社団法人　情報処理学会　研究報告
IPSJ SIG Technical Report

2005－AL－101（2）
2005／5／19

ワイヤレスネットワークにおける エネルギー効率の良いオンラインブロードキャスト

Shay Kutten*　小野廣隆†　David Peleg‡　定兼邦彦　山下雅史

概 要

本報告では，アドホック・ワイヤレスネットワークにおいて未知の位置に配置されたホストにメッセージを送信する，エネルギー効率の良いオンラインプロトコルの設計について考える．まずこの問題を幾つかの形に定式化したのち，それぞれについて競合比が最適となるオンラインプロトコルを提案する．

Energy Efficient Online Algorithms for Broadcasting in Wireless Networks

Shay Kutten*, Hirotaka Ono†, David Peleg‡, Kunihiko Sadakane†, and Masafumi Yamashita†

Abstract

The paper considers the design of energy-efficient online protocols for the basic problem of message transmission to hosts positioned at unknown distances in ad-hoc wireless networks. The paper formulates a number of variants of this problem and presents optimally competitive algorithms for those variants.

1 Introduction

1.1 Background

We consider problems related to the design of energy-efficient online message broadcasting protocols in ad-hoc wireless networks. Recent developments in portable wireless devices with limited power resources have led to considerable interest in problems involving the construction of energy-efficient multicast trees in the network. Wireless devices can control their transmission power in order to save power consumption whenever the distance to the intended destination of the transmission is known. The attenuation of a signal with power P_s is $P_r = \frac{P_s}{d(s,t)^\delta}$, where $d(s,t)$ is the distance between hosts s and t, and $\delta \geq 1$ is the *distance-power gradient* [3]. A message can be successfully decoded if P_r is no less than a constant γ. Therefore the *transmission range* of a host s, namely, the maximum distance to which a message can be successfully delivered from s, is $(P_s/\gamma)^{1/\delta}$. Power control also has a positive effect on reducing the number of transmission collisions between nearby senders.

The problem studied here concerns a single sender which has to transmit a message to a given collection of receivers in an online setting, namely, when the hosts do not know each other's locations. The goal is to specify a protocol for the sender allowing it to directly broadcast the message to the recipients and receive acknowledgements, while minimizing the total transmission costs. By *direct broadcast* we mean that the sender is required

*Information Systems Group, Faculty of Industrial Engineering and Management, Technion, Haifa, Israel. kutten@ie.technion.ac.il

†Department of Computer Science and Communication Engineering, Kyushu University, Fukuoka, Japan. {ono,sada,mak}@csce.kyushu-u.ac.jp

‡Department of Computer Science and Applied Mathematics, The Weizmann Institute of Science, Rehovot, Israel. david.peleg@weizmann.ac.il

to transmit the message itself to every recipient, namely, multi-hop delivery is not allowed. This restriction may be relevant in situations when the battery resources of the receivers is severely limited and it is desired to minimize their transmissions, or when when the reliability of the hosts is uncertain and only direct messages from the source can be trusted.

1.2 Contributions

Using varying levels of transmission power is important for energy-efficient communication. As far as the authors are aware, there has been no online algorithms with provable worst-case guarantees for energy-efficient broadcasting in ad-hoc wireless networks.

The protocols proposed in this paper are based on computing or estimating the distances from the sender host to the receiver hosts in an energy-efficient way. The most basic case is that of a single sender and a single receiver. The generic doubling protocol employed by the sender is based on repeatedly transmitting the messages to increasingly larger distances, until reaching the receiver. The behavior of this protocol depends on the choice of the sequence of distances, and the problem is to determine them so as to minimize the overall power consumption. If a specific probability distribution may be assumed on the hosts, the algorithm can be optimized [4]. This paper, however, assumes an online setting in which no a priori information is given about the distance from the sender to the receiver. Therefore the worst-case scenario should be considered. This motivation leads us to apply a competitive analysis to the algorithm (cf. [2]). We compare the power consumption of an algorithm with that of the optimal (infeasible) offline algorithm that knows the distance d. We show that the optimal competitive ratio for this problem is $3/2+\sqrt{2}$, i.e., there exists an online algorithm for the problem with this competitive ratio, and no online algorithm has smaller competitive ratio. The problem is somewhat similar to the famous *cow path* online problem [1], but setting the parameter of the algorithm is not obvious.

