## Regular Paper

# Illumination Recovery and Appearance Sampling for Photorealistic Rendering 

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#### Abstract

This paper addresses two issues of image-based modeling for synthesizing photorealistic appearances of real objects under natural illumination conditions: capturing real-world illumination and modeling complex appearances of real objects for variable illumination. The appearance of an object changes significantly depending on its surface reflectance properties and the lighting condition of the scene in which it is placed. It is thus important to provide not only appropriate surface reflectance properties of an object in a scene but also natural illumination conditions of that scene so that a realistic appearance of the object can be synthesized under the provided natural illumination conditions. We first discuss an image-based approach and an inverse lighting approach for capturing and modeling real-world illumination. Then we shift our attention to how to model and synthesize complex appearances of real objects seen under natural illumination conditions.


## 1. Introduction

Surface reflectance properties greatly influence the appearance of an object: the appearance of a metallic surface is completely different from that of a matted surface even under the same lighting conditions. Its appearance also changes significantly under different lighting conditions. For instance, the appearance change of a person's face is often much larger than the difference between two different faces under the same lighting. For the task of object recognition and image synthesis, it is thus important to be able to predict the variation of an object's appearance under varying illumination conditions.

In order to synthesize photorealistic appearances of real objects under natural illumination conditions, we address issues for modeling the appearance of the object for arbitrary illumination as well as modeling real-world illumination. Regarding the issue of capturing and modeling real-world illumination, we study both an image-based approach and an inverse lighting approach.

A technique for measuring real-world lighting from photographically acquired images of the scene is called image-based lighting. As one of the efficient methods in image-based lighting, we measure the illumination distribution of a real scene automatically from a pair

[^0]of omni-directional images based on the proposed omni-directional stereo algorithm in Section 2. The proposed method especially shows solutions to the difficulties that are left behind in the previously proposed image-based lighting approaches: how to construct a geometric model of a scene, and how to capture a wide field of view of a scene. We also attempt realtime image synthesis of virtual objects with natural shading and cast shadows onto a real scene whose illumination condition is dynamically changing.
The second approach, inverse lighting, assumes the knowledge of 3D shapes and reflectance properties of objects in a scene and inversely recovers incident light distribution from a photograph of the scene. One of the main advantages of inverse lighting over the imagebased lighting is that it does not require additional images for capturing illumination of a scene, but instead uses the appearance of objects located in a scene for recovering illumination of the scene.

We specifically investigated the effectiveness of using occluding information of incoming light in estimating an illumination distribution of a scene; we present a novel method for recovering an illumination distribution of a scene from image brightness inside shadows cast by an object of known shape in the scene in Section 3.

After describing the two approaches for recovering real-world illumination from images of a scene, we will shift our attention to how to model and synthesize complex appearances of real objects for variable illumination. As
has been noted, the appearance of an object changes significantly under different illumination conditions; even so, it was shown through the frequency-space analysis of reflection that the appearance of an object can be well approximated with a linear subspace spanned by basis images of the object defined in the frequency domain.

We carefully studied the issue of sampling of objects' appearances for variable illumination, and we present a novel method for analytically obtaining a set of basis images of an object from input images of the object taken under a point light source in Section 4.

Once methods for modeling the appearances of real objects for arbitrary illumination and modeling real-world illumination are established, we will be able to synthesize photorealistic appearances of real objects under natural illumination conditions. Their synthesized appearances may be used for many different applications, such as object recognition, archives of digital museums, and seamless integration of synthetic objects into real photographs or video images.

## 2. Image-based Lighting for Measuring Real-world Illumination

Image-based lighting techniques have been developed successfully with practical applications $\left.\left.{ }^{3}\right), 8,(11), 25\right)$. Pioneering work in this field was proposed by Fournier et al. ${ }^{11)}$. Fournier et al.'s method takes into account not only direct illumination but also indirect illumination by using the radiosity algorithm, which is commonly used for rendering diffuse interreflection. This method is effective for modeling subtle indirect illumination from nearby objects. However, this method requires the user to specify the 3D shapes of all objects in the scene. This object selection process could be tedious and difficult if a scene were full of objects. Also, since this method computes global illumination using pixel values of an input image, it is required that the image have a reasonably wide field of view. Even so, this method cannot correctly model direct illumination from outside of the input image unless a user specifies the positions of all lights.

