セル理論　-実空間・サイバースペース融合の基本モデル-

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要旨

実空間とサイバー空間の融合空間上のサイバー世界は、e-business、e-finance、e-manufacturing 等として、急激に発展する現代実社会の基本的インフラストラクチャを形成している。発展の不変量基づく同値関係を見いだすと、それにより融合空間は直和同値類に自動的に分割される。サイバー世界の特徴に基づき、多様な同値関係が特定できる。セル構造空間として融合空間を規定することにより、同値類の商空間（等化空間）において実空間とサイバー空間を統合するセル接着関数を定義し、サイバー世界の変化の不変量をホモトピー不変量として保存する基本モデルを提示できる。これにより、サイバー世界の急激な発展の変化の本質をホモトピー不変量としてデータベース化することが可能となる。これにより融合空間の本質の理解を進め、かつその統合実現を系統的に進めることが可能となる。セルモデルは、この意味で、複雑な融合空間上の「サイバー世界管理システム」実現の基本モデルである。

Cellular Theory　- A Basic Model for Integrating Real- and Cyber Spaces -

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Abstract

Cyberworlds such as e-business, e-financing and e-manufacturing spanning in the integrated spaces of real- and cyber- spaces have been serving as the infrastructure of the fast evolving modern real worlds. Given an integrated space, it is automatically decomposed into a disjoint union of equivalence classes by finding its invariant via an equivalence relation. Equivalence relations characterize the nature of individual cyberworlds. By specifying integrated spaces as cellular structured spaces, we can define cell attaching functions that integrate real- and cyber- spaces in quotient spaces (identification spaces) of equivalence classes, and can derive a basic model that preserves the invariants of the rapid changes of cyberworlds as homotopy invariants. This serves to realize database systems to manage the essentials of the rapid evolution of cyberworlds. The cellular model clarifies the essential nature of integrated spaces and allows systematic integration. Cellular model is, in this sense, the basic model for realizing "cyberworlds management systems" in complex integrated spaces.

1. Epilogue

Cyberworlds are information worlds being formed on the web *either intentionally or spontaneously, with or without design*. Cyberworlds as information worlds are either *virtual* or *real*, and can be both. New worlds such as cyberworlds demand a theoretical ground to get them modeled properly. In terms of information modeling, the ground is far above the level of integrating spatial database models and temporal database models. To capture the characteristics of the information on the web that is dynamically changing, we take *invariants* as the ground. In science, invariants have been the core of scientific laws. For example, classical mechanics is built on mass and energy as invariants. On this, our approach is fundamentally different from the currently popular XML-based or semistructured data-based web information modeling [15]. Considering cyberworlds as a type of spaces that include time as an irreversible space, we show that an appropriate choice of invariants that consists of *dimensions* as *degrees of freedom* and their *connectivity* to tell how different dimensional spaces are connected. This paper is based on the previous publication of us[16] with major changes in Chapter 5 on a situation modeling of web information as non-inductive information schema integration and information integration based on a cellular model that is completely revised. The typos have been corrected. We have also developed varieties of applications of the cellular model and present them as a set of papers in Research Reports of the IPSJ DBS Technical Committee 126th Research Meeting, together with this paper as the major reference.

Generally speaking, what we need to do to model cyberworlds consists of the following four steps.

First, we characterize cyberworlds to identify the differences from and commonality with the real world we live. The most distinct difference is in the speed of growth, and hence in the complexity. This means extreme concurrency linking *local* worlds into *global* web worlds and also speed close to that of light. Light speed on the web signifies the web power far beyond any great powers in human history [1-4, 7]. Everybody working on the web in the world is a constructor and destructor of cyberworlds.

Secondly, we then find appropriate modeling methods to characterize the differences and commonality. Because of the extreme complexity and the speed of changes, the modeling methods need to be based on *a hierarchy of abstractions* to minimize the size of modeling, and also the hierarchy needs to be *an incrementally modular abstraction hierarchy of invariants* to identify the unchanging properties from the rapidly varying cyberworlds.