Furthermore, we study the generalization of this problem where there is more than one receiver. This is a propoer extension of the cow path problem. For this problem we also propose a competitive online algorithm and prove its optimality. Interestingly, the competitive ratio of the generalized problem is the same, namely, $3/2+\sqrt{2}$. The algorithm and its analysis appear in Section 4.

The rest of the paper is organized as follows. In Section 1.3 we discuss the model employed in the paper. Section 2 formally presents our problems and algorithms. Section 3 establishes the competitive ratio for the single receiver case. In Section 4 we propose algorithms for multiple receiver case.

1.3 Model

The model considered in this paper is the following. Each host has a unique id and a global clock, and they can schedule their transmission time in a collision-free manner. Messages are always delivered correctly to the destination. Below we justify our rational for using this model, and particularly the assumptions of synchronous communication and failure-freedom.

Let us first discuss our assumption of a synchronous communication model. Note that this model is a reasonable approximation under some natural timing assumptions and assuming the availability of time-out mechanisms. The problem can also be considered in the alternative asynchronous communication model, in which no assumptions are made concerning the timing and operation rates of the participants. However, in this model there is no way to limit the number of times that collisions occur between messages sent by receivers. As a result, no online algorithm can achieve a constant competitive ratio. In contrast, in a synchronous model it is possible to use the global clock in order to schedule the hosts in a collision-free manner as follows. Assume that each host has a unique id. Without loss of generality assume also that the sender has id 1 and the receivers have id's 2 to n. Suppose that the sender broadcasts a message at time $t+1$. Then each receiver with id i sends an acknowledgment at time $t+i$ if it receives the message. Because the sender knows which receivers have sent acknowledgements, in the next round the sender can send the list of id's of receivers that have received the message in the last round. Each receiver can now determine the time to send an acknowledgment to advance the completion time by using only that information. As our criterion is to minimize the total power consumption, and time efficiency is ignored, this scheduling is sufficient. Clearly, if the problem requires optimizing both the power consumption and the delivery (completion) time, then other schedulings should be considered.

Similarly, our focus on the failure-free model (where messages are always delivered correctly to the destinations) stems from the fact that in a model allowing arbitrary message loss, the worst case competitive ratio cannot be bounded. The development of suitable models for studying fault-tolerant variants of the problem is left for future research.

2 Problems and algorithms

The generic protocol employed by the sender s in the case of a single receiver t is given in the following procedure.

Procedure *SendMessage*(t,*msg*)

```
i ← 1; f ← true
while f do
    Transmit ⟨msg, p_i⟩ with power p_i.
    Wait.
    if received acknowledgment from t then
        f ← false;
    end if
    i ← i + 1;
end while
```

The algorithm is illustrated in Figure 1.

Clearly, the behavior of any algorithm for the sender can be specified as a sequence of increasing power costs $\{p_i\}$, for $i = 1, 2, \ldots$, and hence its performance depends on the choice of this sequence. An algorithm A succeeds on the first step J such that $(p_J/\gamma)^{1/\delta} \geq d$. For lack of distance information, the receiver will rely on the information contained in the received message and transmit its acknowledgement with the same power. The cost of algorithm A is thus $cost(A) = \sum_{i=1}^{J} p_i + p_J$ where the second term is for the acknowledgment. Therefore the problem is to determine this sequence so that the overall power consumption is minimized.

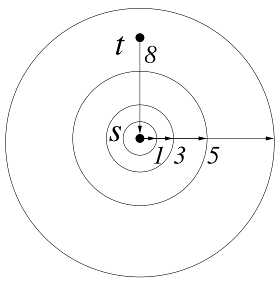

図 1: Illustration of the doubling algorithm. Numbers represent transmission times, with the transmission range doubled by the sender s at each round until reaching the receiver t.

