Kirby is PSPACE-complete

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1 Introduction

Recently, the computational complexity of video games have been analyzed [2]. We investigate the computational complexity of Kirby [1]. Kirby is one of side-scrolling character action games. In this game, a player operates the character Kirby. He can walk, run, jump and "hover" by inhaling the air. More precisely, inhaling a lot of air makes Kirby light, then Kirby can repeatedly hover in the air.

Our contribution is to show the computational complexity of Kirby. By applying the general framework [2] for showing PSPACE-hardness, we prove the PSPACE-completeness of Kirby.

2 Preliminary

Kirby is a side-scrolling character action game [1]. In this video game, a user operates the character, named Kirby. The task of this game is to move from a start point to a finish point and defeat an enemy character in the finish point. Kirby can walk, run, and jump. One of his primitive ability is inhaling. When he inhales the air, he can hover in the air. By repeatedly hovering in the air, he can fly.

There are many Kirby games. In this paper, we focus on Kirby Super Star. ¹ Now, we define our problem KIRBY-SS, as follows.

Problem: KIRBY-SS

Instance: A stage S, start and finish points s, t. **Question:** Is s reachable to t in S?

In this paper, we show the hardness of KIRBY-SS. using polynomial-time reduction. To construct the reduction, we explain some rules of KIRBY-SS, which are used in our reduction, below.

Kirby can get copy abilities by inhaling an enemy. There are many types of copy abilities. The copy abilities can also be obtained by touching copy essences, which are objects in the game. The obtained ability is valid until different types of copy essences are touched. In our reduction, we use only copy essences to give Kirby copy abilities. Hence, we explain how to get abilities from copy essences and omit to get abilities by inhaling enemies. Here, we introduce three copy abilities which are used in our reduction.

Fire ability: While Kirby have the Fire ability, he can become a fireball and go straight to the left or right for a fixed distance in an invincible state. While Kirby is a fireball, a player cannot operate him, that is he just goes straight for a fixed distance.

Stone ability: While Kirby has the Stone ability, he can become a stone and falls downward in an invincible state. While Kirby is a stone, Kirby just falls until a player stops a stone state.

Beam ability: While Kirby has the Beam ability, he can attack using a beam. (Actually, we utilize Beam ability to just remove either Fire ability or Stone ability from Kirby.)

A Gordo is an enemy character. The movement of Gordo is simple. A pattern of Gordo's movement is just staying in the designated place. Another movement pattern is to first go straight to the left (and right). When the Gordo touch a wall, then he turns and goes straight to the right (and left). Kirby cannot defeat a Gordo in any way, that is, Gordo is invincible. If a Gordo exists in a horizontal path with only one-character height, Kirby requires the Fire ability to pass through the Gordo without damage. Similarly, if a Gordo exists in a narrow vertical path with only onecharacter width, then Kirby require the stone ability to fall down through the Gordo without damage.

A *switch* is one of objects. While Kirby presses the switch (just stand on the switch), a designated one shutter opens. When Kirby leaves from the switch, the shutter closes.

We use the above rules (three abilities, Gordo, and switch) to construct a reduction for showing PSPACEhardness in Section 3.

By applying the result by Aloupis et al. [2], the membership of KIRBY-SScan be proved.

Lemma 1 ([2]) KIRBY-SS is in PSPACE.

3 PSPACE-hardness of Kirby

Aloupis et al. [2] showed the framework for showing PSPACE-hardness of video games. In this section, we show that the framework can be applied KIRBY-SS. The framework just requires the following gadgets: Start and Finish point gadgets, Door gadgets, and Crossover gadgets. Implementations of these gadgets implies the PSPACE-hardness of the target video game.

Start and Finish point gadgets: Kirby has health. To reduce his health gauge, the start point has some Gordos such that he cannot avoid them, as illustrated in Figure 1. Kirby has to get damaged from Gordos to go to the exit of Start point gadget. When Kirby get damaged, he turned to be invincible for a certain amount of time. By using this game rule, Kirby goes to the exit of the Start point gadget. For each Gordo, Kirby continue to get damaged and passes through a

¹ "Hoshino-Kirby super deluxe" in Japanese.