Drettakis, et al. ${ }^{8)}$ extended Fourier, et al.'s work. Drettakis, et al.'s method made the creation of the 3D model much easier using computer vision techniques. They also introduced the use of a panoramic image built by image
mosaicing to enlarge the field-of-view of the input image, and the use of hierarchical radiosity for efficient computation of global illumination. However, this method still requires the user to define the vertices and topology of all objects in the scene, and it is often the case that the achieved field-of-view is not wide enough to cover all of the surfaces in the scene. This causes the same limitation on direct illumination outside the input image as in Fournier, et al.'s method.

Later, Debevec introduced a framework of constructing a light-based model of a real scene and using it for superimposing virtual objects into the scene with consistent shadings ${ }^{3)}$. A light-based model is a radiometric representation of a scene that is constructed by mapping reflections on a spherical mirror placed in the scene onto a geometric model of the scene. Although this method succeeded in superimposing virtual objects onto an image of a real scene with convincing shadings, it still requires the user's efforts to construct a light-based model: the user must specify a geometric model of the distant scene and select viewing points for observing the mirror so that the reflections on the mirror can cover the entire geometric model of the scene.

In summary, two difficulties in image-based lighting still remain to be solved: how to construct a geometric model of the scene, and how to capture a wide field of view of the scene.

### 2.1 Acquiring Illumination Based on Omni-directional Stereo Algorithm

We confronted these two difficulties and proposed an efficient method for automatically measuring illumination distribution of a real scene by using a set of omni-directional images of the scene taken by a CCD camera with a fisheye lens ${ }^{41 \text { ). }}$

There are three reasons why we use omnidirectional images rather than images taken by a camera with an ordinary lens. First, because of fisheye lens' wide field of view, e.g., 180 degrees, we can easily capture illumination from all directions from a far fewer number of omnidirectional images than we could with images taken by a conventional camera. Second, since a fisheye lens is designed so that an incom-

[^1]ing ray from a particular direction is projected onto a particular point on an imaging plane, we do not have to concern ourselves with computing directions of incoming rays and considering the sampling frequency of the incoming rays. Third, we are also able to use the directions of the incoming rays for automatically constructing a geometric model of the scene with the fisheye lens' wide field of view.

In our method, the geometric model of a scene is first constructed from a pair of omnidirectional images taken from two different locations as follows:
(1) Feature points with high contrast are extracted in the two omni-directional images by using the feature extraction algorithm proposed by Tomasi and Kanade ${ }^{57)}$. ${ }^{\text {* }}$
(2) 3D coordinates of points in the real scene corresponding to the extracted feature points are determined by using our proposed stereo algorithm.
(3) 3D coordinates for the remaining parts of the real scene are approximated by generating a 3D triangular mesh based on the 3D coordinates of the distinct feature points.
Then, radiance of the scene is computed from a sequence of omni-directional images taken with different shutter speeds and mapped onto the constructed geometric model. We refer to this geometric model with the radiance as a radiance map.

A radiance map must be constructed in order to compute a radiance distribution seen from any point in the scene. In other words, without constructing a radiance map, we can determine only the radiance distribution seen from the particular point where the omni-directional image was captured.

Since our method measures the radiance distribution of the scene as a triangular mesh, an appropriate radiance distribution can be used for rendering a virtual object and for generating shadows cast by the virtual object onto the real scene wherever the virtual object is placed in the scene.

Figure 1 shows the obtained triangular mesh representing the radiance distribution as its color texture and synthesized images under the

[^2]

Fig. 1 Measured illumination distribution based on the proposed omni-directional stereo algorithm and synthesized images under the captured illumination distribution.
captured illumination distribution. In the images synthesized by our method, shading of the virtual object blends well into the scene. Also, the virtual object casts a shadow with a soft edge on the tabletop in the same way as do the other objects in the scene.

### 2.2 Image Synthesis under Dynamically Changing Illumination

We also pursued the possibility of real-time rendering of synthetic objects with natural shading and cast shadows superimposed onto a real scene whose illumination condition was dynamically changing in Ref. 49).

In general, high computational cost for rendering virtual objects with convincing shading and shadows, such as interreflections or soft shadows under area light sources, prohibits real-time synthesis of composite images with superimposed virtual objects. From this limitation, simple rendering algorithms supported by commonly available graphics hardware need to be used for the applications that require realtime image synthesis. Computationally expensive rendering algorithms are not usually supported by such graphics hardware, and this leads to some restrictions on achievable image qualities.

