St(P))+Kr$; thus $l(St(Q)) \leq 3l(St(P))$. □

Corollary 3.2 *The Steiner-remote K-set of a general graph can be polynomially approximated with the approximate ratio* 3.

Proof $St(P)$ of a set P of nodes in G can be realized as a minimum Steiner tree of P in the shortest-path graph $D(G)$. Since $D(G)$ is metric, the corollary follows from Theorem 3.1. □

Corollary 3.3 *The TSP-remote K-set of a general graph can be polynomially approximated with an approximate ratio* 3.

Proof Let $l(TSP(P)$ be the length of the optimal TSP tour of the set P of centers of a central K circles covering. Clearly, $l(TSP(P)) \geq Kr$. Suppose Q is the optimal TSP-remote K-set. Connecting each point of Q to its nearest point in P by a pair of directed edges (with different directions), we can make a TSP tour of $P \cup Q$ from $TSP(P)$. Its length is at most $l(TSP(P)) + 2Kr$. Thus, $3l(TSP(P)) \geq l(TSP(P \cup Q)) \geq l(TSP(Q))$. □

Intuitively, the analysis of the approximation ratio in Theorem 3.1 of the remote K-set generated by the greedy furthest-point algorithm looks loose; however, it is asymptotically optimal.

Theorem 3.4 *The approximation ratio of the MST-remote K-set generated by the greedy furthest-point algorithm for metric graphs can be as large as* $4 - o(1)$.

Proof The lower bound construction is a little complicated, and omitted in this version. See [11]. □

Although the above lower-bound is applicable only to the set of centers of the K-central-circle covering generated by the greedy furthest-first algorithm, we conjecture that 4, rather than 2 obtained in Proposition 2.2, is an asymptotically tight approximation ratio of the problem itself.

4 Euclidean case

Let S be a set of n points $\{p_1, \ldots, p_n\}$ in the plane. Let us consider the complete graph G on n nodes S whose edge (p_i, p_j) has as its weight the Euclidean distance $d(p_i, p_j)$. We consider approximation algorithms for computing the remote K-set and the Steiner-remote K-set of this graph.

The K-central-circle covering defined in the previous section gives a geometric covering of S by K circles, each of which has a point of P as its center, and is of radius r. Since $l(St(P)) \geq \sqrt{3}l(MST(P))/2$ (Du-Hwang [6]) for the Euclidean case, we immediately obtain the following.

Corollary 4.1 *Suppose P is the set of centers of any K-central-circle covering. Then, $MST(P)$ is a $\frac{4K-2}{\sqrt{3}(K-1)}$-approximation of the optimal MST-remote K-set, and $St(P)$ is a $\frac{2K+\sqrt{3}(K-1)}{\sqrt{3}(K-1)}$-approximation of the optimal Steiner-remote K-set.*

Thus, the approximation ratios are asymptotically at most $4/\sqrt{3} \approx 2.309$ for the MST-remote K-set problem, and $(2+\sqrt{3})/\sqrt{3} \approx 2.155$ for the Steiner-remote K-set problem. Unlike in the metric case, it seems that the approximation ratio depends on the choice of the K-central-circle covering. For the example in Figure 3, the worst central-circle covering has a $(2\sqrt{3}+4)/3) \approx 2.448$ approximation ratio, which is near to the upper bound $14/3\sqrt{3} \approx 2.694$ for the MST-remote 4-set.

However, if we consider the central-circle covering created by the greedy furthest-point algorithm, we can obtain a slightly better analysis (which we omit due to the space limitation) than Corollary 4.1. A key difference from the metric case is that the length of a minimum Steiner tree of m points

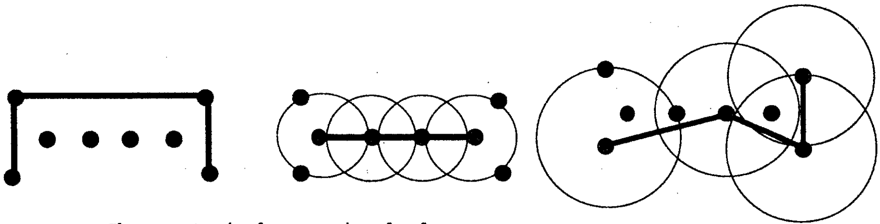

Figure 2: Approximation by circle covers

in a unit circle must be less than $m - 0.1(m-3)$ if $m > 3$ for the Euclidean case (we omit the proof). We can further squeeze the approximation ratio by modifying the algorithm itself; indeed, this can be attained by modifying the output of the greedy algorithm.

Theorem 4.2 *There exists a polynomial time approximation algorithm that has an asymptotic approximation ratio* $(1.95)(2/\sqrt{3}) = 2.2517$ *for the remote* K*-set problem.*

We omit the proof (because it is quite lengthy) of the above theorem in this version.

5 Tree case

In this section, we consider a graph in which the set of edges with finite weights forms a tree. Let T be a weighted tree on n nodes. We assign ∞ to each edge of the cograph T^c of T in the complete graph of order n, and define $G(T) = T \cup T^c$. We first give an $O(n)$ time algorithm for the Steiner-remote K-set of $G(T)$.