Third, we then turn the modeling methods into a design. It is a challenging task to realize such invariants-based modeling methods into one design. Generally, the design requires an appropriate choice of invariants, followed by a particular information structures and operations. For instance, an abstraction hierarchy of invariants is designed as an *inheritance hierarchy of invariants*. Still researches on this belong to open problems. So far, our researches have led us to a pair of invariants: *dimensions* as degrees of freedom and their *connectivity* via cell attachment identifying equivalence relations as invariants. The information structures are *cellular spatial structures* and their operations such as *cell composition* and *cell decomposition* [7, 8].

Fourthly and finally, we implement the design as an information model named cellular model. The cellular model encompasses the capabilities of existing various data models, and also guarantees the continuity to preserve cell boundaries, cell dimensionality and cell connectivity. It is expected that the cellular model represents cyberworlds consistently and proves their validity. The ways the cellular model works include bottom up, top down, and middle to top and bottom approaches.

2. An abstraction hierarchy of invariants

We need to confirm the way we are looking at information modeling. Modeling stands for a key step in scientific research. Science, natural science in particular, has been built around the notion of invariants to model the real world we live. Science models objects by classifying objects and phenomena by *invariants*. In physics, energy and mass had been invariants until the relativity theory broke the boundary. In mathematics, modeling of mathematical objects is conducted to classify mathematical objects into *equivalence classes* as a disjoint union of the subsets of objects by an *equivalence relation* that represents a mathematical invariant. An example of an abstraction hierarchy of equivalence relations is:

1 Set theoretical equivalence relations;
2 Extension equivalence relations, homotopy equivalence relations as a special case;
3 Topological equivalence relations, graph

theoretical equivalence relations as a special case;

4 Cellular spatial structure equivalence relations;

5 Information model equivalence relations;

6 View equivalence relations.

In terms of the abstraction of invariants hierarchically organized from general to specific to realize *modular and incremental design* and hence an *inheritance hierarchy of invariants* of cyberworlds, the following is a reasonable case of an abstraction hierarchy based on the abstraction hierarchy of equivalence relations in mathematics:

1 A set level;

2 An extension level, a homotopy level as a special case;

3 A topology level, a graph theoretical level as a special case;

4. A cellular structured space level;

5 An information model level;

6 A presentation level.

3. A cellular model

For cyberworlds modeling, "a cellular structured space level" based on *cellular spatial structures* [5-8, 14] such as CW-spaces gives a far more versatile basis than those based on a graph theoretical level that is common in conceptual- and data- modeling [9, 10], allowing an information model to specify objects in cognitive- and computational- spaces as cells with or without boundaries. Cells with boundaries are closed, and cells without boundaries are open. Here an *n*-dimensional cell, an *n-cell*, is a space topologically equivalent to an *n*-dimensional ball where *n* is an integer $\mathbb{Z}$, namely $n \in \mathbb{Z}$. We denote an open *n*-cell e^n and a closed *n*-cell $\mathcal{B}^n$. An *interior* of $\mathcal{B}^n$ is denoted as $Int\ \mathcal{B}^n = \mathring{B}^n$, and

$\partial\mathcal{B}^n = \mathcal{B}^n - \mathring{B}^n = S^{n-1}$

is the *boundary* of $\mathcal{B}^n$, and it is an (*n-1*)-dimensional *sphere* S^{n-1}. Cellular modeling allows *cell composition* and *decomposition* while maintaining cell dimensions and connectivity as invariants; object identification is carried out systematically through an *identification mapping* (often called a *quotient mapping*) [7]. Here, *dimensions* mean the *degrees of freedom*. Later we show that database schema composition (also called schema integration) and schema decomposition (also called schema disintegration) are special cases of cell composition and cell decomposition.

