# A Computational Geometry Method for Detecting an Invisible Object in Precision Manufacturing Process

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This paper addresses a method for detecting an invisible object efficiently from a point of view of computational geometry. We discuss the problem by taking the light path alignment process as example, which is considered critical in opto-electronics industry. The feature of our research is found in the arrangement scheme of measuring probes to reduce total number of the probes. Our scheme is expected to shorten the time of detection compared with conventional scheme.

# 精密製造工程における不可視物体の計算幾何学的検知手法

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光通信用デバイスの製造工程の中で重視されている光軸調芯工程を例に、不可視物体を計算幾何学的視点から効率良く検知する手法を紹介する。本研究の特徴は、センサの計測点を物体の形状を考慮し配置することにより計測点の総数を減らすことにある。本手法の適用により、従来法に比較して検知時間の短縮が期待できる。

## 1 Introduction

Today's optical network comprises many kinds of devices necessary to generate, modulate, guide, amplify, switch and detect light signal. The functions of those devices are analogous to the ones used in the conventional copper wire network. In contrast with the electrical devices, the opto-electrical devices requires the high accuracy of light path alignment to reduce power loss. Because light path alignment with sub-micron accuracy is considered a norm at each interface on different devices, the process has become a bottle-neck, and the improvement of efficiency the process greatly influences total production time.

The light path alignment process comprises two subsequent processes. Firstly a process called "blind search" is used to detect the light and secondly another process called "fine search", to find the peak power position. Figure 1 illustrates the blind-search process which uses an optical fiber as a sensing probe. The intensity of the laser light has a Gaussian-like distribution and can be roughly estimated using analytical methods [5]. When the lens converges the light, the energy distribution takes a conic shape like Fig. 2. In the blind-search process, typical assembly systems use *single-plane scheme* that defines a search plane perpendicular to the axis of the light and then shifts the sensing probe by even pitches to measure the power. In this paper, we are going to introduce a new blind-search method called *dual-plane scheme* that exactly reduces the number of probes and thus, the search time is reduced.

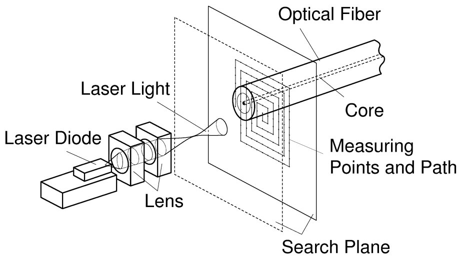


Figure 1: Illustration of the blind-search process in the light path alignment application.

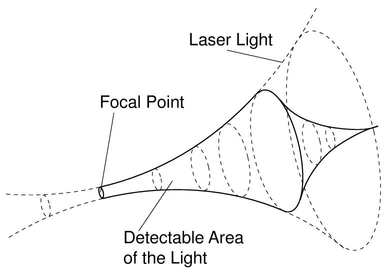


Figure 2: The object to be detected.

One of the earliest topics in computational geometry is the Art Gallery problem that requires the minimum number of guards to watch an art gallery of a polygonal shape[3, 4]. It has been generalized to a watchman route problem [1] for finding a shortest possible route to find an intruder hidden in an art gallery. The problem has also been extended to the problem of detecting a mobile intruder hidden in an art gallery [6]. The problem to be discussed in this paper is also an extension of the art gallery problem in yet another direction. We want to find or detect a rigid object hidden somewhere in an art gallery by arranging probes appropriately over the gallery. Our problem is closely related to the problem of covering a region by some simple geometric objects (see e.g., [2]).

# 2 Problem Description

A **rigid object** $\mathcal{B}$ is hidden somewhere in a region. Implicitly we assume one point $o$ in its interior as a **reference point** (the origin). We call a region in which $o$ can lie **reference point region** $\mathcal{R}$. We want to arrange fewest possible probes in a **probe region** $\mathcal{P}$, in which the probes can be placed, so that we can detect $\mathcal{B}$ wherever it is hidden. We assume that we can determine whether a point $q$ lies in the interior of $\mathcal{B}$ by a **predicate** $F(q)$ that can be computed in linear time in the length of the predicate. We also assume implicitly that a rigid object $\mathcal{B}$ has a simple shape and thus the inclusion test can be done in constant time. If the object $\mathcal{B}$ is a triangle $(p_1, p_2, p_3)$ in the plane, then the predicate is

$F(q)$: $\triangle(p_1, p_2, q) \geq 0$, and $\triangle(p_2, p_3, q) \geq 0$, and $\triangle(p_3, p_1, q) \geq 0$,

where $\triangle(p, q, r)$ is positive if the three points are arranged in a counter-clockwise order, 0 if they lie on a line, and negative if they are in a clockwise order. We also assume that the three points $p_1, p_2$ and $p_3$ are arranged in a counter-clockwise order.

