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警備員巡回路問題と動物園巡回路問題に対する近似アルゴリズム

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警備員巡回路問題とは, 与えられた多角形Pに対し, Pの任意の点が巡回路上の少なくとも1点から見えるような最短の巡回路を見つけることである. 最短警備員巡回路を求める $O(n^4)$ 時間のアルゴリズムが既に報告されたが, そのアルゴリズムは複雑で実用に堪えるものではない. 一般的に, $O(n \log n)$ 時間で警備員巡回路問題を多角形P内にある線分の集合を訪れる最短路を求める問題に帰着することができる. 本論文では, その線分の集合を訪れる (警備員) 巡回路を求める線形時間のアルゴリズムを提案する. 求められた巡回路の長さが最短警備員巡回路の長さの $\sqrt{2}$ 倍に越えないことが保証される. 提案されたアルゴリズムは簡単で実現しやすい. 更に, この近似手法は, 警備員巡回路問題の変形版である, 動物園巡回路問題にも適用できる. これまでに, 動物園巡回路問題に対する近似度6のアルゴリズムが提案されていた. 我々の結果は今までの結果を大幅に改善した.

Approximation Algorithms for the Watchman Route and Zookeeper's Problems

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Abstract

We propose a simple approximation scheme for computing shortest watchman routes in simple polygons. Given a simple polygon P with n vertices and a starting point s on its boundary, the *watchman route problem* asks for a shortest route in P through s such that each point in the interior of the polygon can be seen from at least one point along the route. It is well-known that the watchman route problem can be reduced in $O(n \log n)$ time to that of computing the shortest route which visits a set of line segments in polygon P. We present an $O(n)$ time algorithm for computing a route that visits the set of line segments and is of at most $\sqrt{2}$ times the length of the shortest watchman route. The best known algorithm for computing an exact shortest watchman route through s takes $O(n^4)$ time. In addition, it is too complicated to be suitable in practice. Thus, our result gives a significant improvement over the previous result. Besides, our approximation scheme can be applied to the *zookeeper's problem*, which is a variant of the watchman route problem. The previously known factor of approximation for the zookeeper's problem is 6.

1 Introduction

Motivated by the relations to the well-known *Art Gallery* and *Traveling Salesperson* problems, much attention has been devoted to the problem of computing the *shortest watchman route* in a simple polygon such that each interior point of the polygon is visible to at least one point on the route [2, 6, 8, 9, 10]. Recently, an $O(n^4)$ time dynamic programming algorithm was presented to solve the problem in the case where a starting point s on the polygon boundary is given [10], and an $O(n^5)$ time algorithm is developed for the case where no starting point is given [8].

A variant of the watchman problem, called the *zookeeper's* problem, has also been studied [3, 4, 5, 7]. Given a simple polygon (*zoo*) P with a set $\mathcal{P}$ of disjoint convex polygons (*cages*) inside it, each sharing one edge with P, the zookeeper's problem asks for the shortest route that visits (without entering) at least one point of each cage in $\mathcal{P}$. The best algorithms for computing a shortest zookeeper's route with and without a starting point take $O(n \log^2 n)$ time [4] and $O(n^2)$ time [7], respectively. An approximation solution to the zookeeper's problem (with a starting point) guaranteed to be at most 6 times longer than the shortest zookeeper's route is given in [5].

All known exact algorithms for the watchman route and zookeeper's problems make use of the unfolding technique, which unfolds a polygon or the polygon triangulation by reflecting it on some line segments in the polygon $O(n)$ times. These algorithms are not only complicated but also far from practical. If the input coordinates have L bits of precision, then the output has $O(nL)$ bits [4]. Hence, such algorithms do not work well in practice.

In this paper, we propose a simple approximation scheme for computing shortest watchman routes in simple polygons. It is well-known that the watchman route problem can be reduced in $O(n \log n)$ time to that of computing the shortest route which visits a set of line segments in polygon P. We present an $O(n)$ time algorithm for computing a route that visits the set of line segments and is of at most $\sqrt{2}$ times the length of the shortest watchman route. It is not only much faster than the best exact algorithm, but also vastly simpler. Our approximation scheme can also be used to compute a zookeeper's route of at most $\sqrt{2}$ times the length of the shortest zookeeper's route.