Let look at examples of *dimensions*. For instance, in cyberworlds an object with no attribute has no degree of freedom to change the attribute values and hence the dimension of the object of the cyberworlds is *0*, and we present it as a point at a presentation level. *Attributes* are mutually independent sets to specify qualities or characteristics inherent to objects. Given an object with one attribute, we can change the values of the attribute and hence the degree of freedom and the dimension is *1*; we can present this case as a line. Likewise, objects with two and three attributes have two and three degrees of freedom (dimensions) are *2* and *3*, and can present them as a surface and a ball. An object with *n* attributes has *n* degrees of freedom, and hence its dimension is *n*; we can present it as an *n*-dimensional ball. The relational model presents an object with n attributes as a relational schema and instantiates it as a table with *n* columns [11]. The relational model is based on Cartesian products of sets, and hence it is at the set theoretical level of the abstraction hierarchy.

The *connectivity* is defined by a continuous and surjective mapping called an *attaching map* (an *adjunction map*, an *adjoining map* or a *gluing map*). "A map f: X → Y is *surjective*" means $(\forall y \in Y)\ (\exists x \in X)\ [\ f(x) = y\ ]$. "A map f: X → Y is *continuous*" means "a subset $A \subset Y$ is open in Y if and only if $\{f^{-1}(y) \mid y \in A\}$ is open in X".

Given two disjoint topological spaces X and Y,

$$Y \sqcup_f X = Y \sqcup X / \sim$$

is an attaching space (an adjunction space, or an adjoining space) obtained by *attaching* (*gluing*, *adjuncting*, or *adjoining*) X to Y by an *attaching map* (*adjunction map*, or an *adjoining map*) f (or by identifying points $x \in X_0 \mid X_0 \subset X$ with their images $f(x) \in Y$, namely by a surjective map f)

$$f:\ X_0 \to Y.$$

$\sqcup$ denotes a disjoint union and often a + symbol is used instead (sometimes it is called an "exclusive or"). $\sim$ is an equivalence relation. An *equivalence relation* is simply a relation that is reflexive, symmetric and transitive. It can be a set theoretical equivalence relation, a topological equivalence relation, a geometrical equivalence relation or a homotopic equivalence relation. The transitivity divides the space naturally into a disjoint union of subspaces called *equivalence* *classes*.

Let us look into equivalence relations and equivalence relations here a little bit in more detail as a foundation to model cyberworlds clearly. For a binary relation $R \subseteq X \times X$ on a set X, R is:

reflexive if $(\forall x \in X)$ [xRx]: *reflexivity*;

symmetric if $(\forall x, y \in X)$ [$xRy \Rightarrow yRx$]:*symmetry*;

transitive if $(\forall x, y, z \in X)$ [[$xRy \Rightarrow yRz$] $\Rightarrow$ xRz]:*transitivity*.

R is called an *equivalence relation* (in a notation $\sim$) if R is reflexive, symmetric and transitive.

A subset of X defined by $x / \sim\, = \{y \in X: x \sim y\}$ is called the *equivalence class* of x. Here a class actually means a set; it is a tradition, and hard to be changed at this stage. The set of all the equivalence classes $X/\sim$ is called the *quotient space* or the *identification space* of X.

$$X / \sim\, = \{x / \sim\, \in 2^X \mid x \in X\} \subseteq 2^X.$$

From the transitivity, for each $x \in X$, $x /\sim\, \neq \phi$, the followings hold:

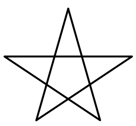
$x \sim y \Leftrightarrow x /\sim\, = y /\sim$, and

$x \nsim y \Leftrightarrow x /\sim\, \cap y /\sim\, = \phi$.

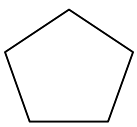
This means a set X is *partitioned* (also called *decomposed*) into non-empty and *disjoint* equivalence classes. If we denote an equivalence class by $x / \sim$, it is, then,

$x / \sim\, = \{y \in X \mid x \sim y\}$.

