6B-04

A Study on the Leapfrogging Strategy for the Quantum Approximate Optimization Algorithm on the Max-cut of n-regular Graph Instances

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Abstract

The quantum approximate optimization algorithm (QAOA) has numerous promising applications on solving the combinatorial optimization problems on the near-term Noisy Intermediate Scalable Quantum (NISQ) devices. QAOA has a quantum-classical hybrid structure, with the quantum part consisting the parameterized alternating operator ansatz, and the classical part consist of an optimization algorithm optimizing the parameters to maximize the expectation value. This value depends highly on the parameters. This implies that a set of good parameters leads to an accurate solution of the given problem. However, at large circuit depth, it is difficult to achieve global optimization due to the multiple occurrence of local maxima. Therefore, we study the so-called leapfrogging strategy on solving the Max-cut problem for 3-regular graphs, which reuses the optimized parameters in larger graphs.

1 Introduction

Since the introduction of QAOA by Farhi et al. [1], it is widely known for its efficiency in solving combinatorial optimization problem on quantum computers. Among the problems, the Max-cut problem is heavily studied for its simple formulation and its deep relation to the Ising model. Since the Max-cut problem is an NP-complete problem, it is known that we are unable to solve it efficiently using classical computers, unless P=NP. Although QAOA does not give the exact solution to the Max-cut problem, the algorithm provides a heuristic approach for the problem. However, several hurdles continue to exist in obtaining a "good" solution for QAOA Max-cut. Solving large problems (graphs with large number of nodes) requires the deep circuit, but QAOA has difficulty at deep circuit due to the existence of many local maxima on the hypersurface of the expectation function.

In this paper, we use the leapfrogging strategy introduced [2] on regular graphs to tackle the problem, and verify our results using the Qiskit Aer QASM simulator.

2 QAOA Max-cut on regular graphs

QAOA is inspired by the Quantum Adiabatic Algorithm [3], which focuses on evolving the initial Hamiltonian H_B to the problem Hamiltonian H_C, satisfying

$$\tilde{H}(s) = (1-s)H_B + sH_C \tag{1}$$

where $s(t) \to 1$ as $t \to \infty$. The evolution in (1) is then discretized, which results in the idea of QAOA. In QAOA, the alternating unitary operators involving H_B and H_C are applied to the initial state to simulate the evolution of the system in (1):

$$|\psi_p(\gamma,\beta)\rangle = e^{-i\beta_p B}e^{-i\gamma_p C}\dots e^{-i\beta_1 B}e^{-i\gamma_1 C}|+\rangle^{\otimes n} \tag{2}$$

where $B = H_B$ and $C = H_C$. For the Max-cut problem of a graph $G = (V, E)$, they are given as

$$C = \frac{1}{2}\sum_{(j,k)\in E}(I - Z_j Z_k) \tag{3}$$

$$B = \sum_{j\in V} X_j \tag{4}$$

X_j and Z_j are the Pauli operators acting on the j-th qubit. After applying the operators as in (2), we calculate the expectation of the operator C with respect to the ansatz state $|\psi_p(\gamma,\beta)\rangle$:

$$F_p(\gamma,\beta) = \langle\psi_p(\gamma,\beta)|C|\psi_p(\gamma,\beta)\rangle \tag{5}$$

Since (5) is parameterized by the angles γ and β, we can use a classical optimization algorithm to search for the maximum M_p.

$$M_p = \max_{\gamma,\beta} F_p(\gamma,\beta) \tag{6}$$

The approximation ratio α is defined as

$$\alpha = \frac{M_p}{C_{\text{true}}} \tag{7}$$

where C_{true} is the true Max-cut value for the graph to be solved.

The positive integer p is known as the circuit depth, and it is shown theoretically that as $p \to \infty$, $M_p \to C_{\text{true}}$. However, for example, previous report [4] states that at large p, it is difficult to maximize F_p due to: 1. The point to be optimized is more likely to be trapped inside a local maxima; 2. The optimizer takes longer as it has more parameters to consider. These drawbacks discourage us to increase the circuit depth.

Farhi et al. [1] observe that at $p = 1$, for regular graphs, for each local term in (5), the operators which are not involved in the edge (j, k) will be canceled out and does not contribute to F_1. This further simplifies F_1 to the linear combination of the expectation value of the 3 types of subgraphs in 3-regular graphs.

$$\begin{aligned} F_1(\gamma_1, \beta_1) =& w_{\text{I}} F_{\text{I}}(\gamma_1, \beta_1) + w_{\text{II}} F_{\text{II}}(\gamma_1, \beta_1) \\ &+ w_{\text{III}} F_{\text{III}}(\gamma_1, \beta_1) \end{aligned} \tag{8}$$

I, II and III represent the 3 types of subgraphs, w is their respective total weights and F is their local expectations. Brandao et al. [2] then propose a leapfrogging strategy to inherit optimized angles from typical graph instance with smaller number of nodes to that with larger number of nodes. As the number of nodes gets larger, the instance will have higher proportion of type III (tree-like) subgraphs compared to I and II. This causes F_1 to concentrate towards F_{III}. Therefore, the leapfrogging works better for larger number of nodes and larger p as the graphs and subgraphs will be more tree-like.

3 Results and discussions

We solve the Max-cut on unweighted 3-regular instances with 6, 8, 12 and 16 nodes using QAOA. We choose the graphs such that they have higher proportion of the type III subgraph. We then compare the approximation ratio α between random initial angles and the leapfrogging method. In the random initialization method, we use 20 random angles as the starting point to the QAOA and optimize them using the Nelder-Mead optimizer. In the leapfrogging method, we first use the random method on solving the 6-node graph. Then, we pass the angles that output the highest α in the 6-node graph to the 8-node graph. We optimize those sets of angles and pass them to the 12-node graph, and then repeat the same for 16.

Figure 1 shows the result for the QAOA simulation. The value of α for the leapfrogging method is higher than the α averaged out of 20 times for the random initialization method. The advantage of the leapfrogging method is that it does not require trying multiple different angles each time a graph instance is solved to obtain the global maxima.

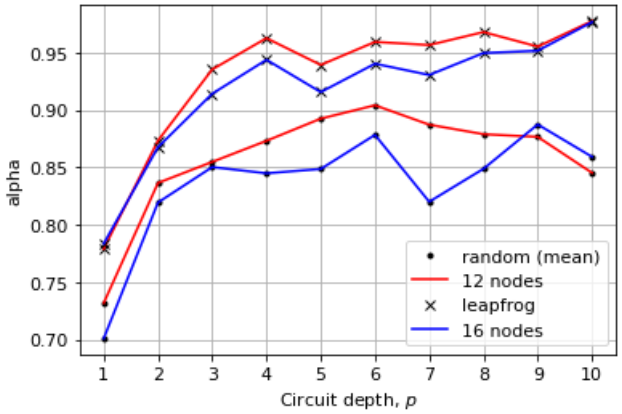

Figure 1: Comparison between the random and the leapfrog method for a 12 node graph and a 16 node graph.

4 Future works

We have applied the leapfrogging strategy to 3-regular graphs and the result shows it is able to approximate better than the random approach. In the future, we will study whether there are other classes of graphs that has potential to be improved by leapfrogging.

References

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