We assume an online setting in which no a priori information is given about the distance from s to t, and a worst-case scenario should be considered. We compare the power consumption of an algorithm with that of the optimal offline algorithm that knows the distance d. This offline algorithm sets $p_i = \gamma d^\delta$; then it terminates at the first step and its energy consumption is minimized, hence its cost is $cost^* = 2\gamma d^\delta$. Of course this is impossible to achieve online in the absence of knowledge on the distances, and our aim is to reduce the cost of our algorithm as much as possible. The problem is formalized as follows.

Problem Broadcast+Ack-2 ($BA2$): There is a sender s and a receiver t at distance $d \geq 1$, which is unknown. The sender runs an algorithm A to send the message. Once it is delivered, the receiver should return an acknowledgment. The goal is to minimize the total power consumption for the sender and the receiver.

Competitiveness: The *competitive ratio* of algorithm A is $\rho(A) = \sup\{cost(A)/cost^*\}$ where the supremum is taken over all inputs.

The problem studied in this paper is to determine the sequence $\{p_i\}$ that minimizes the competitive ratio. For this problem, we use the following simple algorithm.

Theorem 2.1 The optimal competitive ratio for problem $BA2$ is $\rho = \frac{3}{2}+\sqrt{2}$, i.e., for problem $BA2$, there exists an online algorithm whose competitive ratio is ρ, while there is no online algorithm whose competitive ratio is smaller than ρ.

The proof is given in Section 3.

Furthermore, the following generalization of problem 1 (BA2) is a propoer extension of the cow path problem where we consider the case there is more than one receiver.

Problem Broadcast+Ack-n (BAn): There are n hosts in the region, including a sender s and $n-1$ receivers $r_1, \ldots, r_{n-1}$ at different distances from s. The input can be specified as a configuration $\langle \bar{n}, \bar{d} \rangle$ where $\bar{n} = (n_1, \ldots, n_k)$, $\bar{d} = (d_1, \ldots, d_k)$, $\sum_{i=1}^{k} n_i = n-1$ and $d_i < d_{i+1}$ for $1 \leq i \leq k$. In this configuration, the $n-1$ receivers are organized so that there are n_i receivers at distance d_i from s, for $1 \leq i \leq k$. The sender knows n, but does not know the groupings nor the distances. The sender should broadcast a message to all the receivers and get acknowledgments from all of them. The goal is to minimize the total power consumption for the sender and the receivers.

For this problem we also propose an online algorithm of optimal competitive ratio.

Theorem 2.2 *For problem BAn, there exists an online algorithm with competitive ratio $\frac{3}{2} + \sqrt{2}$, and there is no online algorithm with competitive ratio smaller than $\frac{3}{2} + \sqrt{2}$.*

The algorithm and its analysis appear in Section 4.

3 Proof for single receiver case

In this section we prove Theorem 2.1. First we show an upper bound of the competitive ratio of algorithm $DA[\beta]$.

Proposition 3.1 The doubling algorithm $DA[\beta]$ achieves the competitive ratio $\frac{\beta(2\beta-1)}{2(\beta-1)}$ for the problem $BA2$. □

Proof: Let d denote the distance between the sender s and the receiver t. If the algorithm $DA[\beta]$ terminates at step J, then necessarily $(\beta^{J-1}/\gamma)^{1/\delta} < d \leq (\beta^J/\gamma)^{1/\delta}$. The cost of $DA[\beta]$ is

$$\sum_{i=0}^{J} \gamma\beta^i + \gamma\beta^J = \frac{\gamma(\beta^{J+1}-1)}{\beta-1} + \gamma\beta^J,$$

while the optimal cost is at least $2\gamma\beta^{J-1}$. Hence the competitive ratio is at most $\frac{\beta(2\beta-1)}{2(\beta-1)}$. □

By letting $\beta = 1 + \frac{1}{\sqrt{2}}$, the competitive ratio is at most $\frac{3}{2} + \sqrt{2}$. We now show Theorem 2.1 which gives a lower bound of the competitive ratio.

Proof of Theorem 2.1: Proposition 3.1 guarantees the existence of an online algorithm whose competitive ratio is $\frac{3}{2}+\sqrt{2}$. Thus, we now concentrate on showing that a lower bound on the competitive ratio for problem $BA2$ is $\frac{3}{2}+\sqrt{2}$.