Thus, a rigid object $\mathcal{B}$ is specified as

$$\mathcal{B} = \{q | F(q)\}. \tag{1}$$

By $\mathcal{B}(p, \theta)$ we denote the object $\mathcal{B}$ translated to the point $p$ (so the reference point is located at $p$) and then rotated by an angle $\theta$ in a counterclockwise direction around the reference point. Then, the corresponding predicate becomes

$$q \in \mathcal{B}(p, \theta) \Leftrightarrow F(T_{-\theta}(q - p)), \tag{2}$$

where $T_\theta(q)$ is the point determined by counterclockwise rotation of the point $q$ around the

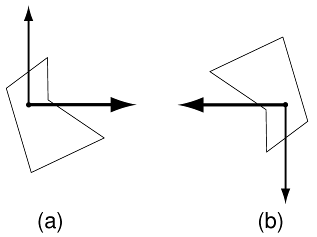


Figure 3: The image of an object: (a) given rigid object ($\mathcal{B}$) and (b) its image ($\mathcal{B}^{-1}$).

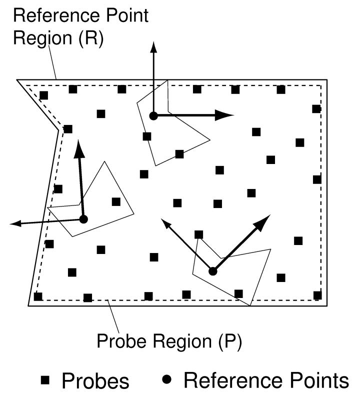


Figure 4: Hidden objects in a reference-point region $\mathcal{R}$ and a set of probes in a probe region $\mathcal{P}$.

origin by the angle $\theta$. We assume that point inclusion is also tested in time proportional to the complexity of the object.

An object $\mathcal{B}(p, \theta)$ can be detected if at least one probe is contained in the interior of the object, that is,

$$p_i \in \mathcal{B}(p, \theta) \Leftrightarrow F(T_{-\theta}(p_i - p)) \tag{3}$$

for some probe $p_i$.

We say that a set of probes is feasible if they detect an object wherever it is located. We want to find a minimum feasible set of probes. A key idea behind our scheme presented in this paper is to define the **image** $\mathcal{B}^{-1}$ of a rigid object $\mathcal{B}$. If no rotation is allowed, then it is defined by

$$\mathcal{B}^{-1} = \{p | F(-p)\}. \tag{4}$$

Figure 3 shows the image of a polygonal object $\mathcal{B}$, that is point-symmetric to the original shape. For the time being, we do not allow rotation.

Now, our problems are described as follows:

**Problem 1: Feasibility Test**
**INSTANCE:** A rigid body $\mathcal{B}$ characterized by a predicate defined by polynomial inequalities with respect to a reference point $o$, a reference point region $\mathcal{R}$, and a probe region $\mathcal{P}$.
**QUESTION:** Is there a feasible set of probes? Or, in other words, is it possible to arrange the probes so that they can detect a hidden object wherever it is located?

**Problem 2: Optimal Feasible Set**
**INSTANCE:** A rigid body $\mathcal{B}$, a reference point region $\mathcal{R}$, and a probe region $\mathcal{P}$.
**QUESTION:** Find a minimum feasible set of probes if there exists one.

Figure 4 illustrates a set of probes arranged in the probe region $\mathcal{P}$ which is usually contained in the reference point region $\mathcal{R}$. The figure includes three objects with different angles.

From the above definitions, we have the following basic observations.

