Application of Back-Pressure Algorithm to Traffic Signal Control in Road Networks of Finite Road Capacity

YING LIU †1,a)　JUNTAO GAO†1,b)　YISHAN LIN †2,c)　MINORU ITO†1,d)

Back-pressure algorithm has been increasingly attractive to reduce traffic congestion for road networks. Recent work has shown the performance superiority of back-pressure based traffic signal control algorithms. However, these back-pressure based traffic signal control algorithms either assume each road can hold infinite vehicles (infinite road capacity) or need to have prior knowledge of vehicle turning ratios, all of which are not realistic for applications. In this paper, we propose a back-pressure based traffic signal control algorithm that can efficiently reduce traffic congestion, and thus vehicle delay, for realistic road networks with finite road capacity and without prior knowledge of vehicle turning ratios. As validated by simulations, our algorithm reduces average vehicle delay by 66.7% under moderate vehicle arrival rate when compared to fixed cycle traffic signal control.

1. Introduction

Traffic congestion has become a significant problem due to increasing vehicles every year, and has a negative impact to all drivers due to delayed travel times [1]. In metropolitan urban road networks, vehicles move according to traffic signals (red means one vehicle must stop, green means one vehicle may go and yellow means one vehicle should be prepared to stop). Fixed-cycle traffic signal control is widely applied in reality. However, it is of low efficiency in scheduling traffic, usually causing traffic congestion. So it is of high interest to find a more efficient traffic signal control algorithm.

Some researchers have proposed adaptive traffic signal control systems, which have currently been implemented in many major cities [3-4]. However, they can just update some control variables including phases, period of time, and offsets [2]. More recently, several related studies for traffic signal control have been conducted, in which the backpressure routing [5] has been adopted to control traffic signals at road junctions for reducing traffic congestion [6-11]. Back-pressure routing is an algorithm originally for routing packets based on queue length differentials (also called pressure gradients) in wireless communication networks [12]. For back-pressure routing in road networks, the pressure of a road is defined as the number of vehicles at that road, then traffic flows from a high-pressure upstream area to a low-pressure downstream area. In other words, the vehicles flow to the roads with more remaining capacity in the network. This algorithm can achieve throughput-optimality and be implemented in distributed manner with low computational complexity [11]. However, these back-pressure based traffic signal control algorithms for urban road networks assume that each road can hold infinite vehicles (infinite road capacity) [6-9], or have a strong assumption that the prior knowledge of vehicle turning ratios (the ratio of vehicles that will turn right, turn left and go straight after entering a road segment) is known in advance [10], all of which are not realistic for applications.

In this paper, we propose a back-pressure based traffic signal control algorithm (BP-TSC) to solve such problems in realistic urban road networks. BP-TSC can efficiently reduce traffic congestion for road networks with finite road capacity and do not need prior knowledge of vehicle turning ratios. The rest of this paper is organized as follows. In Section 2, we introduce system model, including road network model, vehicle arrival and routing process, etc. In Section 3, we describe back-pressure based traffic signal control algorithm (BP-TSC) in details. In Section 4, we do simulations to evaluate the vehicle delay performance under our BP-TSC algorithm. We conclude the whole paper in Section 5.

2. System Model

2.1 Road Network Model

A road network G consists of N roads and M junctions which are respectively denoted as a road set $R = \{R_1, R_2, \ldots, R_N\}$ and a junction set $J = \{J_1, J_2, \ldots, J_M\}$. For road R_i, the road length, speed limit and capacity are denoted as d_i, v_i and C_i, respectively. Vehicles enter the network from one origin road R_i and depart the network from another destination road R_j, and they may pass multiple roads between origin R_i and destination R_j. Further, each road R_i is divided into three lanes, denoted as L_{ij}, representing the lane in which vehicles waiting at road R_i will move to an adjacent road R_j, as shown in Fig. 1, and we assum that each road possess three lanes.