Proposition 5.1 *The Steiner-remote* K*-set of* $G(T)$ *can be computed in* $O(n)$ *time.*

Proof Obviously, we should select K leaves. If K exceeds the number of leaves of T, every set of K nodes containing all leaves makes the optimal Steiner-remote K-set. If $K = 2$, the problem is the diameter path problem on a tree, and can be solved in linear time. We can apply the incremental strategy developed by Peng et al. [14] for computing the K-tree core. A key fact is that any optimal Steiner-remote $(K-1)$-set must be contained in an optimal Steiner-remote K-set. A direct modification of the algorithm of Peng et al. [14] runs in $O(\min\{Kn, n \log n\})$ time, and the one of the improved algorithm of Shioura and Uno [16] runs in $O(n)$ time. □

The MST-remote K-set problem (with respect to MST) of $G(T)$ is a nonsence problem, since we can almost always find a subset P whose MST has infinity weight. If we modify the definition of the remote K-set P so that $l(MST(P))$ is maximized on the condition that $l(MST(P)) \neq \infty$ (we call it connectivity condition), $MST(P)$ must be the induced subtree of P in T; thus, the problem becomes a special case (where all edge weights are non-positive) of the *weighted* $(K-1)$*-cardinality tree* problem defined by Fischetti et al. [9] if we reverse the sign of all weights of T. We can thus apply Fischetti et al's $O(K^2n)$ time dynamic programming algorithm. Moreover, we can improved it to $O(Kn)$ (omitted in this version). The same algorithm can compute MST-remote K-sets (with connectivity condition) of decomposable graphs, such as series parallel graphs, in $O(Kn)$ time.

6 Concluding remarks

Immediate open problems are: proving (or disproving) Σ^2-hardness of Steiner-remote K-set problem, proving NP-hardness of the Euclidean remote K-set problem, and giving better bounds for the approximation ratios for each problem. In particular, a good approximation algorithm for the Steiner-remote K-set problem will be very useful in applications. Also, a fast approximation algorithm is needed; when we apply approximate TSP-remote K-sets to large-scale-TSP heuristics, subquadratic time algorithm is definitely required.

Among several other minimum strucrures than those dealed with in this paper, for example, the problem of computing the K-set maximizing the minimum weight matching looks interesting.

References

[1] A. Aggarwal, H. Imai, N. Katoh, and S. Suri, Finding k Points with Minimum Diameter and Related Problems, *J. of Algorithms* **12** (1991), 38-56.

[2] A. Aggarwal, M. Klawe, S. Moran, P. Shor, and R. Wilber, Geometric Applications of a Matrix-Searching Algorithm, *Algorithmica* **2** (1987), 195-208.

[3] A. Aggarwal, B. Schieber, and T. Tokuyama, Finding a Minimum Weight K-link Path in Graphs with Monge Property and Applications, *Proc 9th ACM Symp. on Computational Geometry*, (1993) 189-197.

[4] S. Arora, C. Lund, R. Motowani, M. Sudan, and M. Szegedy, Proof Verification and Hardness of Approximation Problems, *Proc. 33rd IEEE FOCS* (1992), 14-23.

[5] J. Boyce, D. Dobkin, R. Drysdale, and L. Guibas, Finding Extremal Polygons, *SIAM J. on Computing* **14** (1985), 134-147.

[6] D.-Z. Du and F. K. Hwang, An Approach for Proving Lower Bounds: Solution of Gilbert-Pollak's Conjecture of Steiner Ratio, *Proc. 31st IEEE FOCS* (1990), 76-85.

[7] D. Eppstein, New Algorithms for Minimum Area k-gons, *Proc. 3rd ACM-SIAM SODA* (1992), 83-87.

[8] T. Feder and D. H. Greene: Optimal Algorithms for Approximate Clustering. *Proc. 20th ACM STOC*, (1988), 434–444.

[9] M. Fischetti, H. W. Hamacher, K. Jørnsten, and F. Maffioli, Weighted K-Cardinality Trees: Complexity and Polyhedral Structure, *Networks* **24** (1994), 11-21.

[10] N. Garg and D.S. Hochbaum, An $O(\log k)$ Approximation Algorithm for the k Minimum Spanning Tree Problem in the Plane *Proc. 26th ACM STOC* (1994), 432-438.

[11] M. Halldorsson, K. Iwano, N. Katoh, and T. Tokuyama, Finding subsets maximizing minimum structures, manuscript, to appear in ACM-SIAM Symp. on Discrete Algorithms.

[12] S. Misono and K. Iwano, Circuit Board Drilling Problem, Technical Report, Infom. Proc. Soc. Japan, 93-AL-33 (1993) 95-102.

[13] S. Misono and K. Iwano, Experiments on TSP Real Instances, *manuscript*, 1993.

[14] S. Peng, A. B. Stephens, and Y. Yesha, Algorithms for a Core and k-Tree Core of a Tree, *J. Algorithms* **15** (1993), 143-159.

[15] R. Ravi, R. Sundaram, M. V. Marathe, D. J. Rosenkrants, and S. S. Ravi: Spanning Tree Short or Small, *Proc. 5th ACM-SIAM SODA* (1994), 546-555.

[16] A. Shioura and T. Uno, A Linear Time Algorithm for the k-Tree Core of a Tree. *Preprint* (1994)