オペレーティング・システム 26－8
（1985. 3. 1）

TRANSIENT ANALYSIS OF CARRIER SENSE MULTIPLE ACCESS WITH COLLISION DETECTION VIA DIFFUSION APPROXIMATION

Hideo Miyahara,*　Toshihiro Matsumoto**　and　Kensuke Takashima*

*Faculty of Engineering Science, Osaka University　**Matsushita Electric Industrial Co.,Ltd.

1. Introduction

Random access methods like ALOHA and Carrier Sense Multiple Access with Collision Detection (CSMA/CD) have favorable features in the way of simplicity of packet transmission and better performance especially in light traffic (these are inherent characteristics in random access), and therefore, are effectively applicable to data communication system with multiple access channels such as mobile radio, satellite communication and local area networks. However, there is probability that the system will go into an unstable operating point where throughput decreases and packet delay increases. This situation occurs when the system receives a sudden traffic change, even if operated within low utilization on the average. Therefore, some control mechanism is necessary to make the system go back to a stable operating point. In this paper, we present an approximate method for predicting dynamic behavior of CSMA/CD. We apply a diffusion approximation in obtaining the distribution of the number of backlog packets (NBP). In the system, we treat the case where the rate of packets newly generated at each terminal is varying with time, and derive a diffusion equation with respect to the probability density function (PDF) of the NBP at time t. We apply this equation to an example model and show the following performance measures are attainable by numerrically solving a diffusion equation with time and state dependent coefficients that satisfies a certain boundary condition. (i) The PDF of the NBP in steady state. (ii) The PDF of the NBP at an arbitrary time point when the system starts a given initial condition. (iii) The PDF of the NBP at an arbitrary time point during or after the time at which the system receives a sudden traffic. (iv) Throughput and average packet delay characterisics.

2. CSMA/CD

2.1 The Model

We assume that there are M terminals and that are operated independently of each other. The time axis is divided into slots whose length is defined to be equal to end-to-end propagation delay, τ, and T time slots are required to transmit one packet. Each terminal in thinking mode generates a new packet to be transmitted with probability $\sigma(t)$ per slot at time t and sense the channel to see whether a carrier is on at the beginning of a time slot. If it detects no carrier on the channel, it begins to transmit its packet in that slot and if otherwise it enters backlog mode. Terminal in backlog mode senses the channel with probability p at the beginning of a time slot and with probability (1-p) it delays the sensing one slot after. If no carrier is detected when it senses, it then begins to transmit its backlog packet in that slot. If it detects some carrier it again enters backlog mode. A terminal is assumed to know whether a packet which has been transmitted from the terminal has collided with other packets at γ slots after start of transmission. If it detects collision it enters the backlog mode. That is, we consider the model to be of the p-persistent CSMA/CD protocol.

2.2 Diffusion approximation

A cycle is defined as the time interval from the instant when the channel status changes from busy to idle to the instant of the next such change of the channel status. There are four possible system states after a successful transmission as shown in Figs.1(a)-1(d). In the following, we assume that $\sigma(t)=\sigma$ during a cycle time, i.e., $\sigma(t)$ changes its value at only the time when a cycle begins. The distribution of the idle time I in Figs.1(a)-1(d) is geometric with parameter P_{send}. When the number of backlog packets at the beginning of a cycle is equal to i, P_{send} is given by

$$P_{send}=\text{prob.[at least one terminal transmits its packet within one slot]}$$
$$=1-(1-\sigma)^{M-1}(1-p)^{i}$$

The mean, $\bar{I}$, and the variance, σ_I^2 of I are given

$$\bar{I}=1/P_{send}$$
$$\sigma_I^2=(1-P_{send})/P_{send}^2$$

The probability that each state in Figs.1(a)-1(d) occurs, given that idle time ends, can be obtained as follows.