Each lane is modeled as a queue. System time is slotted as $t \in \{0, 1, 2 \ldots\}$, where each slot indicates a certain period of time. $Q_{ij}(t)$ denotes the number of vehicles queued at lane L_{ij}. Therefore, $Q_i(t) = \sum_j Q_{ij}(t)$ is the number of vehicles waiting at road R_i.

2.2 Vehicle Arrival and Routing Process

For the arrival and routing process of vehicles, we define $A_{ij}(t)$ as the number of vehicles that just enter the network and initially located at L_{ij} in time slot t. Since some vehicles may

†1 Graduate School of Information Science, Nara Institute of Science and Technology, Japan
†2 Institute of Computer Science and Engineering, National Chiao Tung University, Taiwan
a) liu.ying.lm3@is.naist.jp
b) jtgao@is.naist.jp
c) yishanl@cs.nctu.edu.tw
d) ito@is.naist.jp

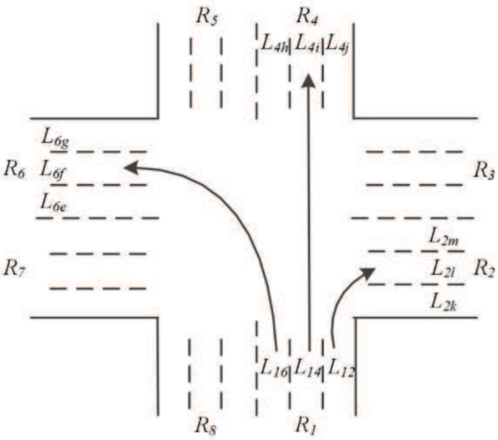

Fig. 1 Possible traffic movement from road R_1 at junction J_i.

enter L_{ij} from other roads, we define $f_{ki\to ij}$ (t) to represent the number of vehicles moving from L_{ki} to L_{ij} in time slot t, so $\sum_k f_{ki\to ij}(t)$ represents the total number of vehicles moving to L_{ij} in time slot t.

When a vehicle enters L_{ij} in time slot t (either just arriving or entering from other roads), the value of $Q_{ij}(t)$ is increased by 1. All queues are divided into three groups: ingress queues (storing exogenous arriving vehicles), sink queues (from which vehicles depart) and common queues (storing endogenous arriving vehicles). The queue dynamics (the variation of $Q_{ij}(t)$) can be descried as follows:

$$Q_{ij}(t+1) = Q_{ij}(t) - \sum_k f_{ij\to jk}(t) + A_{ij}(t) \quad \text{for ingress queue} \quad (1)$$

$$Q_i(t) = 0, \forall t \in \{0, 1, 2, \cdots\} \quad \text{for sink queue} \quad (2)$$

$$Q_{ij}(t+1) = Q_{ij}(t) - \sum_k f_{ij\to jk}(t) + \sum_k f_{ki\to ij}(t) \quad \text{for common queues} \quad (3),$$

where $Q_i(t)$ equals to 0 because vehicles have already left the network from R_i.

2.3 Phase of Traffic Signal

At each junction, if a vehicle moves from road R_i to road R_j, then such a movement is called a traffic movement from R_i to R_j. Some traffic movements can occur simultaneously and are considered as a "traffic phase". Upstream road set U_i and downstream road set D_i of junction J_i are defined as follows: road R_i belongs to U_i if and only if a vehicle can travel through R_i then junction J_i and enter next road R_j. Here, R_j is said to be in D_i. For example, R_1 belongs to U_i; R_2, R_4 and R_6 are in D_i. Let P_i = { p_i^1, p_i^2, …, p_i^{max} } be the set of all possible phases at a junction J_i. $p_i(t)$ denotes the phase activated during the time slot t for junction J_i. $\mu_{jk}(p_i(t))$ represents the maximum number of vehicles leaving from R_j to R_k (i.e., from L_{jk}) if phase $p_i(t)$ is activated. However, there may be small number of vehicles at L_{jk}, the actual vehicles leaving L_{jk} may be less than $\mu_{jk}(p_i(t))$. The actual number of vehicles leaving from L_{jk} during slot t is determined by

$$f_{jk}(t) = \min\{C_k - Q_k(t), Q_{jk}, \mu_{jk}(p_i(t))\} \quad (4),$$

where C_k is the capacity of road R_k. If $Q_k(t) = C_k$, $Q_k(t)$ is full, indicating that no more vehicles can enter R_k.