Let us look at simple examples. *Cardinality* is a case of set theoretical equivalence relations, and divides sets into a disjoint union of the sets of the same cardinality. In graph theory, *isomorphism* is an equivalence relation, and divides a set of graphs into a disjoint union of *isomorphic* graphs. Popular isomorphic graphs are:



and .



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In Euclidean geometry, given a set of figures, a *congruence relation* divides them into a disjoint union of the subsets of congruent figures as a quotient space; a *similarity* relation divides them into a disjoint union of the subsets of similar figures as a quotient space. Congruence and similarity relations are cases of affine transformations. A *symmetry* relation in group theory divides a set of figures into a disjoint union of the subsets of symmetric figures as a quotient space. In finite state automata, an *accepted language* defined as the set of all strings accepted by automata serves as an equivalence relation and divides the automata into a disjoint union of equivalent automata.

We have already seen a case of an attaching map already. Now, let us state a general definition of an attaching map here. The set of all equivalence classes is denoted as $X / \sim$, and is called the *quotient space* or the *identification space* of X

$$X / \sim\, = \{ x / \sim\, \in 2^X \mid x \in X\} \subseteq 2^X.$$

An *attaching map* f is a surjective (onto) and continuous map

$f: X_0 \rightarrow Y$,

where $X_0 \subset X$.

$X \sqcup Y / \sim$ is a quotient space, and

$X \sqcup Y / \sim\, = X \sqcup Y / (x \sim f(x) \mid \forall x \in X_0) = X \sqcup_f Y$.

Here is a special case for later use for information schema integration and information integration by information mining on the web. Let S^{n-1} be the boundary of a closed *n*-cell $\mathcal{B}^n$, namely $\partial\mathcal{B}^n$. That is,

$$S^{n-1} = \partial\, \mathcal{B}^n = \mathcal{B}^n - Int\, \mathcal{B}^n = \mathcal{B}^n - e^n.$$

Let an attaching map f be a surjective (onto) and continuous map

$f: S^{n-1} \rightarrow X$.

An adjunction space Y is defined as a quotient space

$$Y = X \sqcup_f \mathcal{B}^n = X \sqcup \mathcal{B}^n / \{f(u) \sim u \mid u \in S^{n-1}\}.$$

Given two homotopic maps f and g

$f, g: S^{n-1} \rightarrow X$,

then $X \sqcup_f \mathcal{B}^n$ and $X \sqcup_g \mathcal{B}^n$ have the same *homotopy type* (or, are *homotopically equivalent*)

$$X \sqcup_f \mathcal{B}^n \simeq X \sqcup_g \mathcal{B}^n.$$

Given a cyberworld X as a topological space, from X, we can *inductively* compose, according to J. H. C. Whitehead [12, 13], a finite or infinite sequence of cells X^p that are subspaces of X, indexed by integer $\mathbb{Z}$, namely $\{ X^p \mid X^p \subseteq X, p \in \mathbb{Z}\}$ called a *filtration*, such that

X^p covers X (or X^p is a *covering* of X), namely,

$$X = \bigcup_{p \in \mathbb{Z}} X^p,$$

and X^{p-1} is a subspace of X^p namely,

$$X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{p-1} \subseteq X^p \subseteq \ldots \subseteq X.$$

(this is called a *skeleton*). The skeleton with a dimension at most p is called a *p*-skeleton. X^0, X^1, X^2, …, X^{p-1}, and X^p are sub-cyberworlds of a given cyberworld X. A space topologically equivalent to a filtration is called a *filtration space*.

There are important cellular spaces in applications. They include CW-complexes and manifolds. If a filtration space is finite, it is equivalent to a *CW-space*. Further, if a CW-space is diffeomorphic, it is equivalent to a *manifold space*.

