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Multi-objective Optimization Models for Many-to-one Matching Problems

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Abstract: This paper is concerned with many-to-one matching problems for assigning resident physicians (residents) to hospitals according to their preferences. The stable matching model aims at finding a stable matching, and the assignment game model involves maximizing the total utility. These two objectives however are generally incompatible. We focus on a case involving predetermined groups of residents who want to be matched in groups. To pursue these conflicting objectives simultaneously, we propose several multi-objective optimization models for many-to-one matching problems. We first derive a bi-objective optimization model for maximizing the total utility while minimizing the number of blocking pairs to promote stability. We next introduce a small-subgroup penalty, which will be minimized as the third objective for the purpose of matching in groups. Our multi-objective optimization models are formulated by means of the ε -constraint method as scalar objective mixed-integer optimization problems, which can be solved to optimality by using optimization software. The efficacy of our method is assessed through simulation experiments via comparison with the outcomes of two common matching algorithms: the deferred acceptance algorithm and Gale's top trading cycles algorithm. Our results highlight the potential of optimization models for computing good-quality solutions to a variety of difficult matching problems.

Keywords: many-to-one matching, multi-objective optimization, stable matching, assignment game, ε -constraint method

1. Introduction

Many-to-one matching problems arise in a variety of situations such as labor markets [30]. A well-known example is the National Resident Matching Program^{*1}, which provides a mechanism for assigning residents (i.e., new doctors studying for a specialty) to hospitals according to their preferences [28]. Other applications can be found in school admissions [35], student–project allocations [1], and housing allocations [7]. To deal with these matching problems, two standard models have been proposed: the stable matching model [15] and the assignment game model [34].

We consider the problem of matching residents with hospitals. In this problem, each resident can accept at most one residency position, and each hospital can offer a limited number of such positions. A matching is called *stable*^{*2} if it does not have any blocking pairs, because such a resident–hospital pair can form a private arrangement outside of the matching. The stable matching model [15] aims at finding a matching that is stable. Gale and Shapley [15] devised the deferred acceptance (DA) algorithm [29], which generates a stable matching. Another commonly employed algorithm is Gale's top trading cycles (TTC) algorithm [33], which produces a matching that is Pareto-efficient but not necessarily stable.

Another standard matching model is the assignment game

model proposed by Shapley and Shubik [34]. It involves maximizing the total utility gained by residents and hospitals, relying on the assumption that the utility can be exchanged between a resident–hospital pair in the matching. Although the assignment game model reduces to a linear optimization problem, the matching thus obtained is not necessarily stable when the exchange of utility is not allowed. Additionally, the algorithms designed for the stable matching model often do not maximize the total utility. These facts suggest that the two objectives (i.e., stability and utility) are incompatible in general cases.

This paper examines a particular situation in which residents prefer some colleagues over others [11], [12]. Specifically, we assume that there are predetermined groups of residents who want to be matched in groups. This special matching problem is known as *matching in groups*; see Hogan [19] for details. As examples, women at a college may want to join the same sorority as their friends, and workers may want to be employed by the same firm as their partners. Along the same lines, high schools want to recruit a group of student athletes from the same middle school because a highly cohesive group can improve team performance. This matching problem is related to the roommates problem [7], [15], [27], although the roommates problem aims at forming a stable set of roommate pairs.

The matching problem with couples [5], [24] is a special class of problems of matching in groups. Even in the case of couples, however, there are instances of problems that lack stable match-

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^{*1} <http://www.nrmp.org/about-nrmp/>

^{*2} See Section 2.2 for the precise definition.

ing [8], [20], [21], [28]. This fact clearly illustrates the theoretical difficulty of dealing with predetermined groups of residents in matching problems. For matching with couples, several algorithms work well in practice [4], [31], and various optimization models have been employed [2], [6], [10], [18], [25]. However, to our knowledge, no previous studies have developed an effective algorithm for matching in groups.