2 The reflection principle and its approximation

The previously known watchman route and zookeeper's algorithms primarily make use of the reflection principle. If a and b are two points on the same side of a line L, then the shortest path visiting a, L and b in that order, denoted by $S(a, L, b)$, follows the reflection principle, i.e., the incoming angle of path $S(a, L, b)$ with L is equal to the outgoing angle of $S(a, L, b)$ with L. To actually compute the path $S(a, L, b)$, we first reflect b across L to get its image b', then draw the straight line segment $\overline{ab'}$ from a to b', and finally fold back the portion of $\overline{ab'}$ lying in the other side of L to obtain the desired shortest path $S(a, L, b)$. See Fig. 1a for an example. More generally, to find a shortest path from a to b that visits a line segment l, we first compute the shortest path that passes between the endpoints of segment l and connects a to the reflection b'. The desired shortest path, denoted by $S(a, l, b)$, is then found by folding back the portion of the path lying in the other side of the line passing through l (Fig. 1b).

Let $L(a)$ denote the point of L nearest to a (Fig. 1). Clearly, the path consisting of the line segments $\overline{aL(a)}$ and $\overline{L(a)b}$, denoted by $S'(a, L, b)$, is a good approximation of the path $S(a, L, b)$. Note that three points a, $L(a)$ and b' form an obtuse-angled triangle (Fig. 1). The following lemma tells us that the length of $S'(a, L, b)$ is at most $\sqrt{2}$ times that of $S(a, L, b)$.

Lemma 1 *For an obtuse-angled triangle, the sum of lengthes of two shorter edges is smaller than or equal to $\sqrt{2}$ times the length of the longest edge.*

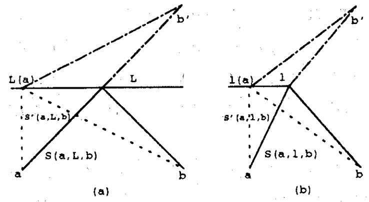

Figure 1: The reflection principle and its approximation.

Proof. For an obtuse-angled triangle, we draw a semicircle using the longest edge as the diameter. Extending either shorter edge to the boundary of the semicircle, we can obtain a right-angled triangle that encloses the obtuse-angled triangle. The lemma is then proved by noticing that the sum of lengths of two shorter edges of a right-angled triangle is the maximum when two acute angles are 45°. □

Analogously, let $l(a)$ denote the point of the line segment l nearest to a (Fig. 1b). Then the path consisting of two segments $\overline{al(a)}$ and $\overline{l(a)b}$, denoted by $S'(a, l, b)$, is also a $\sqrt{2}$-approximation of the path $S(a, l, b)$.

3 Approximating the shortest watchman route

3.1 Basic definitions

We define notation for this section; much of our notation is borrowed from [9, 10]. Let P be a simple polygon with a point s on its boundary. We assume that P is given by the sequence of its vertices in the clockwise order from s. A vertex is *reflex* if its internal angle is greater than π. Polygon P can be partitioned into two pieces by a "cut" that starts at a reflex vertex v and extends either edge incident to v until it first hits the boundary. A cut is said to be a *visibility cut* if it produces a *convex* angle ($< \pi$) at v in the piece of P containing s. For a cut C, the piece of P containing s, denoted by $P(C)$, is called the *essential piece* of C.

We say cut C_j *dominates* cut C_i if $P(C_j)$ contains $P(C_i)$. Clearly, if C_j dominates C_i, any route that visits C_j will automatically visit C_i. A cut is called an *essential cut* if it is not dominated by any other cuts. The watchman route problem is then reduced to that of finding the shortest route visiting all essential cuts. All visibility cuts can be computed in $O(n \log n)$ time using the ray-shooting algorithm, and the essential cuts can then be identified in $O(n)$ time via a clockwise scan. In the rest of this paper, we will consider only essential cuts.