**Lemma 2.1** *For an arbitrary rigid object $\mathcal{B}$ the following always holds*

$$p \in \mathcal{B}(q) \Leftrightarrow q \in \mathcal{B}^{-1}(p). \tag{5}$$

**Proof** $p \in \mathcal{B}(q) \Leftrightarrow F(p - q) \Leftrightarrow q \in \mathcal{B}^{-1}(p)$. □

Once we have the above lemma, the following two lemmas are almost obvious.

**Lemma 2.2** *Given $\mathcal{B}$, $\mathcal{R}$ and $\mathcal{P}$, there is a feasible set of probes if and only if the Minkowski sum $\mathcal{P} \oplus \mathcal{B}^{-1}$ contains $\mathcal{R}$.*

**Proof** The proof immediately follows from the definition of the Minkowski sum:

$P \oplus Q = \{p + q | p \in P \text{ and } q \in Q\}$. □

**Lemma 2.3** *A set $S = \{p_1, p_2, \ldots, p_n\}$ of probes in $\mathcal{P}$ covers a region $\mathcal{R}$ if and only if the union of their images covers $\mathcal{R}$, that is,*

$$\mathcal{R} \subseteq \cup_{p_i \in S} \mathcal{B}^{-1}(p_i). \quad (6)$$

**Proof** "if" part: Equation 6 implies that for any point $p \in \mathcal{R}$ there exists some points $p_i$ such that $p \in \mathcal{B}^{-1}(p_i)$. By the definition, $p \in \mathcal{B}^{-1}(p_i) \Leftrightarrow p_i \in \mathcal{B}(p)$, which implies that the point $p_i$ detects the object $\mathcal{B}(p)$ at $p$.
"Only if" part: If the union of the images does not cover $\mathcal{R}$, there must be a point $p \in \mathcal{R}$ which is not contained in $\mathcal{B}^{-1}(p_i)$ for any point $p_i \in S$. This means that $p_i \notin \mathcal{B}(p)$ for any $p_i \in S$ and thus the set $S$ does not cover $\mathcal{R}$. □

The lemma 2.3 implies that our problem reduces to that of finding the minimum number of images $\mathcal{B}^{-1}$ to cover the entire reference point region $\mathcal{R}$.

# 3 Application to the Light Path Alignment Process

In this paper we consider a rather special case in which a rigid object is a double-sided conic cylinder and the reference region (which is equal to the probe region) is a cuboid of $L \times L$ wide and of unit height. We assume that the height of the cylinder in one side is exactly 1 so that each half of the cylinder is tall enough to cover the height of the cuboid,and the largest and smallest radii of the circles given as cross sections are $R$ and $r$, respectively. The conic cylinder shown in Fig. 5 is a simplified model of the light energy distribution around the focal point of the laser beam.

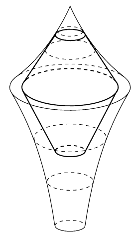

Figure 5: Actual light energy distribution (outside) and conic cylinder (inside).

Our objective is to arrange the smallest number of probes in the probe region (it is equal to the reference region which is a cuboid in this case) so that any hidden object (a focal point of a laser beam in this case) can be detected wherever it is located in the reference region. The traditional heuristic method is characterized as a *single-plane scheme* in which all the probes must be located on a single plane which is parallel to the base face (rectangle in this case) of the reference region. With the scheme we have to cover the rectangle by circles of the smallest radius $r$. Obviously it is not advantageous. We call our new scheme *dual-plane scheme* in which probes are located in two different planes. We will prove an advantage of the dual-plane scheme over the single-plane one by showing that the total number of the probes is considerably reduced. Figure 6 illustrates the concept of our scheme.

The reference point region $\mathcal{R}$ is an axis-parallel cuboid of height 1. Formally, an object

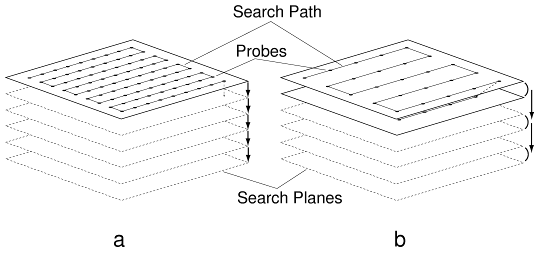


Figure 6: (a) Conventional single-plane scheme and (b) our dual-plane scheme (b).