The aim of this paper is to propose multi-objective optimization models that simultaneously account for several conflicting objectives in many-to-one matching problems. First, we derive a bi-objective optimization model for maximizing the total utility and minimizing the number of blocking pairs. The number of blocking pairs is minimized to find a nearly stable matching based on the characterization [3] of stable matchings. We next introduce a small-subgroup penalty to be minimized as the third objective in the multi-objective optimization model. This penalty increases when a resident group is divided into small subgroups, so minimizing the penalty leads to assigning many members of a group to the same hospital.

The classical approach to dealing with a multi-objective optimization model is the scalarization method, which transforms the model into a scalar objective optimization problem. Various scalarization methods have been proposed [13], [26]. These scalarization methods include weighted sum methods, augmented weighted Tchebychev methods, and ε -constraint methods. We use the ε -constraint method [17], which can yield uniform Pareto-optimal solutions even in the case of a non-convex objective space. By means of this scalarization method, our multi-objective optimization models are formulated as scalar objective mixed-integer optimization problems, which can be solved to optimality by using optimization software.

The efficacy of our method is assessed through simulation experiments following up on previous studies [14], [16], [23]. The simulation results demonstrate that our method is capable of visualizing optimality trade-offs between total utility and the number of blocking pairs in many-to-one matching problems. In addition, our method clearly outperforms the DA and TTC algorithms in terms of total utility. The simulation results also show that our method works well for matching in groups because it finds good matchings without dividing resident groups into subgroups that are too small.

2. Multi-objective Optimization Models

This section presents our multi-objective optimization models for matching problems.

2.1 Assignment Game Model

Let us consider assigning residents to hospitals according to their preferences. We denote by R and H the sets of residents and hospitals, respectively. To decide a matching between the sets, we introduce the binary decision variable $\mathbf{x} := (x_{rh})_{(r,h) \in R \times H}$ such that

$$x_{rh} = \begin{cases} 1 & \text{if resident } r \text{ is assigned to hospital } h, \\ 0 & \text{otherwise,} \end{cases}$$

for all $(r, h) \in R \times H$.

Each resident is also assumed to fill only one residency position, and the number of residency positions offered by hospital h is $q_h \in \mathbb{Z}_+$. A matching is defined based on these conditions as follows.

Definition 1. We call $\mathbf{x} \in \{0, 1\}^{|R| \times |H|}$ a matching if the following constraints are satisfied:

$$\begin{aligned} \sum_{h \in H} x_{rh} &\leq 1 \quad (\forall r \in R), \\ \sum_{r \in R} x_{rh} &\leq q_h \quad (\forall h \in H). \end{aligned}$$

Also assumed is that when resident r is matched with hospital h , the resident and the hospital gain utilities of $u_r(h)$ and $u_h(r)$, respectively. For simplicity, we make the following assumptions throughout the paper.

Assumption 1. The utilities of residents and hospitals satisfy the following conditions:

- (1) $u_r(h) \geq 0$ and $u_h(r) \geq 0$ for all $(r, h) \in R \times H$;
- (2) $u_r(h) \neq u_r(h')$ for all $(r, h, h') \in R \times H \times H$ with $h \neq h'$; and
- (3) $u_h(r) \neq u_h(r')$ for all $(r, r', h) \in R \times R \times H$ with $r \neq r'$.

Accordingly, the resident-hospital pair (r, h) yields the utility of $u_{rh} := u_r(h) + u_h(r)$, and the total utility of a matching $\mathbf{x} \in \{0, 1\}^{|R| \times |H|}$ is given by

$$\sum_{r \in R} \sum_{h \in H} u_{rh} x_{rh}. \quad (1)$$

The assignment game model [34] aims at finding a matching that maximizes the total utility (1). This model can be written as the following integer optimization problem:

$$\text{maximize} \quad \sum_{r \in R} \sum_{h \in H} u_{rh} x_{rh} \quad (2)$$

$$\text{subject to} \quad \sum_{h \in H} x_{rh} \leq 1 \quad (\forall r \in R), \quad (3)$$

$$\sum_{r \in R} x_{rh} \leq q_h \quad (\forall h \in H), \quad (4)$$

$$x_{rh} \in \{0, 1\} \quad (\forall r \in R, \forall h \in H). \quad (5)$$

Note that the binary constraint (5) is imposed for the sake of consistency throughout this paper. Even if this constraint is relaxed to a non-negativity constraint (i.e., $x_{rh} \geq 0$), all optimal basic solutions to the assignment game model are binary valued [22].