An essential cut may intersect with some others and is thus divided into several segments spanning between consecutive intersection points. We call these segments the *fragments* of a cut. We say fragment f (point p) *dominates* cut C if f (p) is not contained in $P(C)$. We also say fragment f *dominates* fragment g if f dominates the cut to which g belongs.

A set of fragments is called the *watchman fragment set* if the cuts dominated by the fragments give the whole set of essential cuts and no one is dominated by any other fragments. It is then easy to see that any route that visits all fragments of a watchman fragment set is a watchman route. With respect to a watchman fragment set, we distinguish a fragment as an *active* or *inactive* fragment according to whether it belongs to the fragment set or not. A cut is *active* if it contains an active fragment. Otherwise, it is *inactive*.

Given a watchman fragment set, we can compute the corresponding optimal watchman route by repeatedly applying the reflection principle [2]. First, the non-essential pieces of all active essential cuts are removed (since the optimum watchman route never enters them) and the resulting polygon P' is triangulated. The active fragments are then used as mirrors to "unfold" the triangulation of P' in the order they appear in the boundary of P'. The problem is now reduced to that of finding the shortest path from s to its image s' in the unfolded polygon. The optimum watchman route is finally obtained by folding back the shortest path. Note that the watchman route computed by the unfolding method is optimum only with respect to the given fragment set.

Usually, a watchman route R (computed by the unfolding method) makes a *reflection contact* with an active cut C, i.e., R comes into cut C at some point and then reflects on C and goes away from that point [9, 10]. We refer the *incoming (outgoing) angle* of route R with respect to cut C to the angle between C and the segment of R coming into (moving away from) C when one follows R in the clockwise direction. The reflection is *perfect* if the incoming angle of route R with cut C is equal to the outgoing angle. It is also possible that the reflection contact degenerates into a line segment, i.e., the route R shares a segment with the cut C.

A watchman route R is said to be ***adjustable*** on an active cut C if the reflection point of R on C can be moved to get a shorter watchman route. In this case, the incoming angle of R with C is not equal to the outgoing angle, and the adjustable direction of R on C is defined as the one from the larger angle to the smaller angle. If a watchman route is adjustable only on an active cut, we call it a ***one-place-adjustable*** route.

3.2 The approximation algorithm

Let m be the number of essential cuts, and $C_1, C_2, \cdots, C_m$ the sequence of essential cuts indexed in the clockwise order of their left endpoints, starting at s. Also, let $s = s_0 = s_{m+1}$, and let the edge containing s be the cuts C_0 and C_{m+1}, and polygon P the essential pieces $P(C_0)$ and $P(C_{m+1})$. Starting from s_0, we first find the point of C_1 that is closest to s_0 in $P(C_0)$. Let s_1 denote the point found on C_1, which is called as the ***image*** of s_0 on C_1. Similarly, we find the images of s_0 on C_2, C_3 and so on. The computation of images of s_0 is terminated when an image s_{i+1} does not dominate all the cuts $C_1, \ldots, C_i$ before it (Fig. 2). Next, we take s_i as a new starting point. The images of s_i on the cuts after it are then computed within $P(C_i)$. Also, the computation of images of s_i is terminated when an image s_{j+1} does not dominate all the cuts $C_{i+1}, \ldots, C_j$. This procedure is repeatedly performed until the image s_m on C_m is found. In particular, we call the images, which are considered as the starting points, the *critical* images. See Fig. 2 for an example, where images s_0 $(= s_5)$, s_2, s_3 and s_4 are critical. (Since s_{m+1} is identical to s_0, it is omitted in our figures.)

Let R' denote the route which is a concatenation of the shortest paths between every pair of adjacent critical images s_i, s_j $(0 \le i < j \le m+1)$. Clearly, R' is a watchman route, which reflects on any cut on which a critical image is defined. Note that the incoming angle of route R' with the cut on which a critical image is defined is smaller than or equal to $\pi/2$ (Fig. 2). We take rout R' as our approximation solution. Let R denote the shortest watchman route through s. Observe that if route R reflects on C_i and the critical image s_i is defined, then s_i is to the left of the reflection point of R on C_i, as viewed from s_0 (Fig. 2). This observation is the key to analyze the efficiency of our approximation solution.