(a) The probability that a packet from a terminal in thinking mode is successful.

$$P_{ST}=\frac{(M-i)\sigma(1-\sigma)^{M-i-1}(1-p)^i}{P_{send}}$$

(b) The probability that a packet from a terminal in backlog mode is successful

$$P_{SB}=\frac{ip(1-p)^{i-1}(1-\sigma)^{M-i}}{P_{send}}$$

(c) The probability that collision occurs and the collided packets are those from the terminals in backlog mode.

$$P_{CB}=\frac{(1-\sigma)^{M-i}[1-(1-p)^i-ip(1-p)^{i-1}]}{P_{send}}$$

(d) The probability that collision occurs and at least one packet from terminals in thinking mode are included in the collided packets.

$$P_{CT}=1-(P_{ST}+P_{SB}+P_{CB})$$

Therefore, the average length of the cycle time, $\bar{C}$, becomes

$$\bar{C}=\bar{C}_S(P_{ST}+P_{SB})+\bar{C}_F(P_{CT}+P_{CB})$$

where $\bar{C}_S=\bar{I}+T+1$ and $\bar{C}_F=\bar{I}+\gamma+1$

First we consider the decreasing process of the number of backlog packets. This corresponds to the backlog packet departure process of the system. Let S_B be the probability that a packet transmission from a terminal in backlog mode is successful. S_B becomes

$$S_B=\frac{P_{SB}}{\bar{C}}$$

The mean interdeparture time between two consecutive backlog packets, m_{db}, becomes

$$m_{db}=\frac{1}{S_B}=\frac{\bar{C}}{P_{SB}} \qquad (1)$$

The variance of interdeparture time of packets, σ_d^2, when the outgoing packets are not discriminated as to whether they have been backlogged or not becomes

$$\sigma_d^2=\frac{P_S\sigma_S^2+P_S(1-P_S)\sigma_F^2+(1-P_S)\bar{C}_F^2}{P_S2}$$

where $P_S=P_{ST}+P_{SB}$ and σ_S^2, σ_F^2, are the variance of the cycle length when it ends with collision and success, respectively.

The variance of interdeparture time of packets from terminals in backlog mode, σ_{db}^2, can be written as

$$\sigma_{db}^2 = \frac{\sigma_d^2}{P_b} + \frac{1 - P_b}{P_b^2} m_{db}^2 \qquad (2)$$

where $P_b = \dfrac{P_{sb}}{P_{ST}+P_{SB}}$

Next we consider the increasing process of the number of backlog packets and obtain the mean and the variance. When a cycle ends with successful packet transmission (which corresponds to the cases of Figs.1(a) and 1(b)), the increment of the number of backlog packets during this cycle is equal to the number of accesses during the time interval (T+1) from the terminal in thinking mode. On the other hand, when a cycle ends with a collision (which corresponds to the case Fig.1(c) and 1(d)), the packets newly arrived during the time interval $(\gamma+1)$ become backlogged. Therefore, the average increment of the number of backlog packet during a cycle can be represented as follows depending on each case of Figs. 1(a) to 1(d).

(a) when a packet from a terminal in thinking mode is successful

$$a_{ST} = (M-i-1)[1-(1-\sigma)^{T+1}]$$

(b) when a packet from a terminal in backlog mode is successful

$$a_{SB} = (M-i)[1-(1-\sigma)^{T+1}]$$

(c) when only packets transmitted from terminal in backlog mode are collided

$$a_{CB} = (M-i)[1-(1-\sigma)^{\gamma+1}]$$

(d) when packets including ones transmitted from the terminals in thinking mode are collided

$$a_{CT} = 1+(M-i-1)[1-(1-\sigma)^{\gamma+1}]$$

Then, the average increment, a, becomes

$$\bar{a} = (P_{ST}a_{ST}+P_{SB}a_{SB}+P_{CB}a_{CS}+P_{CT}a_{CT})/\bar{C}$$

This increasing process can be approximated by a Poisson process with mean $\bar{a}$. (The reason stated in [1] can be applyied to our CSMA/CD model, though ALOHA model is treated in [1]) Then, the mean, m_a, and the variance, σ_a^2, of packet interarrival time, where arrivals increase the number backlogged, are

$$m_a = 1/\bar{a} \qquad (3)$$

$$\sigma_a^2 = 1/\bar{a}^2 \qquad (4)$$

Equations (1) to (4) are obtained under the condition that there are i backlogged packets at the beginning of the cycle, and are the functuions of i and also t. Let R(t) be the number of backlog packet at time t. Although R(t) is a step function having changes at the beginning of each cycle, we approximate it to be a time continuous process, R*(t), under the assumption that the average cycle length is negligibly small compared with the observation time. The infinitesimal mean, $\beta(x,t)$, and the infinitesimal variance, $\alpha(x,t)$, with respect to the NBP can be approximated as follows by replacing i with x in Eqs.(1)to (4).