2.4 Calculation of $\mu_{jk}(p_i(t))$

From (4), we can know that $\mu_{jk}(p_i(t))$ is the upper bound for the number of vehicles leaving L_{jk}. $\mu_{jk}(p_i(t))$ depends on road speed limit of R_j and previous phase state. Roads R_j with high speed limit has higher $\mu_{jk}(p_i(t))$. If the previous phase state is red for road R_j and the current phase state is green for road R_j, vehicles need to accelerate first, which makes $\mu_{jk}(p_i(t))$ small. However, if the previous phase state is grenn for road R_j and the current phase state is also green for road R_j, vehicles can leave road R_i faster thus resulting in higher $\mu_{jk}(p_i(t))$. We set each road to have two values of $\mu_{jk}(p_i(t))$, one is higher, the other is lower, denoted by $\mu_{jk}^{high}(p_i(t))$, and $\mu_{jk}^{low}(p_i(t))$, respectively:

$$\mu_{jk}^{high}(p_i(t)) = \frac{v_j \times slot}{gap} \quad (5)$$

$$\mu_{jk}^{low}(p_i(t)) = \frac{\left(slot - \frac{v_j}{a}\right)\times v_j + \frac{1}{2}\times a \times \left(\frac{v_j}{a}\right)^2}{gap} \quad (6),$$

where v_j is the road speed limit of road R_j, a is the acceleration of vehicles and *slot* denotes the length of a time slot, and *gap* is sum of vehicle length and minimum safe distance between two vehicles.

3. Methods

Consider that each vehicle enters the network with a fixed destination and runs according to fixed shortest-path route. Our proposed BP-TSC consists of the following two stages.

- Stage 1: Each vehicle calculates the shortest path from origin to its destination using Dijkstra's algorithm [6]. Here, the cost of road R_i is defined as

$$T_i = \frac{d_i}{v_i} \quad (7),$$

  that is, time cost of travelling through road R_i.
- Stage 2: For each junction J_i, the following procedure is executed. At realistic roads, left-turning lanes are only for vehicles turning left, vehicles turning right and vehicles going straight may share the same lanes. So we calculate traffic pressure as follows.
  - Calculate the traffic pressure $F_j(t)$ of road $R_j \in U_i \cup D_i$ in time slot t based on the number of vehicles and road capacity. $F_j(t)$ for right-turning and going straight vehicles is defined as:

$$F_j(t) = \frac{Q_j(t)}{C_j} \quad (8),$$

    Traffic pressure $F_{jk}(t)$ for left-turning lane L_{jk} is defined as:

$$F_{jk}(t) = \frac{3\times Q_{jk}(t)}{C_j} \quad (9),$$

    When a vehicle at lane L_{jk} of road R_j turns left, it will enter road R_k, and because each road has three lanes, the vehicle number at lane L_{jk} is multiplied by 3 to make road traffic pressure $F_j(t)$ and lane traffic pressure $F_{jk}(t)$ comparable.
  - The calculation of pressure difference $W_{jk}(t)$ between R_j and R_k in time slot t is defined as(right-turning and going

straight):

$$W_{jk}(t) = max\{F_j(t) - F_k(t), 0\}, \forall R_j \in U_i and R_k \in D_i \quad (10),$$

And pressure difference for left-turning is defined as:

$$W_{jk}(t) = max\{3{\times}F_{jk}(t) - F_k(t), 0\}, \forall L_{jk} \in U_i and R_k \in D_i \quad (11),$$

・Release traffic pressure defined as follows.

$$p_i(t) = \arg\max_{p_i^r \in P_i} \sum_{j,k} W_{jk}(t) \cdot \mu_{jk}(p_i^r) \quad (12)$$

Traffic phase $p_i(t)$ for junction J_i that maximizes the pressure release is activated.