2.2 Bi-objective Optimization Model for Many-to-one Matching

For a matching $\mathbf{x} \in \{0, 1\}^{|R| \times |H|}$, let $m_{\mathbf{x}}(r) \in H$ be the hospital to which resident r is assigned, and $M_{\mathbf{x}}(h) \subseteq R$ be the set of residents that are assigned to hospital h as given below:

$$m_{\mathbf{x}}(r) = h \iff x_{rh} = 1,$$

$$M_{\mathbf{x}}(h) := \{r \in R \mid x_{rh} = 1\}.$$

We also define the preference orders $>_r$ and $>_h$ of resident r and hospital h based on their utilities as

$$h >_r h' \iff u_r(h) > u_r(h'),$$

$$r >_h r' \iff u_h(r) > u_h(r').$$

Note that these are strict linear orders from Assumption 1.

Suppose that there is a resident–hospital pair (r, h) such that resident r is not matched with hospital h but the pair elements prefer each other. In this case, they can form a private arrangement outside of the matching. Such a pair is called a blocking pair, which is formally defined as follows.

Definition 2. Let $x \in \{0, 1\}^{|R| \times |H|}$ be a matching. A pair $(r, h) \in R \times H$ is a blocking pair for x if the following conditions are fulfilled:

- (1) resident r is not assigned to hospital h (i.e., $x_{rh} = 0$);
- (2) resident r is unassigned or prefers hospital h to $m_x(r)$;
- (3) hospital h is undersubscribed or prefers resident r to at least one member of $M_x(h)$.

Since the individual rationality condition is fulfilled from Assumption 1 for all $(r, h) \in R \times H$, a stable matching is characterized by the absence of blocking pairs as follows.

Definition 3. A matching $x \in \{0, 1\}^{|R| \times |H|}$ is stable if it does not have any blocking pairs.

Baïou and Balinski [3] mentioned (without proof) that stable matchings to a many-to-one matching problem can be characterized by the following linear inequalities:

$$q_h x_{rh} + q_h \sum_{j>,h} x_{rj} + \sum_{i>,r} x_{ih} \geq q_h \quad (\forall r \in R, \forall h \in H). \quad (6)$$

For the sake of completeness, we provide a proof of this statement.

Theorem 1 (Baïou and Balinski [3]). Suppose that $x \in \{0, 1\}^{|R| \times |H|}$ is a matching. Then, it is stable if and only if constraint (6) is satisfied.

Proof. We prove the theorem by contraposition:

constraint (6) is violated

$$\iff \exists (r, h) \in R \times H; x_{rh} = 0, \sum_{j>,h} x_{rj} = 0, \text{ and } \sum_{i>,r} x_{ih} \leq q_h - 1$$

$$\iff \exists (r, h) \in R \times H; (r, h) \text{ is a blocking pair for } x$$

(\because Definition 2)

$$\iff x \text{ is not stable. } (\because \text{Definition 3}) \quad \square$$

We use the characterization (6) of stability to minimize the number of blocking pairs. Let us introduce the binary decision variable $w := (w_{rh})_{(r,h) \in R \times H} \in \{0, 1\}^{|R| \times |H|}$. We then make use of the following constraint:

$$q_h x_{rh} + q_h \sum_{j>,h} x_{rj} + \sum_{i>,r} x_{ih} \geq q_h (1 - w_{rh}) \quad (\forall r \in R, \forall h \in H).$$