Let us consider an online algorithm A^* which achieves the optimal competitive ratio $c \leq \frac{3}{2}+\sqrt{2}$. The output sequence of algorithm A^* (namely, the sequence of transmission powers used by the sender) is denoted by $x_1, x_2, \ldots$. Note that for any integer $n \geq 2$, one must consider a scenario where the receiver t is positioned at distance $x_{n-1}+\epsilon$ from the sender s, for an arbitrarily small ϵ. On such scenario, the optimal cost is $2x_{n-1}+2\epsilon$, whereas the online algorithm incurs a cost of $x_1+x_2+\cdots+x_{n-1}+x_n+x_n$. Therefore, by the definition of the competitive ratio, we have the inequalities

$$x_1+\cdots+x_{n-1}+x_n+x_n \;\leq\; 2c(x_{n-1}+\epsilon) \quad (1)$$

for any integer $n \geq 2$ and for arbitrarily small $\epsilon > 0$. Since $A \leq B+\epsilon$ for every $\epsilon > 0$ implies $A \leq B$, we have from inequality (1) that

$$\sum_{i=1}^{n-1} x_i + 2x_n \;\leq\; 2cx_{n-1} \quad (2)$$

for any integer $n \geq 2$. These inequalities yield new necessary conditions as follows. First, taking (2) with $n=2$ we have

$$x_2 \;\leq\; \left(c-\frac{1}{2}\right)x_1, \quad (3)$$

By (3) and using (2) with $n=3$, we similarly have

$$2cx_2 \;\geq\; x_1+x_2+2x_3 \;\geq\; \frac{2}{2c-1}\,x_2+x_2+2x_3,$$

that is,

$$x_3 \;\leq\; \left(c-\frac{1}{2}-\frac{1}{2c-1}\right)x_2.$$

Repeating this argument, we get the inequalities

$$x_{i+1} \;\leq\; \alpha_i x_i, \quad (4)$$

with the coefficients α_i defined as

$$\alpha_i \;=\; c-\frac{\mu_i}{2} \quad (5)$$

where

$$\mu_i = \begin{cases} 1, & \text{if } i=1,\\ \mu_{i-1}/\alpha_{i-1}+1, & \text{if } i \geq 2.\end{cases}$$

Equation (5) can be simplified as

$$\alpha_i \;=\; c+\frac{1}{2}-\frac{c}{\alpha_{i-1}}\;, \quad (6)$$

for any integer $i \geq 2$. Note that $\alpha_i > 1$ holds for any integer i by (4) and $x_i \leq x_{i+1}$.

We now show that this α_i is a Cauchy sequence, namely, $\frac{\alpha_{i+1}}{\alpha_i} \leq 1$. Indeed,

$$\begin{aligned}\frac{\alpha_{i+1}}{\alpha_i}-1 &= \frac{1}{\alpha_i}\left(c+\frac{1}{2}-\frac{c}{\alpha_i}\right)-1\\ &= -c\left(\frac{1}{\alpha_i}-\frac{c-\frac{1}{2}}{2c}\right)^2+\frac{(c-\frac{1}{2})^2-4c}{4c}\;.\end{aligned}$$

Recall that $c \geq 1$ and $c \leq \frac{3}{2}+\sqrt{2}$, by the property of the competitive ratio and Proposition 4.1, respectively. In this range, $(c-\frac{1}{2})^2-4c \leq 0$, thus $\frac{\alpha_{i+1}}{\alpha_i} \leq 1$, and α_i is a Cauchy sequence. Therefore, α_i converges to some value α, which by (6) must satisfy

$$\alpha = c+\frac{1}{2}-\frac{c}{\alpha}$$

and its discriminant satisfies

$$\begin{aligned} D &= \left(c+\frac{1}{2}\right)^2-4c\\ &= \left(c-\frac{3}{2}-\sqrt{2}\right)\left(c-\frac{3}{2}+\sqrt{2}\right) \;\geq\; 0.\end{aligned}$$

As $c \geq 1$, the right term in the product is positive. Hence the left term must also be positive, implying $c \geq \frac{3}{2}+\sqrt{2}$ and thus yielding the desired lower bound on the competitive ratio. □

4 Handling multiple receivers

In this section we consider the case the sender must send the message to more than one receiver.