4. Simulation Results

We compare the vehicle delay performance under BP-TSC and fixed cycle traffic signal control by simulations.

4.1 Simulation Setup

We implemented BP-TSC in SUMO (Simulation of Urban MObility), which can simulate realistic traffic signal control and vehicle travelling scenarios [9].

We consider a road network where all roads are bi-directional and have different road length as shown in Fig. 3. All roads are divided into three lanes in each direction. Vehicles at right-turning lanes can also go straight illustrated in Fig. 4. Each junction has four phases as shown in Fig. 2. The road network is open, vehicles can enter or leave the road network anywhere.

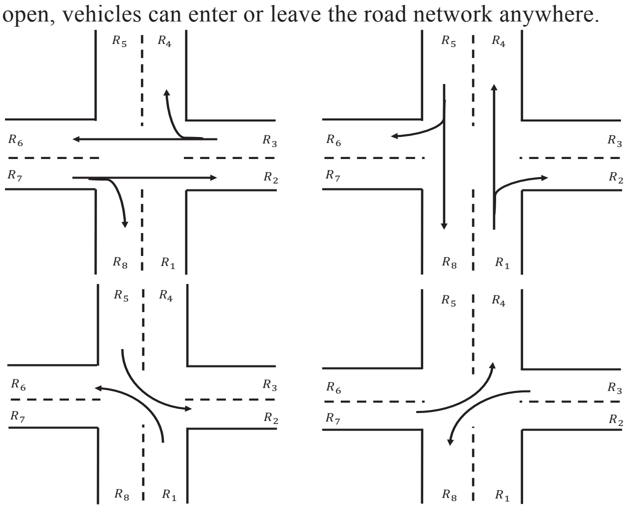

Fig. 2 Four typical phases through a junction.

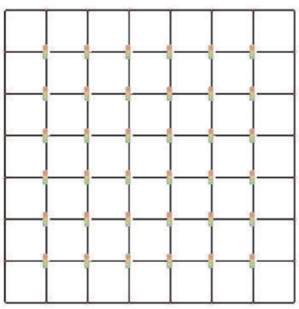

Fig.3 The road network used for simulations

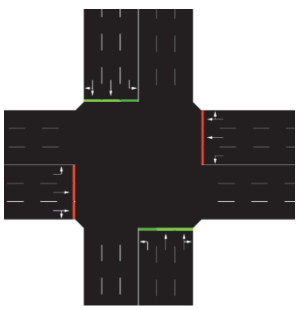

Fig.4 Junction example

Vehicles randomly join the road network with random origin and destinations. Traffic flows are divided into two classes: background traffic flows with random origins and destinations and test traffic flows with fixed origins and destinations. We generate vehicles of two classes with different arrival rates according to geometric distribution. Two experiments have been carried out: (1) By fixing the arrival rate of test traffic flows at λ_{test} =0.1 vehicles per second and varying the arrival rate of background traffic flows λ_{back} = 0.1, 0.12, 0.14, 0.16, 0.18, 0.2 vehicles per second, we measure the end-to-end delay of test vehicles under BP-TSC and fixed cycle traffic signal control; (2) By fixing the arrival rate of background traffic flows at λ_{back} =0.15 vehicles per second and varying the arrival rate of test traffic flows with λ_{test} = 0.01, 0.03, 0.05, 0.07, 0.09, 0.11 vehicles per second, we measure the end-to-end delay of test vehicles under BP-TSC and fixed cycle traffic signal control. Under each simulation setting, we run simulations for 5400 seconds in simulated road network (not the time in real world) and collect vehicle delay data to calculate average vehicle delay.