Note that this constraint is the same as the characterization (6) of stability when $w_{rh} = 0$. Therefore, $w_{rh} = 1$ indicates that (r, h) is a blocking pair, so the number of blocking pairs is given by

$$\sum_{r \in R} \sum_{h \in H} w_{rh}. \quad (7)$$

We consider maximizing the total utility and minimizing the number of blocking pairs at the same time. Our bi-objective optimization model for many-to-one matching is formulated as follows:

$$\text{minimize} \quad \left(- \sum_{r \in R} \sum_{h \in H} u_{rh} x_{rh}, \sum_{r \in R} \sum_{h \in H} w_{rh} \right) \quad (8)$$

$$\text{subject to} \quad \sum_{h \in H} x_{rh} \leq 1 \quad (\forall r \in R), \quad (9)$$

$$\sum_{r \in R} x_{rh} \leq q_h \quad (\forall h \in H), \quad (10)$$

$$q_h x_{rh} + q_h \sum_{j>,h} x_{rj} + \sum_{i>,r} x_{ih} \geq q_h (1 - w_{rh}) \quad (\forall r \in R, \forall h \in H), \quad (11)$$

$$x_{rh} \in \{0, 1\} \quad (\forall r \in R, \forall h \in H), \quad (12)$$

$$w_{rh} \in \{0, 1\} \quad (\forall r \in R, \forall h \in H). \quad (13)$$

2.3 Tri-objective Optimization Model for Matching in Groups

We now address the problem of matching in groups [19]. Let us suppose that there are some disjoint predetermined groups of residents, $R_g \subseteq R$ for $g \in G$ (e.g., couples, friends, and partners). Residents in the same group want to be assigned to the same hospital. To fulfill their hopes, we define a target number $\beta \in \mathbb{Z}_+$ of residents. We then attempt to ensure that even if resident group R_g is divided into some subgroups, the number of residents in each subgroup will be β or more.

To this end, we introduce the binary decision variable $z := (z_{gh})_{(g,h) \in G \times H} \in \{0, 1\}^{|G| \times |H|}$ and the nonnegative decision variable $s := (s_{gh})_{(g,h) \in G \times H} \in \mathbb{R}_+^{|G| \times |H|}$ that indicates a shortage of subgroup members. For the purpose of matching in groups, we make use of the following constraint:

$$\beta z_{gh} - s_{gh} \leq \sum_{r \in R_g} x_{rh} \leq |R_g| z_{gh} \quad (\forall g \in G, \forall h \in H).$$

The right-hand inequality means that if at least one member $r \in R_g$ is assigned to hospital h (i.e., $\sum_{r \in R_g} x_{rh} \geq 1$), then we must have $z_{gh} = 1$. From the left-hand inequality, the number of corresponding subgroup members (i.e., $\sum_{r \in R_g} x_{rh}$) at hospital h must be $\beta - s_{gh}$ or more. We then minimize the *small-subgroup penalty* $\sum_{g \in G} \sum_{h \in H} (s_{gh})^2$, which is the sum of squared shortages of subgroup members relative to the target number β .

Our tri-objective optimization model for matching in groups is formulated as follows:

$$\text{minimize} \quad \left(- \sum_{r \in R} \sum_{h \in H} u_{rh} x_{rh}, \sum_{r \in R} \sum_{h \in H} w_{rh}, \sum_{g \in G} \sum_{h \in H} (s_{gh})^2 \right) \quad (14)$$

$$\text{subject to} \quad \sum_{h \in H} x_{rh} \leq 1 \quad (\forall r \in R), \quad (15)$$

$$\sum_{r \in R} x_{rh} \leq q_h \quad (\forall h \in H), \quad (16)$$

$$q_h x_{rh} + q_h \sum_{j>,h} x_{rj} + \sum_{i>,r} x_{ih} \geq q_h (1 - w_{rh}) \quad (\forall r \in R, \forall h \in H), \quad (17)$$

$$\beta z_{gh} - s_{gh} \leq \sum_{r \in R_g} x_{rh} \leq |R_g| z_{gh} \quad (\forall g \in G, \forall h \in H), \quad (18)$$