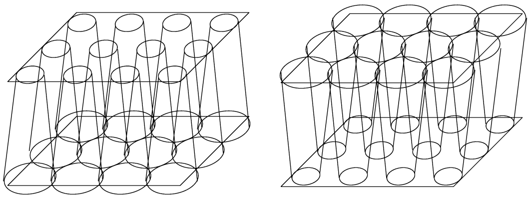

Figure 7: Covering the cuboid region $\mathcal{R}$ by two sets of conic cylinders.

$\mathcal{B}$ is defined by

$$\begin{aligned}\mathcal{B} = \{(x, y, z) | \sqrt{x^2 + y^2} \\ \leq \ (r - R)|z| + R, -1 \leq z \leq 1\},\end{aligned} \quad (7)$$

where $r$ and $R$ are the radii of the top and bottom circles of the object. Since $\mathcal{B}$ is symmetric, $\mathcal{B}^{-1} = \mathcal{B}$.

It has an axis that is parallel to the $z$-axis and it is bounded by a conic surface. Note that, the cross section of $\mathcal{B}$ at any plane perpendicular to the $z$-axis is a disk. A radius of such a disk is largest at $z = 0$ and smallest at $z = 1$ and $z = -1$. The largest and smallest radii are denoted by $R$ and $r$ respectively.

Consider the following arrangement of conic cylinders. An idea here is to place those cylinders on the two planes $z = 0$ and $z = 1$. Precisely, two sets of probe locations are determined as follows.

$$\begin{aligned} S_0(k) &= \{(0, 0, 0), (0, 2d, 0), (0, 4d, 0), \ldots, \\ &\quad (2d, 0, 0), (2d, 2d, 0) \ldots, \\ &\quad (2kd, 2kd, 0)\}, \\ S_1(k) &= \{(d, d, 1), (d, 3d, 1), (d, 5d, 1), \ldots, \\ &\quad (3d, d, 1), (3d, 3d, 1), \ldots, \\ &\quad ((2k+1)d, (2k+1)d, 1)\}. \end{aligned}$$

The set of conic cylinders whose center points (reference points) are located on the plane $z = 0$ are called 0-cylinders and those on the plane $z = 1$ 1-cylinders. 0-cylinders are located on a regular grid of space $2d$. 1-cylinders are also located in a similar manner, but their centers are characterized by odd integers times $d$. The space parameter $d$ is determined by

$$d = \frac{R + r}{2}. \quad (8)$$

The other parameter $k$ is determined to be a smallest integer such that the corresponding set of cylinders cover the entire cuboid. To find such an smallest integer $k$, we have to consider different cases (See Fig. 8). For simplicity, we assume that the base area is a square of side length $L$. In case $kd < L \leq (k + 1)d$, the cuboid is covered with cylinders corresponding to $S_0(k) \cup S_1(k)$ so that total number of cylinders needed is $|S_0(k)|^2 + |S_1(k)|^2 = 2k^2 = 2(\lfloor L/(2d) \rfloor)^2$. In other case $(k+1)d < L \leq (k+2)d$, the cuboid is covered with the ones corresponding to $S_0(k+1) \cup S_1(k)$ so that the total number of disks for covering becomes $|S_0(k+1)|^2 + |S_1(k)|^2 = (k+1)^2 + k^2 = (\lfloor L/(2d) \rfloor + 1)^2 + (\lfloor L/(2d) \rfloor)^2$.

By $C^0_{i,j}$ and $C^1_{i,j}$ we denote the intersections of the cuboid with the 0-cylinder and 1-cylinder, respectively, whose reference points

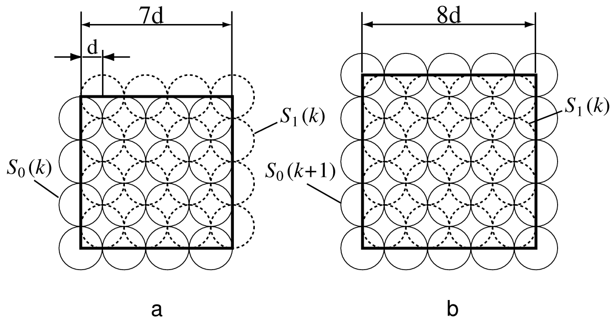


Figure 8: Change in parameter $k$ with different size of base area; (a) upper right corner of cuboid lies on the center of $S_1(k)$ (b) the same corner lies on the center of $S_0(k)$.