4.1 Equal distance receivers

First we consider the easier special case where all the receivers are at the same distance from the sender. That distance is unknown to the participants.

Problem Uniform-Broadcast+Ack-n: ($UBAn$): There are n hosts in the area, of which one is the sender, s, and the other $n-1$ are receivers, $r_1, \ldots, r_{n-1}$. The receivers are all at the same distance d from the sender, but the sender does not know this distance. The sender should broadcast a message to all the receivers and get acknowledgments from all of them. The goal is to minimize the total power consumption for the sender and the receivers.

Note that this problem is identical to problem $BA2$ if $n = 2$. For this problem we claim that the doubling algorithm has optimal competitive ratio.

Proposition 4.1 *For problem $UBAn$, fixing $\beta = 1 + \frac{1}{\sqrt{n}}$, the competitive ratio of Algorithm $DA[\beta]$ is at most $1 + \frac{2}{\sqrt{n}} + \frac{1}{n}$.*

The proof is similar to that of Proposition 3.1. We now show an asymptotically matching lower bound.

Proposition 4.2 *For problem $UBAn$, there is no algorithm with competitive ratio smaller than $1 + \frac{2}{\sqrt{n}} + \frac{1}{n}$.*

Proof: The proof of Theorem 2.1 can be easily extended to the general case. Similar to inequalities (4) and (5), albeit with different α, we have:

$$x_{i+1} \;<\; \alpha_i x_i$$

where

$$\begin{aligned}\alpha_1 &= c - \frac{1}{n},\\ \alpha_i &= c - \frac{1}{n} + 1 - \frac{c}{\alpha_{i-1}},\end{aligned}$$

for any integer $i \geq 2$. The sequence α_i is again a Cauchy sequence because $c \geq 1$ and $c \leq 1+\frac{1}{n}+\frac{2}{\sqrt{n}}$ hold by the property of the competitive ratio and Proposition 4.2, respectively. Thus we have

$$\alpha = c - \frac{1}{n} + 1 - \frac{c}{\alpha},$$

which implies $c \geq 1 + \frac{1}{n} + \frac{2}{\sqrt{n}}$. □

4.2 Variable distance receivers

Next we consider the general Problem BAn as stated in Section 1, where the $n-1$ receivers are placed at different distances from the sender. If the configuration $\langle \bar{n}, \bar{d} \rangle$ is known to all the hosts, then the solution is trivial (the sender transmits once, reaching all the receivers, and each receiver transmits its acknowledgement using the minimal power required). However, in case the configuration is known only to the sender (but not to the other hosts), computing the optimal cost is not obvious. Nevertheless, it can be computed as follows.

Proposition 4.3 *In case the configuration $\langle \bar{n}, \bar{d} \rangle$ is known to the sender, the optimal cost for problem BAn can be computed in linear time.*

Proof: Given a configuration $\langle \bar{n}, \bar{d} \rangle$, we can restrict the distances chosen by the sender s for message transmission to the set $\{d_1, \ldots, d_k\}$, i.e., the transmission powers can be specified from among $p_i = \gamma d_i^\delta$ for $i = 1, \ldots, k$. The *schedule* of s's broadcast can be represented as an ordered set Y of indices corresponding to the distance to which the sender s transmits the message. For example, using the schedule $Y = \{1, 3, 6\}$, the sender s transmits the messages to distances d_1, d_3 and d_6. Given a schedule Y, for each $1 \leq i \leq k$, the n_i receivers at distance d_i from s will receive the message for the first time on round $m(i, Y) = \min\{j \mid j \geq i, j \in Y\}$, and thus use power $p_{m(i,Y)}$ for the acknowledgement. Hence for any schedule Y, the total cost of broadcast (including acknowledgements) is $cost(Y) = \sum_{i \in Y} p_i + \sum_{i=1}^{k} n_i p_{m(i,Y)}$. Subsequently, the goal of this problem is to find a schedule Y^* that minimizes $cost(Y^*)$.