$$x_{rh} \in \{0, 1\} \quad (\forall r \in R, \forall h \in H), \quad (19)$$

$$w_{rh} \in \{0, 1\} \quad (\forall r \in R, \forall h \in H), \quad (20)$$

$$z_{gh} \in \{0, 1\} \quad (\forall g \in G, \forall h \in H), \quad (21)$$

$$s_{gh} \geq 0 \quad (\forall g \in G, \forall h \in H). \quad (22)$$

2.4 ε -constraint Method

We use the ε -constraint method [17] to convert a multi-objective optimization model into a scalar objective optimization model. This method retains one of the objectives and imposes upper-bound constraints on the rest of the objectives.

By means of the ε -constraint method, our tri-objective optimization model (14)–(22) can be reduced to the following scalar objective mixed-integer optimization problem:

$$\text{maximize} \quad \sum_{r \in R} \sum_{h \in H} u_{rh} x_{rh} \quad (23)$$

$$\text{subject to} \quad \sum_{r \in R} \sum_{h \in H} w_{rh} \leq \varepsilon_1, \quad (24)$$

$$\sum_{g \in G} \sum_{h \in H} (s_{gh})^2 \leq \varepsilon_2, \quad (25)$$

$$\text{Eqs. (15)–(22)}, \quad (26)$$

where $\varepsilon_1, \varepsilon_2 \in \mathbb{Z}_+$ are user-defined upper-bound parameters for the number of blocking pairs and the small-subgroup penalty, respectively.

3. Simulation Experiments

This section evaluates the effectiveness of our method through simulation experiments.

3.1 Experimental Design

For our simulation, we generated the utilities of residents and hospitals according to a linear model [14], [16], [23]. From a uniform distribution, we randomly produced a common vector $v \in [0, 1]^{|H|}$ and individual vector $v^{(r)} \in [0, 1]^{|H|}$ for each resident $r \in R$. We then calculated residents' utilities as

$$(u_r(h))_{h \in H} = \alpha v + (1 - \alpha)v^{(r)} \quad (\forall r \in R),$$

where $\alpha \in [0, 1]$ is a user-defined parameter. As α decreases, residents' preferences become more diverse. We tested $\alpha \in \{0.3, 0.6\}$ following up on the previous studies [14], [23]. We also randomly produced hospitals' utilities $u_h(r) \in [0, 1]$ from a uniform distribution for all $(r, h) \in R \times H$.

We compare the performance of the following matching methods.

BO: our bi-objective optimization model (8)–(13);

TO(β, ε_2): our tri-objective optimization model (14)–(22), where $\beta \in \mathbb{Z}_+$ is the target number of subgroup members, and $\varepsilon_2 \in \mathbb{Z}_+$ is the upper bound on the small-subgroup penalty;

DA: deferred acceptance algorithm [15], [29];

TTC: Gale's top trading cycles algorithm [33].

For our multi-objective optimization models, the scalar objective optimization problem (23)–(26) was solved for each $\varepsilon_1 = 0, 1, 2, \dots$ using the Gurobi Optimizer*¹ 9.0.0. After that, we removed solutions that are not Pareto-optimal. Note here that constraint (15) was replaced with an equality constraint ($\sum_{h \in H} x_{rh} = 1$) so that matching will not be prevented by the small-subgroup

penalty even when ε_2 is very small. The resident-proposing versions of DA and TTC algorithms were implemented in the Python programming language.

3.2 Results of Many-to-one Matching

We compare the performance of our bi-objective optimization model (8)–(13) with those of the DA and TTC algorithms for many-to-one matching problems. We considered $(|R|, |H|) \in \{(24, 6), (100, 10)\}$ as the numbers of residents and hospitals. We set the number of residency positions as $q_h = |R|/|H|$ for all $h \in H$. The computed results were averaged over ten trials.