4. Web information modeling, inductive web-information schema integration, and web-information integration via information mining based on a cellular model

The first thing we have to do in web information modeling is the characterization of the nature of the formation of cyberworld as shared information worlds on the web to see how the cyberworlds are emerging and what they are. It is often the case that a cyberworld X is created on the web as a result of varieties of local activities at many web sites. Unlike corporate information, we usually cannot assume that there is an information administrator to give us the initial set of schemas. Through *information mining*, we can dig up particular information at local web sites to reveal what X is. Of course, we do no conduct information mining arbitrarily. After information browsing at web sites, we gather idea on what to be mined and what are expected to emerge from the information at scattered sites on the web by integrating them. This is a type of information mining to be called generally "*information mining by design*" because there is a certain set of rules to be applied as *integration guides* regarding what to be mined and what not to be mined. Such *integration guides* work as *design guides* of what to be integrated and how.

Information mining on the web works perfectly to integrate local web worlds to a global cyberworld, based on the *Whitehead inductive scheme* stated above. To illustrate more concretely how the *inductive integration* goes to obtain an n-dimensional cyberworld X^n, we explain web search and integration processes.

The *inductive integration* consists of two phases: the information schema integration phase and the information integration phase. The first phase, the *information schema integration* phase, proceeds as follows:

1. Retrieve every cell $\mathcal{B}^0_i$ of interest with no attribute at web sites to create a 0-dimensional cyberworld X^0 such that
   $X^0 = \{ \mathcal{B}^0_1, \mathcal{B}^0_2, \mathcal{B}^0_3, \ldots, \mathcal{B}^0_j \}$.
2. Retrieve every combination of cells $\mathcal{B}^1_i$ with one attribute of *interest* at web sites to create a 1-dimensional cyberworld X^1 such that and we attach their disjoint union
   $$\sqcup_i \mathcal{B}^1_i = \mathcal{B}^1_1 \sqcup \mathcal{B}^1_2 \sqcup \mathcal{B}^1_3 \sqcup \ldots \sqcup \mathcal{B}^1_k$$
   to X^0 via an attaching map F by identifying each boundary element (in this case an attribute of $\mathcal{B}^1_i$) $x \in \partial\mathcal{B}^1_i$ of a 1-cell $\mathcal{B}^1_i$ with an attribute in F(x). Then, we obtain a valid 1-dimensional cyberworld X^1 such that
   $$X^1 = X^0 \sqcup_F (\sqcup_i \mathcal{B}^1_i) = X^0 \sqcup (\sqcup_i e^1_i)$$
   where $i = 1, \ldots, k$, and an attaching map F is
   $$F: \sqcup_i \partial\mathcal{B}^1_i \to X^0.$$
3. Suppose, after repeated retrievals and integrations, we have dug up an $(n-1)$-dimensional cyberworld X^{n-1} through information mining. X^{n-1} has $n-1$ attributes. To integrate an n-dimensional cyberworld X^n that has n attributes, we retrieve every combination of n-cells $\mathcal{B}^n_i$ with $n+1$ attributes of interest as before at web sites. Then we attach their disjoint union
   $$\sqcup_i \mathcal{B}^n_i = \mathcal{B}^n_1 \sqcup \mathcal{B}^n_2 \sqcup \mathcal{B}^n_3 \sqcup \ldots \sqcup \mathcal{B}^n_m$$
   to the already build $(n-1)$-dimensional cyberworld X^{n-1} via an attaching map G by *identifying* each boundary element $(n-1)$-attributes out of n attributes of $\mathcal{B}^n_i$) $x \in \partial\mathcal{B}^n_i$ of an n-cell $\mathcal{B}^n_i$ with $n-1$ attributes in G(x). Then, we obtain a valid n-dimensional cyberworld X^n such that
   $$X^n = X^{n-1} \sqcup_G (\sqcup_i \mathcal{B}^n_i) = X^{n-1} \sqcup (\sqcup_i e^n_i)$$
   where $i = 1, \ldots, k$, and an attaching map G is
   $$G: \sqcup_i \partial\mathcal{B}^n_i \to X^{n-1}.$$

This completes the information schema integration.