$$\beta(x,t) = \frac{1}{m_a(x,t)} - \frac{1}{m_{db}(x,t)} \qquad (5)$$

$$\alpha(x,t) = \frac{\sigma_a^2(x,t)}{m_a^3(x,t)} + \frac{\sigma_{db}^2(x,t)}{m_{db}^3(x,t)} \qquad (6)$$

The probability density function, $P(x,t)$, that $R^*(t)=x$ satisfies the following diffusion equation.[1][2]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}[\beta(x,t)P(x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[\alpha(x,t)P(x,t)] \quad (7)$$

The boundary conditions are

$$-\beta(x,t)P(x,t) + \frac{1}{2}\cdot\frac{\partial}{\partial x}[\alpha(x,t)P(x,t)]=0 \quad \text{at } x=0 \text{ and } x=M \text{ for all } t \quad (8)$$

3. Numerical examples

The results shown in this section are obtained by solving the differencial Eq.(7) to satisfy boundary condition of Eq.(8). We use the implicit method called Crank-Nicolson in order to solve the equation numerically.[3] In the following example models we assume that $M=100$, $T=100$ and $\gamma=2$. Fig.2 shows that the drift, i.e., the value of the infinitesimal mean against x, when $\sigma(t)=0.007$ and $p=0.1$. From the definition of the infinitesimal mean in Eq.(5), it can easily be understood that the force which reduces the NBP works on the system in the region where the value of the $\beta(x,t)$ is negative and on the other hand the force which increases the NBP on the system where $\beta(x,t)$ is positive. Therefore, the system has two balancing points, S_1 and S_2 in Fig.2. Fig.3 represents the PDF of the NBP in steady state in exactly the same parameter settings as those in Fig2. Although the PDF of the NBP of the system can be considered to have two peaks in its distribution, the system remains in lower balancing point S_1 in our observation. However, the system might go into higher balancing point S_2 if it receives a sudden traffic change. In order to see this, We consider the case where the input rate to the system varies with time as shown by Fig.4. The Probability $\sigma(t)$ that a terminal in thinking mode transmits a new packet at time t suddenly increases from 0.007 to 0.010 at time $t=t_1$ and is kept at this high value for the duration of 200 packet transmission time. Figs.5 and 6 represent the PDF of the NBP at $t=t_1$, t_2, t_3 and t_4 when the system starts from the empty state, i.e., $P(0,0)=1$. In Fig.5, the retransmission probability p is not changed and is kept at 0.1 for the duration of the observation time. But the retransmission probability p is reduced from $p=0.1$ to $p=0.01$ at $t=t_2$ in Fig.6. In fig.5, although the NBP is almost always less than five and the system is operated in stable reagion for $t<t_1$, both the mean and the variance of the distribution become large after the system receives sudden input traffic change. The PDF of the NBP distributes arround ten at $t=t_2$. The distribution still tends to shift toward right and the system goes into saturating operating point as shown by the curve $t=t_4$ in spite of the fact that over five hundred packet transmission time have passed since the input rate had returned to normal value. On the other hand, when the retransmission probability p is reduced when the system receives high input traffic, the distribution quickly returns to the one before the system receive the traffic change within three hundred packet transmission time. Figs.7 and 8 show the throughput and delay characteristics when the input traffic pattern is given by Fig.4. From these observations, we can see how the retransmission probability affects the stability of the system and that the degradation of the system efficiency due to sudden input traffic increases can be avoided by adjusting the value of retransmission probability.