Figure 1 shows the total utility (1) and the number (7) of blocking pairs in the matchings obtained by each method, where only the solutions with $\varepsilon_1 = 0, 5, 10, \dots$ are displayed in Fig. 1 (c) and Fig. 1 (d). We can see that our method (BO) provided a variety of good matchings, which depended on values of the upper-bound parameter ε_1 . Specifically, we note that BO found stable matchings, which have no blocking pairs, for all problem instances. Additionally, BO could improve the total utility at the expense of stability; Fig. 1 (d) shows that the total utility was increased from 138.1 to 144.8 by permitting 71.7 blocking pairs. Conversely, BO can greatly reduce the number of blocking pairs by permitting a lower total utility as shown by the two points (71.7, 144.8) and (20.0, 143.3) in Fig. 1 (d). These points imply that the number of blocking pairs decreased by 72% from 71.7 to 20.0, whereas the total utility decreased by only 1% from 144.8 to 143.3.

DA also found stable matchings for all problem instances; however, BO had higher total utility without increasing the number of blocking pairs from zero. Figure 1 (a) is a notable example in that DA had a total utility of 33.0 but BO increased this to 33.9 without admitting any blocking pairs. In contrast with BO and DA, the TTC algorithm generated matchings of poor quality in terms of both the total utility and number of blocking pairs.

3.3 Results of Matching in Groups

We examine the effectiveness of our tri-objective optimization model (14)–(22) for matching in groups. We set the numbers of residents and hospitals as $(|R|, |H|) = (24, 6)$. We considered $|G| \in \{2, 6\}$ as the number of resident groups, where each group was composed of the same number of residents (i.e., $|R_g| = |R|/|G|$ for all $g \in G$). We set the number of residency positions as $q_h = |R|/|H| = 4$ for all $h \in H$.

As for our method TO(β, ε_2), we employed $\beta \in \{2, 4\}$ as the target number of subgroup members. We also set the upper-bound parameter as $\varepsilon_2 = 0$ (i.e., $s_{gh} = 0$ for all $(g, h) \in G \times H$) so that the number of subgroup members will always be β or more. When the value of ε_1 was so small that the optimization problem (23)–(26) was infeasible, we instead used the smallest ε_1 such that the problem was feasible. The computed results were averaged over ten trials.

Figure 2 shows the total utility (1) and the number (7) of blocking pairs in the obtained matchings. We firstly focus on the two-group case $|G| = 2$ (i.e., Fig. 2 (a) and Fig. 2 (b)). We can see that TO(2, 0) performed similarly to BO despite the existence of resident groups. Additionally, TO(4, 0) verified that even when $\beta = 4$, the total utility can be increased to more than 30, and the