$\sqcup_i \sqcup \mathcal{B}_{m_i}^{r_i}$ The second phase, the *information integration* phase, is fairly simple but computationally intensive. It proceeds in checking every instance at each step of cell attachment during the schema integration to judge and decide, based on the design guides, whether the instance should be included in the cyberworld being created by cell attachment.

The cyberworld we construct based on the *Whitehead inductive scheme* guarantees the following relation to hold:

$$X^0 \subseteq X^1 \subseteq X^2 \subseteq \ldots \subseteq X^{n-1} \subseteq X^n \subseteq \ldots \subseteq X.$$

From a cyberworld validation point of view, this means given a validated cyberworld, any cyberworlds having the lower dimensions are included in the given cyberworld and valid.

In the above, *identification* is by equivalence classes based on equivalence relations. Clearly "*identification by equivalence classes*" is a *generalization of a join operation* in the relational model [11]. Hence one of the real powers of a cellular model is seen on this aspect. Having highly complex and fast changing cyberworlds on the web, the integration power of cellular model provides web information model with a true theoretical foundation. Also when we said "attributes of interest" to exercise the design guides, "*interest*" means, at least partially, the *choice of equivalence relations for identification*. So "*the choice of equivalence relations for identification*" is the major part of *design guides*. For web-based information systems, *design guides* are either local to govern local sites as intranets (also as community nets) or global to work in borderless cyberworlds. Design guides are a type of *reusable resources* of web-based information systems.

5. A situation modeling of web information as non-inductive information schema integration and information integration based on a cellular model

On the web, usually we encounter with situations where we need to create new cyberworlds from given cyberworlds. This situation is more common in e-business including e-commerce on the web than the previous situation that carries out information mining through induction. Let us look at e-commerce situations to *model web situations* that vary in space and time. There are general needs to find out *e-commerce trading structures on the web* in terms of *information schemas* to build an e-commerce information system. Typical e-commerce situations include:

- Situation 1. An e-customer wishing to buy an e-merchandise browses the web to find an e-shop that offers the best price;
- Situation 2. An e-shop selling e-merchandises browses e-customer lists to expand the sales.

On the web, in the above situations, we are *not* interested in finding out the precise information of an e-shop, an e-customer and an e-merchandise. Let an e-shop, an e-customer and an e-merchandise be p-, q- and q- dimensional cyberworlds denoted as cells $\mathcal{B}_s^p$, $\mathcal{B}_c^q$ and $\mathcal{B}_m^r$.

Situation 1: In Situation 1, an e-customer identifies the merchandise name of an e-shop with that of interest of the e-customer when the e-merchandise has the best price at the e-shop. In this situation the *information schema* is formed by cell decomposition operations followed by identification operations.

A *cell decomposition operation* is a map f

f: $\mathcal{B}^n \rightarrow \sqcup_i \sqcup_{g_i} \mathcal{B}^{u_i}$

that maps a given n-dimensional cell $\mathcal{B}^n$ to a disjoint union of cells $\sqcup_i \sqcup_{g_i} \mathcal{B}^{u_i}$ ($\Sigma_i u_i = n$) such that the attaching map g_i is preserved. The attaching map g_i here is a surjective and continuous identification function

g_i: $\partial\ldots\partial\mathcal{B}^n \rightarrow \mathcal{B}^{u_i}$,

where ∂ is repeated $n - u_i$ times. In general, let us call $\partial\ldots\partial\mathcal{B}_m^{r-t}\mathcal{B}^n$ where ∂ is repeated m times the *m-th order boundary* of $\mathcal{B}^n$. As we elaborate later, attaching map preservation at each cell decomposition is to make cell decomposition homotopic.