4. Conclusion

We derived a diffusion equation which represents the probability distribution function of the number of backlog packets in CSMA/CD system. We can predict the dynamic behavior of the systems by solving a diffusion equation numerically subject to certain boundary conditions. In the example system of CSMA/CD where a time varying input traffic is assumed, we examine how the distribution of the number of backlogged fluctuates according to the fluctuation of the input traffic. We also obtained the backlog distribution in steady state, the throughput-delay characteristics and throughput-load characteristics. The method presented here requires only $O(M)$ memory (M:the number of terminals) in obtaining these performance measures, whereas the method by Markov analysis requires $O(M^2)$. Therefore, the diffusion approximation method presented here can be considered to be effective in an analysis of the dynamic behavior of random access systems with time varying input traffic and/or with many access terminals.

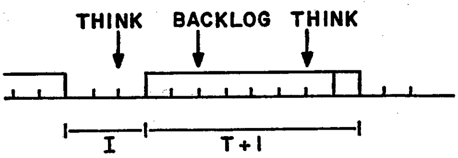

Fig.1(a) Succ. of a new packet

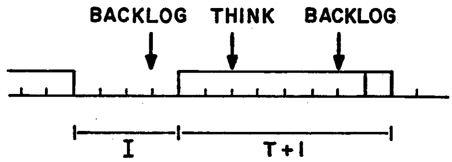

Fig.1(b) Succ. of a backloged packet

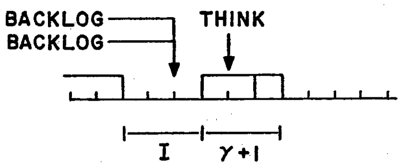

Fig1(c) Colli. excluding new packets

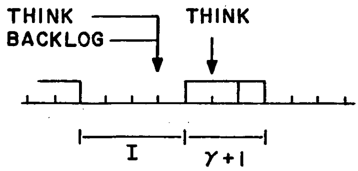

Fig.1(d) Colli. including new packets

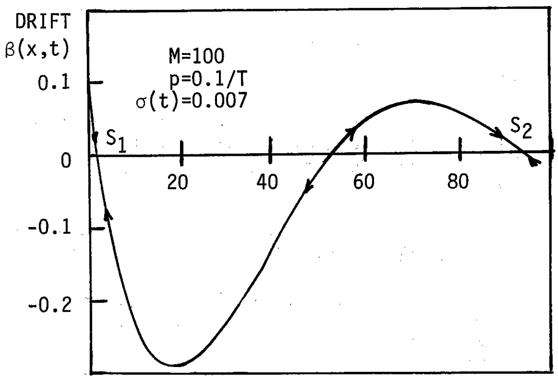

Fig.2 Infinitesimal mean vs. the NBP

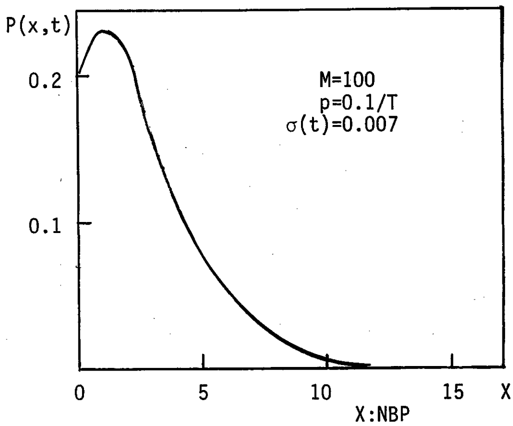

Fig.3 Steady state dist. of the NBP

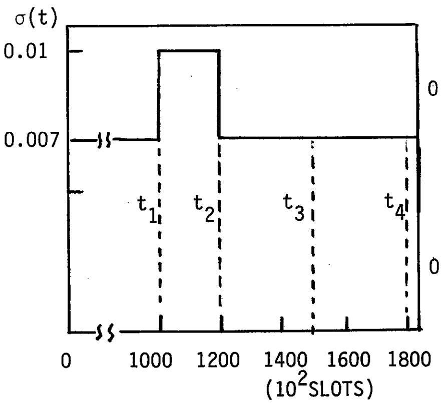

Fig.4 Time varying input traffic

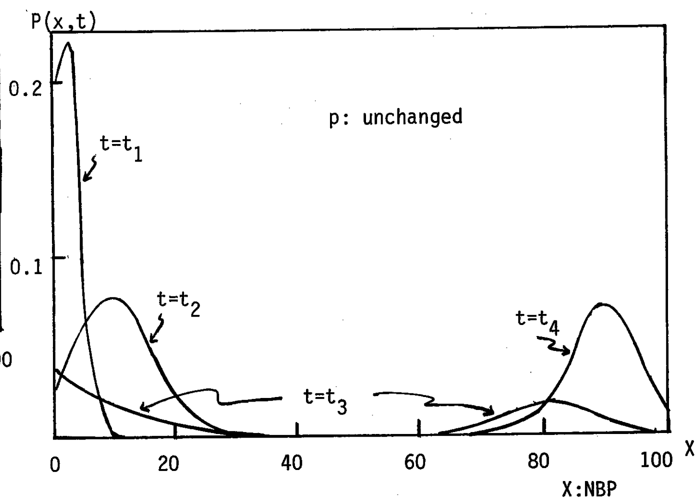

Fig.5 The PDF of the NBP in transient state

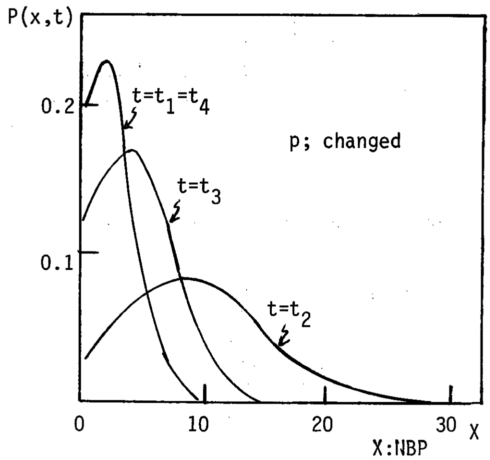

Fig. 6 The PDF of the NBP in transient state

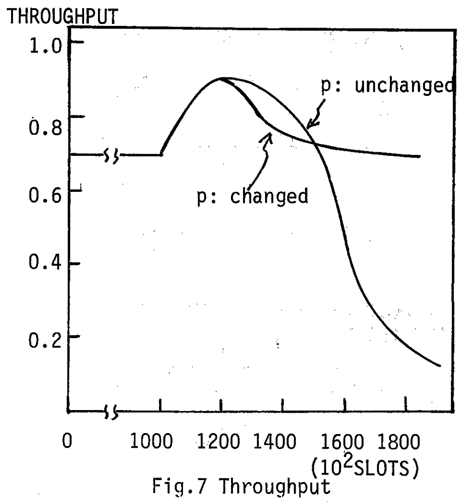

Fig.7 Throughput

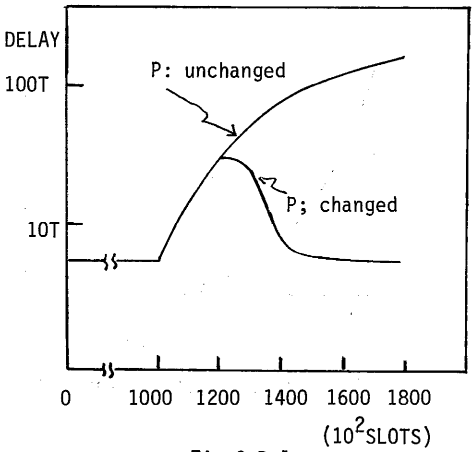

Fig.8 Delay

References

(1) H. Kobayashi et al, "An Approximate Method for Design and Analysis of an ALOHA System," IEEE Trans. Commun. Vol.COM-25, No.1, Jan. 1977

(2) D. R. Cox and M. D. Miller, The Theory of Stochastic Process, New York: Wiley, 1965

(3) Gordon D. Smith, Numerical Solusion of Partial Differencial Equations, Oxford Univ. Press, 1965