*1 www.gurobi.com

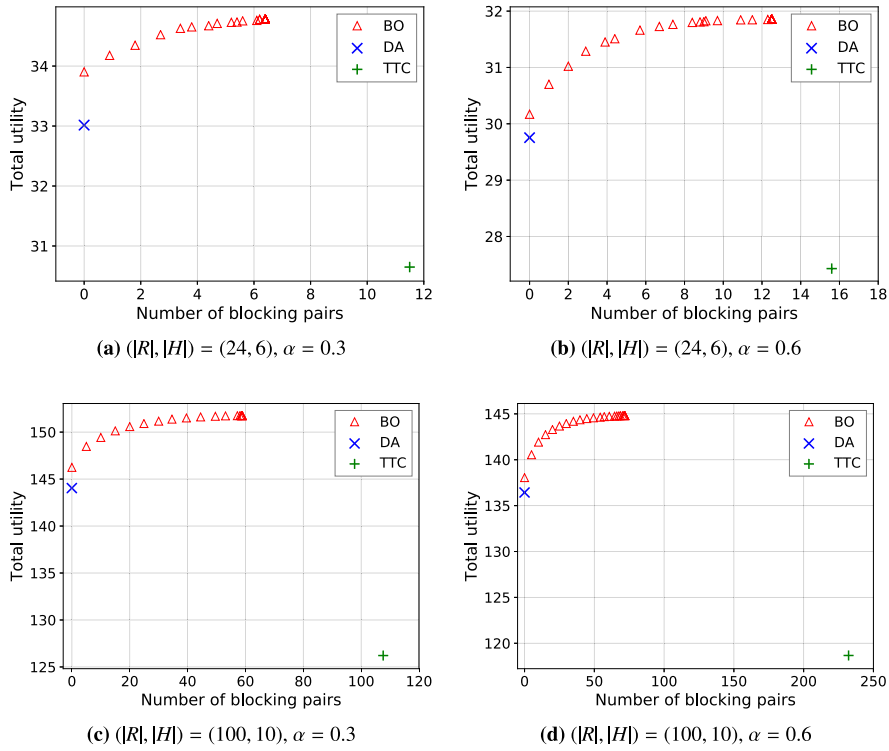


Fig. 1 Results of many-to-one matching.

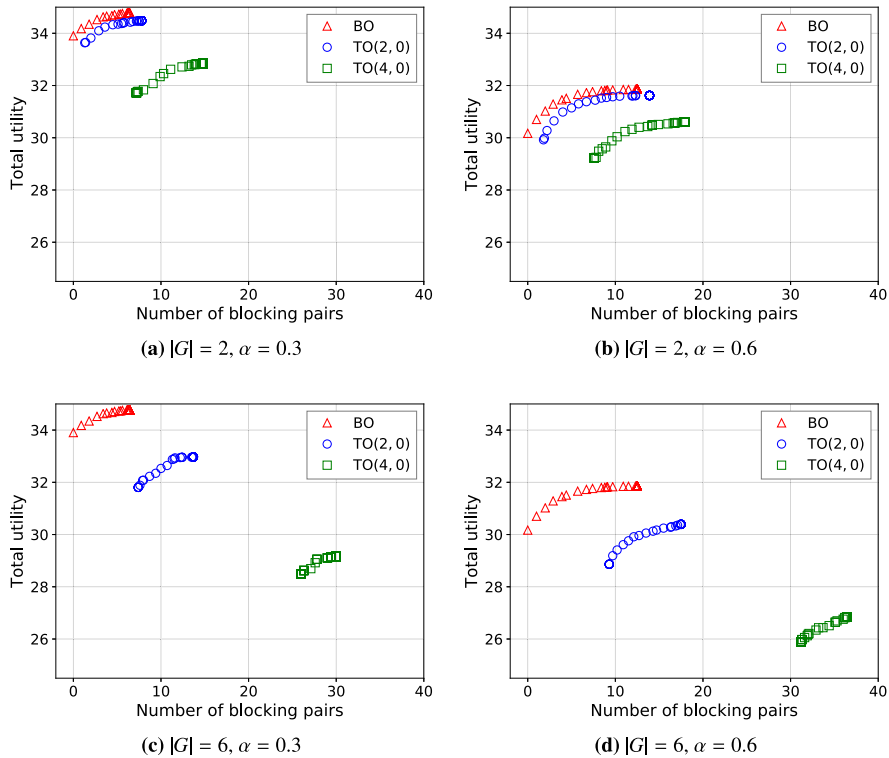


Fig. 2 Results of matching in groups with $(|R|, |H|) = (24, 6)$.

number of blocking pairs can be reduced to less than 10. However, TO(2,0) and TO(4,0) did not reduce the number of blocking pairs to zero thus confirming that finding a stable matching when residents had to be matched in groups was impossible.

We next consider the case $|G| = 6$ (see Fig. 2 (c) and Fig. 2 (d)). Since the number of resident groups was increased, it became very difficult to fulfill their hopes about groups. Consequently,

the difference between BO and TO(2,0) was larger in this case than in the two-group case. We should also notice that TO(4,0) does not divide any resident groups because $|R_g| = \beta = 4$ for all $g \in G$. As a result, TO(4,0) has a small number of options for matchings. For this reason, the variety of matchings obtained by TO(4,0) was smaller in this case than in the two-group case.