In the following, we denote by $|xy|$ the length of a line segment $\overline{xy}$, and $|Z|$ the length of a route Z. A route $R_{i,j}$ from a critical image s_i to the other critical image s_j is called a *partial watchman route* if it lies in $P(C_j)$ and visits all the cuts $C_i, C_{i+1}, \ldots, C_j$ if $i < j$, or the cuts $C_i, C_{i-1}, \ldots, C_j$ otherwise. Note that the route $R_{i,j}$ in $P(C_j)$ may differ from the route $R_{j,i}$ in $P(C_i)$.

Lemma 2 *If route R reflects on cut C_i and the critical image s_i is defined, then s_i is to the left of the reflection point of R on C_i (as viewed from s_0).*

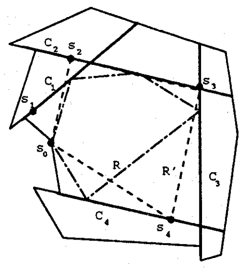

Figure 2: Critical images and routes R', R.

Proof. The proof is by induction on the number k of critical images, excluding s_0 and s_{m+1}. Assume first that $k \geq 2$. (For $k = 1$, the lemma is trivially true as $R = R'$.) To simplify the presentation (i.e., considering the cuts with continuous indices), assume that each cut has a critical image defined on it. Our general method is to find a route that reflects at s_i on C_i and is adjustable only on C_i with the rightward direction.

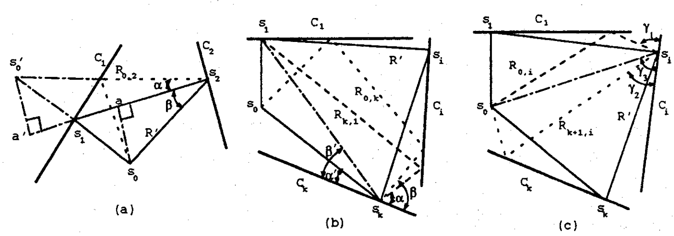

Figure 3: Illustration for the proof of Lemma 2.

For $k = 2$, we first consider a route that concatenates two shortest partial watchman routes $R_{0,1}$ and $R_{3,1}$. Clearly, the concatenated route is adjustable only on C_1. Since either the incoming angle of this route with C_1 is equal to $\pi/2$ or s_1 is the left endpoint of C_1, the adjustable direction on C_1 is rightward.

Next, we show that s_2 is to the left of the reflection point of R on C_2. Without loss of generality, assume that the shortest partial watchman route $R_{i+1,i}$ is just the segment $\overline{s_{i+1}s_i}$ ($i = 0, 1, 2$), and that the segment $\overline{s_0s_1}$ is perpendicular to cut C_1. From the definition of critical images, the shortest partial watchman route $R_{0,2}$ cannot be the shortest path from s_0 to s_2, i.e., it has to reflect on at least one cut. Also, we assume that the route $R_{0,2}$ makes a perfect reflection with cut C_1. In Fig. 3a, the point s'_0 is obtained by reflecting s_0 across C_1, and a (a') is the point on the line through $\overline{s_1s_2}$ such that the angle $\angle s_0as_2$ ($\angle s'_0a's_2$) is $\pi/2$. Let α denote the (acute) angle between $\overline{s_1s_2}$ and the last segment of the route $R_{0,2}$, and β the angle between $\overline{s_1s_2}$ and $\overline{s_0s_2}$. Since triangle $\Delta_{s_0s_1a}$ is congruent with triangle $\Delta_{s'_0s_1a'}$, we have $|s_0a| = |s'_0a'|$. Furthermore, since $\tan(\alpha) = \frac{|s'_0a'|}{|s_2a'|}, \tan(\beta) = \frac{|s_0a|}{|s_2a|}$ and $|s_2a'| > |s_2a|$, we have $\alpha < \beta$. It follows that s_2 is to the left of the reflection point of R on C_2.