What we do now in this Situation 1 is to come out with a *situation model* of the Situation 1 as follows:

1. Cell decomposition: Decompose an s-cell $\mathcal{B}_s^p$ as an e-shop, a c-cell $\mathcal{B}_c^q$ as an e-customer and an m-cell $\mathcal{B}_m^r$ as an e-merchandise such that we separate equivalent cells $\mathcal{B}_a^t$ (where $a = s, c, m$) from the rests to identify e-commerce trading related attributes. The cell decomposition operations are:

   f_s: $\mathcal{B}_s^p \rightarrow \mathcal{B}_s^{p-t} \sqcup_g \mathcal{B}_s^t$,

   f_c: $\mathcal{B}_c^q \rightarrow \mathcal{B}_c^{q-t} \sqcup_h \mathcal{B}_c^t$, and

   f_m: $\mathcal{B}_m^r \rightarrow \mathcal{B}_m^{r-t} \sqcup_k \mathcal{B}_m^t$,

   where g, h and k are the identification functions such that

   g: $\partial\ldots\partial\mathcal{B}_s^p \rightarrow \mathcal{B}_s^t$ (∂ is repeated $p - t$ times),

   h: $\partial\ldots\partial\mathcal{B}_c^q \rightarrow \mathcal{B}_c^t$ (∂ is repeated $q - t$ times), and

   k: $\partial\ldots\partial\mathcal{B}_m^r \rightarrow \mathcal{B}_m^t$ (∂ is repeated $r - t$ times)

   to preserve the homotopy.

   For example, if the attributes are a merchandise name, a merchandise identifier and merchandise price, $\mathcal{B}_a^t$ is $\mathcal{B}_a^3$.
2. Cell composition via cell attachment: To a c-cell $\mathcal{B}_c^q$ as an e-customer, attach an m-cell $\mathcal{B}_m^r$ as an e-merchandise and an s-cell $\mathcal{B}_s^p$ as an e-shop, via attaching maps

   p_m: $\partial\ldots\partial\mathcal{B}_c^q \rightarrow \mathcal{B}_m^t$ (∂ is repeated $q - t$ times), and

p_s: $\partial\ldots\partial\mathcal{B}_c^q \to \mathcal{B}_s^t$ (∂ is repeated q - t times)

by identifying equivalent cells $\mathcal{B}_a^t$ (where a = s, c, m).

Now, since the *information schema* is composed as the *e-commerce trading structures on the web*, we can search the attribute 'merchandise price' for the lowest value in the e-commerce trading related attributes $\mathcal{B}_s^t$ of one e-shop $\mathcal{B}_s^p$ after another.

Situation 2: In Situation 2, the e-merchandises being and to be handled by the e-shop $\mathcal{B}_s^p$ are of varied types in their attributes and hence are in their cellular structures. We first conduct web search of e-customers $\mathcal{B}_c^q$ who post on the web the purchase merchandise candidate lists $\mathcal{B}_{m_i}^{r_i}$ (that usually gives only partial information on the merchandises).

1. Cell decomposition: We decompose the searched e-customers $\mathcal{B}_c^q$ into the purchase merchandise candidate lists $\sqcup_i\mathcal{B}_{m_i}^{r_i}$ and the rest via a cell decomposition operation f_c

   f_c: $\mathcal{B}_c^q \to \mathcal{B}_c^{q-t} \sqcup_g (\sqcup_i\mathcal{B}_{m_i}^{r_i})$

   where t $=\Sigma_i r_i$, and g

   g: $\partial\ldots\partial\mathcal{B}_c^q \to \sqcup_i \mathcal{B}_{m_i}^{r_i}$ (∂ is repeated q - t times)

   is the identification function to preserve the homotopy against the cell deformation via a cell decomposition operation. Apparently, the e-merchandises on the e-customer purchase merchandise candidate list $\mathcal{B}_{m_i}^{r_i}$ are the subset of the e-merchandises on the web $\sqcup_i\mathcal{B}_{m_i}^{r_i} \subseteq \mathcal{B}_m^r$ where $\Sigma_i r_i \leq r$.