We conclude this section by giving typical examples of match-

Table 1 Numbers of resident subgroup members for the smallest value of ε_1 with $(|R|, |H|) = (24, 6)$, $|G| = 6$, and $\alpha = 0.6$.

h	BO						TO(2, 0)						TO(4, 0)							
	Group g						Group g						Group g							
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6		
1	2	1	1	0	0	0	1	2	0	0	0	2	0	1	4	0	0	0	0	0
2	0	0	0	2	1	1	2	0	0	0	2	0	2	2	0	0	0	4	0	0
3	0	1	1	0	0	2	3	0	0	2	0	0	2	3	0	0	0	0	4	0
4	1	1	1	1	0	0	4	2	0	0	2	0	0	4	0	0	4	0	0	0
5	0	0	1	1	2	0	5	0	2	2	0	0	0	5	0	0	0	0	0	4
6	1	1	0	0	1	1	6	0	2	0	0	2	0	6	0	4	0	0	0	0

Table 2 Numbers of resident subgroup members for the largest value of ε_1 with $(|R|, |H|) = (24, 6)$, $|G| = 6$, and $\alpha = 0.6$.

h	BO						TO(2, 0)						TO(4, 0)							
	Group g						Group g						Group g							
	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6		
1	2	1	1	0	0	0	1	2	2	0	0	0	0	1	4	0	0	0	0	0
2	0	0	0	0	1	3	2	0	0	0	2	0	2	2	0	0	0	0	0	4
3	0	0	1	1	1	1	3	0	0	2	0	0	2	3	0	0	4	0	0	0
4	2	2	0	0	0	0	4	2	0	0	2	0	0	4	0	4	0	0	0	0
5	0	0	1	2	1	0	5	0	0	2	0	2	0	5	0	0	0	4	0	0
6	0	1	1	1	1	0	6	0	2	0	0	2	0	6	0	0	0	0	4	0

ing results obtained by our methods for the smallest and largest values of ε_1 . **Tables 1** and **2** list the numbers of resident subgroup members (i.e., $\sum_{r \in R_g} x_{rh}$) assigned to hospital $h \in H$ from group $g \in G$. Since BO does not take such resident groups into account, it developed many one-member subgroups. TO(2, 0) aggregated these one-member subgroups appropriately to form two-member subgroups. TO(4, 0) promoted further aggregation so that all subgroups were composed of four members.

4. Conclusion

This paper dealt with the many-to-one matching problems for assigning residents to hospitals according to the preferences of residents and hospitals. We proposed the bi-objective optimization model for maximizing the total utility and minimizing the number of blocking pairs. We also focused on the problem of matching in groups, where residents want to be matched in groups. For this purpose, we proposed the tri-objective optimization model that employs the small-subgroup penalty as the third objective. These multi-objective optimization models were formulated as scalar objective mixed-integer optimization problems using the ε -constraint method and solved by optimization software.

We demonstrated through simulation experiments that our method generated a variety of good matchings for many-to-one matching problems depending on the upper-bound parameter values. More importantly, our method visualizes the optimality trade-offs between the total utility and the number of blocking pairs, which is the most beneficial feature that the DA and TTC algorithms lack. We also confirmed that our method worked well for matching in groups. These results support the efficacy of optimization models for computing good-quality solutions to a variety of difficult matching problems.

We used the the optimization software based on the branch-and-bound algorithm. This algorithm consists of systematic enumeration of candidate solutions and terminates with a certificate proving optimality of the obtained solution. On the other hand, the evolutionary computation is capable of efficiently comput-

ing a set of near-optimal solutions to multi-objective optimization problems [9], [36]. The evolutionary computation will be required for large-scale problems that cannot be handled by the branch-and-bound algorithm.

A future direction of study will be to speed up solving our mixed-integer optimization problems. To this end, the stable admissions polytope [3], [32], which is the convex hull of stable matchings of a many-to-one matching problem, should prove effective in minimizing the number of blocking pairs. We will also consider another optimization formulation for matching in groups. For instance, it is possible to quantify residents' utilities for being matched in groups and incorporate them into optimization models.

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