are located at the points $(id, jd, z)$, that is,

$$
\begin{aligned}
C^0_{i,j} \;&=\; \{(x,y,z)|0 \le z \le 1,\\
&\quad (x-id)^2+(y-jd)^2\\
&\quad \le (R-(R-r)z)^2\},\\
&\quad (i,j)=(0,0),(0,2d),(0,4d),\ldots,\\
&\quad (2d,0),(2d,2d),\ldots,(2kd,2kd) \text{ and}\\
C^1_{i,j} \;&=\; \{(x,y,z)|0 \le z \le 1,\\
&\quad (x-id)^2+(y-jd)^2\\
&\quad \le (r+(R-r)z)^2\},\\
&\quad (i,j)=(d,d),(d,3d),(d,5d),\ldots,\\
&\quad (3d,d),(3d,3d),\ldots,\\
&\quad ((2k+1)d,(2k+1)d).
\end{aligned}
$$

**Lemma 3.1** *A cuboid of a square base face and of height* 1 *can be covered by* 0*-cylinders and* 1*-cylinders placed at the locations specified by* $S_0(k)$ *and* $S_1(k)$*, respectively, on the planes* $z = 0$ *and* $z = 1$*, respectively.*

**Proof** We shall show how the cross section of the cuboid at $z = z_0$, $0 \le z_0 \le 1$ is covered by those cylinders. When $z_0 > 1/2$, the 1-cylinders cover more space than 0-cylinders. The radius $r_1(z_0)$ of a 1-cylinder at $z = z_0$ is given by

$$r_1(z_0) = r + (R-r)z_0. \tag{9}$$

Similarly, the radius $r_0(z_0)$ of the circle of a 0-cylinder at $z = z_0$ is given by

$$r_0(z_0) = R - (R-r)z_0. \tag{10}$$

If $r_1(z_0) > \sqrt{2}d$, that is, $z_0 > (\sqrt{2}d - r)/(R-r)$, then the cross section of the cuboid at $z = z_0$ is covered by 1-cylinders. For $d < r_1(z_0) \le \sqrt{2}d$, that is, $1/2 < z_0 \le (\sqrt{2}d - r)/(R-r)$, the farthest points from a center $(id, jd)$ of a 1-cylinder $C^1_{i,j}$ are $\{((i \pm 1)d, (j \pm 1)d)\}$.

Consider the intersection at $z = z_0$ between two cylinders $C^1_{i,j}$ and $C^1_{i,j+2}$, which is given by $(id + \sqrt{r_1(z_0)^2 - d^2}, (j+1)d)$. This point is covered by the cross section of the cylinder (or exactly disk) $C^0_{i+1,j+1}$ because

$$
\begin{aligned}
& \{(i+1)d - id - \sqrt{r_1(z_0)^2 - d^2}\}^2\\
+\;& \{(j+1)d - (j+1)d\}^2 - r_0(z_0)^2\\
=\;& \{d - \sqrt{r_1(z_0)^2 - d^2}\}^2 - (2d - r_1(z_0))^2\\
=\;& -2d\{\sqrt{r_1(z_0)^2 - d^2} + 2(d - r_1(z_0))\}\\
=\;& -2d(2d - 2r_1(z_0) + \sqrt{r_1(z_0) - d^2}) < 0.
\end{aligned}
$$

The last inequality is verified as follows. Let $r_1 = r_1(z_0)$ and $f(r_1) = 2d - 2r_1 + \sqrt{r_1 - d^2}$. Differentiating the function $f$ w.r.t. $r_1$ we have $f'(r_1) = -2 + \frac{r_1}{\sqrt{r_1^2 - d^2}}$.
So, the function $f(r_1)$ takes an extreme value when $r_1 = \frac{2\sqrt{3}}{3}d$. The extreme value is positive since $f(\frac{2\sqrt{3}}{3}d) = (2-\sqrt{3})d > 0$

We also see that $f(d) = 0$ and $f(\sqrt{2}d) = (3 - 2\sqrt{2})d > 0$. All these observations suggest $f(r_1) > 0$. If $d < r_1(z_0) \le \sqrt{2}d$, then $-2d(2d - 2r_1(z_0) + \sqrt{r_1(z_0) - d^2}) < 0$. □