The computation is based on a recursive formula for the cost function. Suppose that the sender s transmits the messages to an l-th distance in the schedule Y for $\langle \bar{n}, \bar{d} \rangle$. Then the cost function is expanded as

$$\begin{aligned} cost(Y) \quad = \quad & \left(\sum_{\substack{i \in Y \\ i \leq l}} p_i + \sum_{i=1}^{l} n_i p_{m(i,Y)} \right) \\ & + \left(\sum_{\substack{i \in Y \\ i > l}} p_i + \sum_{i=l+1}^{k} n_i p_{m(i,Y)} \right). \end{aligned}$$

Since the left and right parts in this equation use disjoint index sets, we have

$$\begin{aligned} OPT(\langle \bar{n}, \bar{d} \rangle) \quad = \quad & \min_{l=1,\ldots,k} \{ OPT(\langle \bar{n}, \bar{d} \rangle^{(1,l)}) \\ & + OPT(\langle \bar{n}, \bar{d} \rangle^{(l+1,k)}) \}, \end{aligned}$$

where $\langle \bar{n}, \bar{d} \rangle^{(a,b)}$ denotes $\langle (n_a, \ldots, n_b), (d_a, \ldots, d_b) \rangle$ and $OPT(\langle \bar{n}, \bar{d} \rangle^{(a,b)})$ denotes the optimal energy consumption for $\langle \bar{n}, \bar{d} \rangle^{(a,b)}$. Based on these observations, we can construct a dynamic programming algorithm as follows. Denote the set of indices from i to k by $S_i = \{i, i+1, \ldots, k\}$. For every i, we look for the schedule Y_i which minimizes the energy consumption for $\langle \bar{n}, \bar{d} \rangle$ over all schedules $Y \subseteq S_i$. Note that $Y_k = \{k\}$ and $Y_1 = Y^*$. One can easily verify that

$$Y_i = \begin{cases} Y_{i+1} \cup \{i\}, & \text{if } cost(Y_{i+1}) > cost(Y_{i+1} \cup \{i\}) \\ Y_{i+1} & \text{otherwise,} \end{cases}$$

which yields a dynamic programming algorithm for solving static BAn in linear time. □

It is easy to show the following.

Proposition 4.4 *For problem BAn, there is no algorithm with competitive ratio smaller than $\frac{3}{2} + \sqrt{2}$.*

Proof: Let us consider a scenario where $n-2$ receivers are positioned near the sender and one receiver is very far away. Then the optimal total cost is dominated by the power required to transmit the message to the farthest receiver and for that receiver to transmit its acknowledgment. By Proposition 4.2 for $n = 2$, $\frac{3}{2} + \sqrt{2}$ is also a lower bound for BAn. □

Next we consider an upper bound for the problem BAn. We propose the following algorithm.

Define the parameter $\beta_k = 1 + \frac{1}{\sqrt{k}}$ for every $k \geq 2$.

Algorithm 2(Dynamic Doubling Algorithm)
ProcedureDDA(n,msg)

$p \leftarrow \gamma$;
while $n > 1$ **do**
  Transmit $\langle msg, p \rangle$ with power p.
  Wait.
  $l \leftarrow$ #received acknowledgment packets;
  $n \leftarrow n - l$;
  $p \leftarrow \beta_n p$;
end while

Note that each receiver sends exactly one acknowledgment for a particular message *msg*.

To prove Theorem 2.2, we now show that Algorithm DDA achieves a competitive ratio of $\frac{3}{2}+\sqrt{2}$ for problem BAn.

Proof of Theorem 2.2: A lower bound of $\frac{3}{2}+\sqrt{2}$ is shown in Proposition 4.4. We show that Algorithm DDA has the same competitive ratio. Let p_i^* $(i = 1, 2, \ldots, l)$ be a sequence of transmission powers used the sender s in the optimal cost algorithm. If a receiver r_i receives the message, it sends an acknowledgment with power p_k^* where k is the minimum index such that $p_k^* \geq \gamma d(s, r_i)^\delta$. Note that because a message from s is delivered to multiple hosts the receivers do not know the exact distance to the sender. Therefore the optimal cost is identical to that of the following instance:

> There are n_i receivers at distance d_i for $i = 1, 2, \ldots, l$ where d_i satisfies $p_i^* = \gamma d_i^\delta$.