For $k \geq 3$, as done above, it can be shown that s_1 is to the left of the reflection point of R on C_1. We now show that s_k is to the left of the reflection point of R on C_k. Let α (resp. β) denote the angle between C_k and the first (resp. last) segment of the shortest partial watchman routes $R_{k,1}$ (resp. $R_{0,k}$), and α' (resp. β') the angle between C_k and the first segment of the shortest path from s_k to s_0 (resp. from s_k to s_1). See Fig. 3b. Since the images are computed in the clockwise order, we have $\beta' \geq \alpha'$. If we consider s_1 as a starting point, ignoring s_0, it then follows from the induction hypothesis that $\alpha \geq \beta'$. Consider further a partial watchman route that concatenates two shortest partial watchman routes $R_{0,1}$ and $R_{k,1}$. Clearly, this route is adjustable only on C_1 at s_1, and the adjustable direction on C_1 is rightward. (Route $R_{0,k}$ can be obtained by performing this adjustment and the following ones. See Fig. 3b). So we have $\beta \geq \alpha$. It follows that $\beta \geq \alpha'$ and thus s_k is to the left of the reflection point of R on C_k.

Finally, we show that s_i $(1 < i < k)$ is to the left of the reflection point of R on C_i. Let γ_1 (resp. γ_2) denote the angle between C_i and the last segment of the shortest partial watchman route $R_{0,i}$ (resp. $R_{k+1,i}$). See Fig. 3c. Consider a route that concatenates $R_{0,i}$ and the shortest path from s_i to s_0. Let γ_3 denote the outgoing angle of this one-place-adjustable route with C_i. Applying the induction hypothesis, we have $\gamma_1 > \gamma_3$. On the other hand, the last segment of $R_{k+1,i}$ has to lie in between C_i and the first segment of the shortest path from s_i to s_0 (Fig. 3c). Hence, $\gamma_3 > \gamma_2$, and thus $\gamma_1 > \gamma_2$. So s_i is to the left of the reflection point of route R on C_i. This completes the proof. □

We are now ready to give the main result of this paper.

Theorem 1 *For any instance of the watchman route problem, $|R'| \leq \sqrt{2}|R|$. Moreover, the watchman route R' can be found in $O(n)$ time, provided that the set of essential cuts is given.*

Proof. It follows from Lemma 2 that routes R and R' have at least one intersection point after a critical image (excluding s_0). Let t_1 and t_2 denote two intersection points of routes R and R' immediately before a critical image s_i $(1 \leq i \leq m)$ and its sucessor (the critical image immediately after s_i), along route R', respectively. See Fig. 4. (Note that t_1 and t_2 may be s_0 and s_{m+1}, respectively.) Let R_{t_1,t_2} and R'_{t_1,t_2} denote the parts of routes R and R' from t_1 to t_2, respectively. Clearly, the first part of the theorem is true, provided that $|R'_{t_1,t_2}| \leq \sqrt{2}|R_{t_1,t_2}|$.

Consider the situation where route R' does not touch the boundary of the polygon P, excluding the endpoints of essential cuts. Assume first that route R_{t_1,t_2} reflects on C_i. Let t'_2 denote the point obtained by reflecting t_2 across C_i. Since the incoming angle of route R' with cut C_i is smaller than or equal to $\pi/2$, the angle $\angle t_1 s_i t'_2$ is larger than or equal to $\pi/2$. It then follows from Lemma 1 that $|t_1 s_i| + |s_i t'_2| \leq \sqrt{2}|t_1 t'_2|$. Since $|R'_{t_1,t_2}| = |t_1 s_i| + |s_i t'_2|$ and $|t_1 t'_2| \leq |R_{t_1,t_2}|$, we have $|R'_{t_1,t_2}| \leq \sqrt{2}|R_{t_1,t_2}|$. See Fig. 4a.

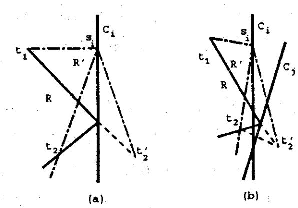

Figure 4: Illustration for the proof of Theorem 1.