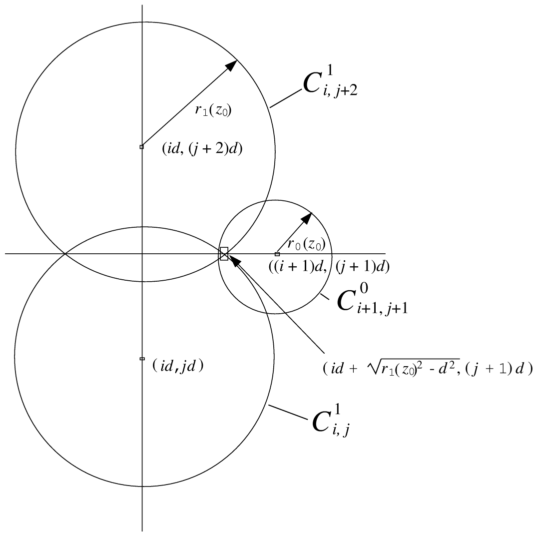


Figure 9: Cross sections of 0-cylinders and 1-cylinders when $1/2 < z_0 \leq (\sqrt{2}d - r)/(R - r)$.

Our discussion is also applicable to the case of the arrangement in discrete space. It is worth mentioning this because the light path alignment process is driven by pulse motors and the probe positions are set by number of pulses. In this case, we take $\lfloor d \rfloor$ instead for the pitch of conic cylinders and rearrange them (See Fig. 10). The proof of covering can be made by substituting $\lfloor d \rfloor$ to $d$ in Lemma 3.1. The rearrangement causes wider overlap between conic cylinders and thus results in the increase of probe points.

Now, we can compare the performance of our dual-plane scheme with that of the single-plane scheme in which probes are arranged so that the smallest circles cover the rectangular reference point region.

$$(k+1)^2 + k^2 = (\lfloor L/(2d) \rfloor + 1)^2 + \lfloor L/(2d) \rfloor^2. \quad (11)$$

On the other hand, the number of circles re-

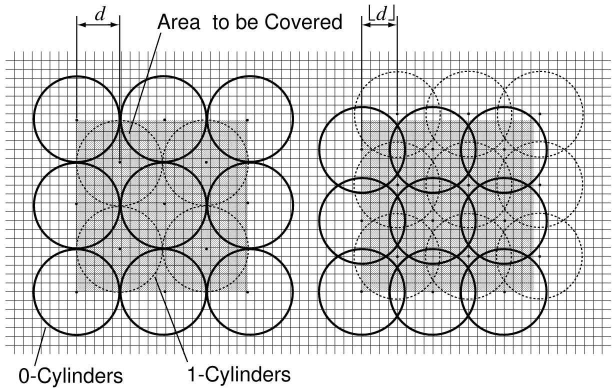


Figure 10: Rearrangement of Conic Cylinders from Continuous (left) to Discrete (right) Space.

quired by the single-plane scheme is

$$(k'+1)^2 + k'^2 = (\lfloor L/(2r) \rfloor + 1)^2 + \lfloor L/(2r) \rfloor^2. \quad (12)$$

Thus, the ratio is given by

$$\begin{aligned} & (k'^2 + (k'+1)^2)/(k^2 + (k+1)^2) \\ \simeq \; & \lfloor L/(2r) \rfloor^2 / \lfloor L/(2d) \rfloor^2 \simeq (d/r)^2 \\ = \; & ((R+r)/(2r))^2 \\ = \; & (1 + R/r)^2/4. \end{aligned} \quad (13)$$

So, if $R = 3r$, then the ratio is $(1+3)^2/4 = 4$.

# 4 Conclusions

In this paper, we have considered the problem of arranging fewest possible probes to find a hidden geometric object in a given region. This problem is closely related to an industrial application of the light path alignment problem. To reduce the time of the blind-search process in the application, we introduced the *dual-plane scheme* using morphic characteristic of the object. Our scheme succeeded in improving the performance of the search process.

We are going to consider next the sequence of measurement to shorten average detection time. Also, in practical applications we have to deal with small rotation of objects with additional freedom of two rotations, which has been left as another open problem.

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