2. Cell composition via cell attachment: Attach the searched e-customers $\mathcal{B}_c^q$ to e-merchandises $\mathcal{B}_m^r$ identifying the attributes of e-merchandises $\mathcal{B}_m^r$ equivalent to those of the purchase candidate merchandises $\sqcup_i\mathcal{B}_{m_i}^{r_i}$ via an identification map (quotient map) f_m such that

   f_m: $\partial\ldots\partial\mathcal{B}_c^q \to \partial\ldots\partial\mathcal{B}_m^r$ /~

   (∂ is repeated q $-\Sigma_i r_i$ times in $\partial\ldots\partial\mathcal{B}_c^q$ and r $-\Sigma_i r_i$ times in $\partial\ldots\partial\mathcal{B}_m^r$, and $\partial\ldots\partial\mathcal{B}_m^r = \sqcup_i \mathcal{B}_{m_i}^{r_i}$)

   to obtain the full information on potential e-customers

   $\mathcal{B}_m^r\sqcup_{f_m}\mathcal{B}_c^q$

   that contains the purchase candidate e-merchandise information for the e-shop $\mathcal{B}_s^p$ to expand the sales. With this sales information, the e-shop now becomes

   $\mathcal{B}_s^p \sqcup (\mathcal{B}_m^r\sqcup_{f_m}\mathcal{B}_c^q)$.

6. A homotopy theoretical framework of a cellular model for spatial/temporal information and spatial/temporal operations

Standing at the gate of the next millennium, it is truly fortunate to live at this critical moment to be able to influence the real world we live in a fundamental way. Establishing the science of the web and cyberworlds that are expected play the major roles in the next millennium will eventually be the most important to build the web-based information technology. Information models for the web and cyberworlds are key elements in that context because cyberworlds are information worlds.

It is also fortunate that we have necessary mathematical frameworks to create the science we are talking about as cellular spatial structures, and also homotopy theory that we sketch below.

Homotopy theory serves as the foundation of cellular spatial structures in the sense that we rely on it when we deal with cyberworld change in space and time [7] to accommodate spatio-temporal information and spatio-temporal operations. Let us consider the changes of a mapping function f relating a topological space X to another topological space Y. After the change, f becomes another mapping function g. In short, we are designing the continuous deformation of f into g where

$$f, g: X \to Y.$$

We consider the deformation during the normalized interval I = [0, 1] that can be a time interval or a space interval. Let us denote the unchanging part A of the topological space X as a subspace $A \subset X$. Then, what we are designing is a *homotopy* H, where

$$H: X \times I \to Y$$

such that

$$(\forall x \in X)\ (H(x, 0) = f(x) \text{ and } H(x, 1) = g(x)), \text{ and}$$

$$(\forall a \in A, \forall t \in I)\ (H(a, t) = f(a) = g(a)).$$

f is said *homotopic* to g *relative to* A, and denoted as

$$f \simeq g \text{ (rel A)}.$$

Now here is a new design problem. That is, how we can design two topological spaces X and Y to be *homotopically equivalent* $X \simeq Y$, namely *of the same homotopy type*. It is done by designing

$$f : X \to Y \text{ and } h: Y \to X$$

such that

$$h \circ f \simeq 1_X \text{ and } f \circ h \simeq 1_Y,$$

where 1_X and 1_Y are identity maps
$1_X : X \rightarrow X$ and $1_Y : Y \rightarrow Y$.

We can change cell dimensions homotopically. Homotopy equivalence is more general than topology equivalence. Homotopy equivalence can identify a change of any cyberworld that is topologically not any more equivalent after the change. While a cyberworld goes through changes by various operations and processes, the changing processes are specified by a homotopy and validated by homotopy equivalences. For instance, we can see why we preserve an attaching map when we perform each cell decomposition; it is to make cell decomposition homotopic so that we can reverse the decomposition. This *homotopic invariance* is of central importance because it turns cellular information on the web, that has been dynamically deformed by cellular operations, *reusable*.

As a matter of fact, researching on homotopic information models is a challenging area to find out the science of information models. It provides an interesting subject to see what information operations are homotopically equivalent.

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