On the other hand, the cost of any online algorithm for the original instance is at most the cost for the new instance, it is enough to show the claim for the latter instance. For this instance, Algorithm DDA first uses $\beta = 1 + \frac{1}{\sqrt{n}}$. Then the ratio of the cost to send to hosts with distance

d_1 to the optimal cost is at most $\frac{\beta^2/(\beta-1)+n_1\beta}{n_1+1}$. From Proposition 4.1 this value is minimized if $\beta = 1 + \frac{1}{\sqrt{n_i}}$, and it is increasing as β decreases. Therefore its maximum value is achieved if $\beta = 1 + \frac{1}{\sqrt{n}}$, and it is upper bounded by $1 + \frac{2}{\sqrt{n}} + \frac{1}{n}$, which is no greater than $\frac{3}{2} + \sqrt{2}$.

Next Algorithm DDA uses $\beta = 1 + \frac{1}{\sqrt{n-n_1}}$. Because the transmission power for the first step is larger than γ, the maximum ratio is at most $1 + \frac{2}{\sqrt{n-n_1}} + \frac{1}{n-n_1}$, which is again no greater than $\frac{3}{2} + \sqrt{2}$. For the rest of the execution of the algorithm we always have the maximum ratio smaller than $\frac{3}{2} + \sqrt{2}$. □

Problem BAn corresponds to the following problem:

Problem Weight-Broadcast+Ack-n ($WBAn$): There are one sender and k receivers $r_1, \ldots, r_k$. The receivers are at different and unknown distances from the sender, and r_i needs power $n_i\gamma d^\delta$ to send a message to a host with distance d. The problem is to minimize the total power consumption for the sender and the receivers. Therefore we can handle the case of individual power consumption rates (i.e., where each host has a multiplicative weight defining its power consumption) in the same framework.

The competitive ratio can be further improved if some information is known about the distances of receivers.

Proposition 4.5 *Consider problem BAn assuming that the grouping vector $\bar{n}$ is known in advance, while the distance vector $\bar{d}$ is unknown. In this setting, the competitive ratio of Algorithm DDA is* $1 + \frac{2}{\sqrt{n_k}} + \frac{1}{n_k}$.

Proof: Consider the scenario where the last n_k receivers are very far and all the other receivers are positioned near the sender. The algorithm will reach all the receivers except for the farthest group and get their acks in one step, and from that point on, it will behave like Algorithm $DA[\gamma_{n_k}]$. □

5 Concluding remarks

Using multiple levels of transmission power is important for energy-efficient and collision-free communication. As far as the authors know, there has been no online algorithms with provable worst-case guarantees for energy-efficient broadcasting in ad-hoc wireless networks. Our algorithm for sending a message to a receiver with unknown distance has the optimal competitive ratio $\frac{3}{2} + \sqrt{2}$. Our algorithms for broadcasting a message to multiple receivers also have optimal competitive ratio. Interestingly, the competitive ratio of both problems are the same. We believe that our algorithms can potentially be made practical for actual wireless networking.

Acknowledgments

The authors would thank Mr. M. Mesbah Uddin and Prof. Hiroto Yasuura of Kyushu University, who suggested the problem. This work is supported in part by the Grant-in-Aid of the Ministry of Education, Science, Sports and Culture of Japan.

参考文献

[1] R. Baeza-Yates, J. Culberson and G. Rawlins, Searching in the plane. *Information and Computation* 106:234–252, 1993.

[2] A. Borodin and R. El-Yaniv, *Online Computation and Competitive Analysis*, Cambridge Univ. Press, 1998.

[3] G.S. Lauer. *Packet radio routing*, Chapt. 11, *Routing in communication networks*, M. Streenstrup (ed.), pages 351–396. Prentice-Hall, 1995.

[4] R. Mathar and J. Mattfeldt. Optimal Transmission Ranges for Mobile Communication in Linear Multihop Packet Radio Networks. *Wireless Networks*, 2:329–342, 1996.