Figure 1: Start point gadget for KIRBY-SS.



Figure 2: Crossover gadget for KIRBY-SS.

Gordo. Gordos are placed in this gadget such that Kirby died if he touches Gordo once more.

Finish point gadget is just a warp star (which is a goal point) of a stage.

Crossover gadgets: We construct Crossover gadgets for KIRBY-SS, which is illustrated in Figure 2. Our Crossover gadget has two crossing paths. The one path has its entrance and exit in the top left and the bottom right, respectively. In the center of this gadget, which is the intersection of two paths, Gordo is placed. Since the health gauge of Kirby is very lower (the health gauge of Kirby is reduced in Start point gadget), Kirby have to avoid to get damaged from the Gordo. Hence, Kirby gets Stone ability, goes down through the Gordo using Stone ability without damage. Then, after Kirby gets Beam ability, he goes to the bottom right exit. The purpose of getting Beam ability is to make Kirby lose Stone ability. Note that, while the state of Kirby is a stone, he only goes down. Hence, he can go neither left nor right in the intersection place of this gadget. Another path has its entrance and exit in the top right and the bottom left, respectively. To pass through this path, Kirby uses Fire ability. Using Fire ability, Kirby goes left in an invincible state without getting damaged from the Gordo.

Door gadgets for Kirby: The Door gadget for Kirby is shown in Figure 3. The Door gadget has the following three paths.

The open path has its entrance and exit in bottom left. In the travasal of the open path, first, Kirby gets Fire ability, then he passes through the Gordo g_1 . The switch X is placed in the bottom center of the gadget. While Kirby presses the switch X, the shutter with the label X opens. While the shutter opens, the Gordo g_2 in the traverse path moves right. When the Gordo touches the right wall, Kirby stops to press the switch and the shutter closes. Since the Gordo is placed right, the traverse path turns to be passable. After that, Kirby goes back to the entrance of open path. Note that Kirby cannot go to the close path from the open path, since Gordos g_3 and g_4 are placed.

The close path has its entrance and exit in the middle right and the top right, respectively. Now, we first assume that the Gordo g_2 is placed on the right



Figure 3: Door gadget for KIRBY-SS.

of the shutter with the label X. To go to the exit of the close path, Kirby have to make the Gordo g_2 move left. To do that, first Kirby goes to the switch passing through the Gordo g_3 using Stone ability. Second, by pressing the switch, Kirby makes the Gordo g_1 move left. Finally, Kirby goes back to the close path passing through the Gordo g_4 using Stone ability. If the Gordo g_2 is placed on the left of the shutter with the label X, Kirby can go to the exit straightforwardly.

The traverse path has its entrance and exit in the top left and the middle left, respectively. Kirby can pass through the traverse path if and only if the Gordo g_2 is placed in the right of the shutter with the label X. As explained above, to make the Gordo move, Kirby must pass through the open path before.

Using these gadgets, we have a polynomial-time reduction from Quantified 3SAT [2]. Thus, we have the following lemma.

Lemma 2 KIRBY-SS is PSPACE-hard.

By Lemmas 1 and 2, we have our theorem.

Theorem 3 KIRBY-SS is PSPACE-complete.

Our proof can be applied to the other Kirby game: Kirby & The Amazing Mirror.² Let KIRBY-AM be the problem for this game. Now, we have the following corollary.

Corollary 4 KIRBY-AM is PSPACE-complete.

References

- [1] Official home of Kirby. https://kirby. nintendo.com.
- [2] G. Aloupis, E. D. Demaine, A. Guo, and G. Viglietta. Classic nintendo games are (computationally) hard. *Theoretical Computer Science*, 586:135–160, 2015.

²Fire ability in KIRBY-SS have to be replaced with Burning ability in KIRBY-AM.