4.2 Simulation Results

Fig. 5 and Fig. 6 are the snapshots of traffic at a junction, providing an intuitive view of the traffic situation at the junction. Because fixed cycle traffic signal control cannot adjust its phase selections according to real-time traffic, it is possible to stop a traffic flow with large number of vehicles, but let go a flow with few vehicles, which is extremely inefficient. Under the same simulation setting, BP-TSC is able to adjust phases according to real-time traffic, always releasing traffic with higher pressure. Fig. 5 and Fig. 6 clearly show that BP-TSC greatly reduces congestion.

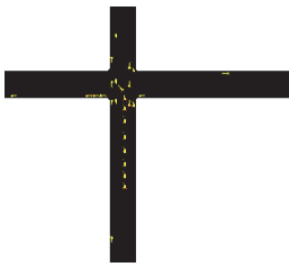

Fig.5 Snapshot of BP-TSC

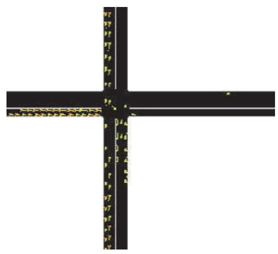

Fig.6 Snapshot of fixed circle control

Fig. 7 and Fig. 8 depict average delay of test vehicles under different signal control methods. In Fig.7, we can observe that as test vehicles arrival rate increases, average delay of test vehicles under BP-TSC increases only slightly, whereas average delay under fixed cycle traffic signal control increases almost linearly. Under moderate test vehicles arrival rate 0.2, our algorithm reduces average vehicle delay by 66.7% when compared to fixed cycle traffic signal control. This indicates that our BP-TSC method effectively reduces traffic congestion and thus vehicle delay. The superiority of BP-TSC over fixed cycle traffic signal control can also be observed in Fig. 8, where the average vehicle delay is reduced by 51.1% under different background vehicles arriving rates.

Fixed cycle control

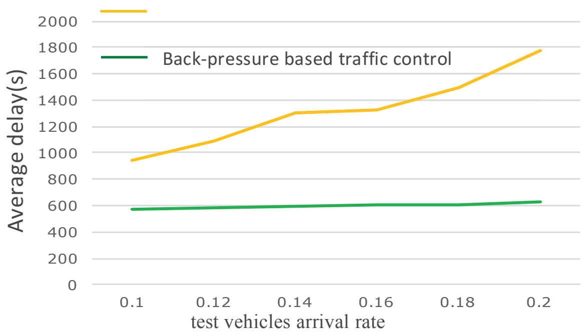

Fig 7. Comparison of the average vehicle delay under BP-TSC and fixed circle traffic signal control with fixed background traffic arrival rate.

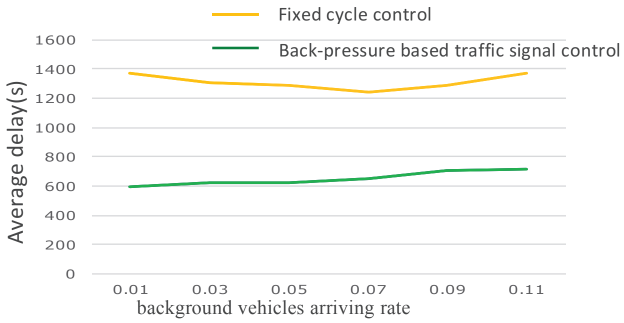

Fig 8. Comparison of the average vehicle delay under BP-TSC and fixed circle traffic signal control with fixed test traffic arrival rate.

5. Conclusion

In this paper, we proposed a back-pressure based traffic signal control algorithm, BP-TSC. Our algorithm effectively reduces traffic congestion and vehicle delay of realistic road networks with finite road capacity and without prior knowledge of vehicle turning ratios. In the future, we will consider joint traffic signal control and adaptive vehicle routing based on back-pressure algorithm.

Acknowledgments

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