If route R_{t_1,t_2} does not reflect on C_i, then it has to reflect on a cut C_j, whose active fragment dominates cut C_i. In this case, we reflect t_2 across C_j to obtain the point t'_2. See Fig. 4b. By noticing the fact that $|s_i t_2| < |s_i t'_2|$, we have $|R'_{t_1,t_2}| \leq \sqrt{2}|R_{t_1,t_2}|$.

Consider now the situation where route R' touches some parts of the polygon boundary. Assume that the shortest path between two adjacent critical images s_h and s_i, with $h < i$, wraps around some reflex vertices of polygon P (i.e., the path makes left turns at these vertices when one follows it in the clockwise direction). See Fig. 5. Let r and r' denote the last and penultimate reflex vertices of P touched by the shortest path between s_h and s_i. (Vertex r' may degenerate into vertex r.) If $t_1 = r$, then as shown above, $|R'_{t_1,t_2}| \leq \sqrt{2}|R_{t_1,t_2}|$. Otherwise, we extend the segment $\overline{rs_i}$ until the foot of a perpendicular to the extension and through t_1 is reached (Fig. 5). (It can always be done because r is a reflex vertex and s_i is invisible from t_1.) Let R''_{t_1,t_2} denote the route which concatenates route R'_{t_1,t_2}, the extended segment along $\overline{rs_i}$

and the segment connecting t_1 with the endpoint of that extension. Since the extended segment along $\overline{rs_i}$ intersects with route R_{t_1,t_2}, we have $|R'_{t_1,t_2}| < |R''_{t_1,t_2}| \leq \sqrt{2}|R_{t_1,t_2}|$. It completes the proof of the first part of the theorem.

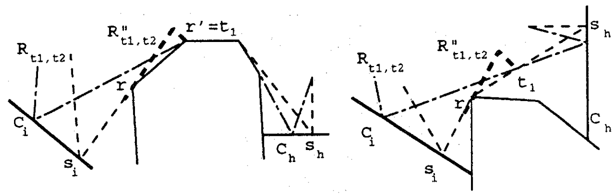

Figure 5: Route R' wraps around the polygon boundary.

Let us consider the time taken to compute the watchman route R'. First, triangulate the polygon P using the linear time algorithm [1]. To efficiently compute the group of images $s_1, \ldots, s_i$, where s_i is the maximum image dominating all the cuts $C_1, \ldots, C_{i-1}$ before it, we figure out a polygon P_i $(1 \leq i \leq m)$. Let LS_i and RS_i denote the left and right *sleeve* from s_0 to the left and right endpoints of C_1, respectively. Sleeve LS_i (RS_i) is the union of the triangles, which contains the shortest path from s_0 to the left (right) endpoint of C_1. We define polygon P_i as the region enclosed by LS_i and RS_i. (Usually, it contains the region $LS_i \cup RS_i$.) Clearly, LS_i, RS_i and P_i can be found in the time linear to their sizes.

The images s_1, ..., s_i can be computed as follows. We first find the shortest paths from s_0 to two endpoints of C_1. These two paths share parts of their length; at some vertex v they part and proceed separately to their destinations. The region bounded by C_1 and the shortest paths from v to two endpoints of C_1 is usually called a *funnel*. Each of the funnel paths is *inward convex*: it bulges in toward the funnel region. Next, we compute the shortest paths from s_0 to the intersection points of C_1 with other cuts in the increasing order of indices. By maintaining the part of the current cut C_k which dominates all previous cuts, we can compute the image of s_0 on C_k and report whether or not all previous cuts are dominated by s_k in $O(1)$ time. Hence, the images $s_1, \ldots, s_i$ can be found in the time linear to the size of P_i.

The next group of images can be found by taking s_i as s_0 and the portion of C_{i+1} contained in $P(C_i)$ as C_1. In this way, all images can be computed. Observe that the sum of sizes of the polygons P_j used to compute all images is linear to n. This is because a triangle appears at most six times in these polygons P_j. (Imagine that one walks in the first polygon P_i from s_0 to some point of C_1. Consider a pair of edges of a triangle T encountered by the walk. This pair of edges of T can appear at most twice in all polygons P_j.) Hence, the time complexity of our approximate algorithm is $O(n)$. □

4 Approximating the shortest zookeeper's route

We extend our result obtained in the previous section to the zookeeper's problem. Let $P_1, \ldots, P_m$ denote the cages indexed in a clockwise scan of the boundary of zoo P, starting at s. Also, let $s = s_0 = s_{m+1}$. Differing from the essential cuts defined in the previous section, all cages in $\mathcal{P}$ are disjoint. So we find a point s_1 on the boundary of P_1 that is closest to s_0 in the interior of zoo P (i.e. $P - \mathcal{P}$), a point s_2 on the boundary of P_2 that is closest to s_1, and so on. The point s_{i+1} is also called as the *image* of s_i on the boundary of P_{i+1}. Putting the shortest paths between s_i and s_{i+1} for all i, $0 \leq i \leq m$, gives a zookeeper's route R'.

Theorem 2 *For any instance of the zookeeper's problem, $|R'| \leq \sqrt{2}|R|$, where R denotes the shortest zookeeper's route through s. Moreover, the zookeeper's route R' can be found in $O(n)$ time.*

Proof. Let us consider the edges where images are defined as pseudo-cuts and the whole interior of zoo P as the essential piece of a pseudo-cut. Similar to the proof of Lemma 2, it can be shown that image s_i is to the left of the reflection point of route R on the boundary of P_i.

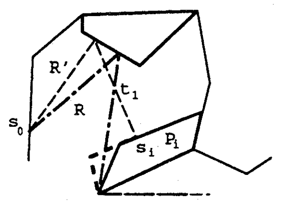

Figure 6: Illustration for the proof of Theorem 2.

Let t_1 and t_2 denote the intersection points of R and R' immediately before image s_i and its successor, along R', respectively. If route R'_{t_1,t_2} reflects at a point on the boundary of cage P_i, then similar to the proof of Theorem 1, we have $|R'_{t_1,t_2}| \leq \sqrt{2}|R_{t_1,t_2}|$. If route R' wraps around the boundary of cage P_i, we can similarly enlarge the route R'_{t_1,t_2} so that the length of the enlarged route is at most $\sqrt{2}|R_{t_1,t_2}|$ (Fig. 6). Therefore, we obtain that $|R'| \leq \sqrt{2}|R|$.

Finally, the image s_i can also be computed by considering the shortest paths from the previous image s_{i-1} to two extreme endpoints of cage P_i. Clearly, the time taken to compute all images as well as the route R' is $O(n)$. □

References

[1] B. Chazelle, Triangulating a simple polygon in linear time, *Proceedings, 31th Annu. IEEE Symp. Found. of Comput. Sci.*, 220-229, 1990.

[2] W.P.Chin and S.Ntafos, Optimum watchman routes, *Inform. Process. Lett.* **28** (1988) 39-44.

[3] W.P.Chin and S.Ntafos, The zookeeper route problem, *Inform. Sci.* **63** (1992) 245-259.

[4] J.Hershberger and J.Snoeyink, An efficient solution to the zookeeper's problem, *Proc. of the 6th Canadian Conf. on Comput. Geom.*, 104-109, 1994.

[5] H.Jonsson, On the zookeeper's problem. In *Proc. 15th Europ. Workshop on Comput. Geom.* (1999) 141-144.

[6] B. Nilsson, Guarding art galleries; Methods for mobile guards, Ph. D. thesis, Lund University, Sweden, 1995.

[7] X.Tan, Shortest zookeeper's routes in simple polygons, *Inform. Process. Lett.* **77** (2001) 23-26.

[8] X.Tan, Fast computation of shortest watchman routes in simple polygons, *Inform. Process. Lett.* **77** (2001) 27-33.

[9] X.Tan, T.Hirata and Y.Inagaki, An incremental algorithm for constructing shortest watchman routes, *Inter. J. Comput. Geom. Appl.* **3** (1993) 351-365.

[10] X.Tan, T.Hirata and Y.Inagaki, Corrigendum to an incremental algorithm for constructing shortest watchman routes, *Int. J. Comput. Geom. Appl.* **9** (1999) 319-323.