Alternative approaches have been proposed for re-rendering a scene as a linear combination of a set of pre-rendered basis images of the scene ${ }^{5), 6), 31)}$. These approaches are based
on the linearity of scene radiance with respect to illumination intensities. Since this linearity holds for scenes with complex geometry and complex photometric properties such as interreflections between objects and cast shadows, a photo-realistic appearance of a scene can be synthesized based on this simple linear combination framework.

Most of the previously proposed methods, however, have been developed for the task of interactive lighting design. Therefore, basis lights under which a set of basis images are rendered are intentionally positioned at the desired locations, so that a scene under desired lighting configurations can be efficiently synthesized. Recently, Debevec, et al. introduced a method for re-rendering a human face based on a linear combination of face images taken under densely sampled incident illumination directions in Ref. 4). This method further included a model of skin reflectance to estimate the appearance of the face seen from novel viewing directions and under arbitrary illumination.

We generalized the approach based on the linearity of scene radiance with respect to illumination radiance and presented an efficient technique for superimposing synthetic objects with natural shadings and cast shadows onto a real scene whose illumination was dynamically changing. ${ }^{*}$ The main advantage of the proposed method was that image quality was not affected by the requirement for real-time processing, since reference images were rendered off-line. This enabled us to employ computationally expensive algorithms for providing reference images, and this resulted in achieving high quality in the final composite images of the scene.

Taking advantage of the linear relationship between brightness observed on an object surface and radiance values of light sources in a scene, our proposed method synthesizes a new image for novel lighting conditions as described in the following steps.
Step 1: The entire illumination of a scene is approximated as a collection of area sources $L_{i}(i=1,2, \cdots, n)$, which are equally distributed in the scene (Fig. 2-1).
Step 2: Two images that are referred to as reference images are rendered under each area

[^3]

Fig. 2 Basic steps of the proposed method.
light source: One with a virtual object superimposed onto the scene $O_{i}$, and the other without the object $S_{i}$ (Fig. 2-2).
Step 3: Scaling factors of the light source radiance values $r_{i}(i=1,2, \cdots, n)$ are measured by using an omni-directional image of the scene taken by a camera with a fisheye lens (Fig. 2-3). Step 4: New images $I_{o}{ }^{\prime}$ and $I_{s}{ }^{\prime}$, which should be observed under the current illumination condition, are obtained as a linear combination of $O_{i}$ 's and $S_{i}$ 's, respectively, with the measured scaling factors $r_{i}$ 's (Fig. 2-4).
Step 5: Using $I_{o}{ }^{\prime}$ and $I_{s}{ }^{\prime}$, the virtual object is superimposed onto the image of the scene along with natural shading and shadows that are consistent with those of real objects (Fig. 2-5). The ray casting algorithm ${ }^{61)}$ is imposed here; if an image pixel corresponds to the virtual object surface, the color of the corresponding pixels in $I_{o}{ }^{\prime}$ is assigned as the value of the pixel. Otherwise, the effects on the real objects caused by the virtual object, i.e., shadows and secondary reflection, are added by multiplying the pixel


Fig. 3 Image acquisition system and images synthesized by our method: a color camera with a fish-eye lens is used for capturing illumination of the scene. The bus shown in the top image is replaced with the house in the bottom image for this experimental set-up.
value by the ratio of $I_{o}{ }^{\prime}$ to $I_{s}{ }^{\prime}$ (Fig. 2-5).
Our image acquisition system and several examples of synthesized composite images are shown in Fig. 3. In this experiment, SGI Onyx2 with 6 CPUs was used for capturing a sequence of input images and for synthesizing final composite images. For efficient generation of the composite images, the following two strategies were used.
(1) As described above, each pixel in the composite image requires a different type of operation, depending on which object surface the pixel corresponds to. The computational cost of synthesizing each image pixel in the composite image is taken into account for distributing computation evenly among all of the 6 CPUs. Because effects due to the light sources with low radiance values are negligible in the final composite image, the reference images rendered under light sources whose radiance values are lower than a predefined threshold value are omitted when a linear combination of the reference images is computed. In this way, we can reduce the number of reference

Table 1 Processing time due to the number of reference images.

| No. of reference images | Processing time (sec) |
| :---: | :---: |
| 6 | 0.08 |
| 25 | 0.2 |
| 90 | 0.33 |
| 190 | 0.55 |
| 400 | 1.2 |

images used for synthesizing final composite images, and achieve the required processing time by adjusting the threshold value.
In the synthesized composite images shown in Fig. 3, the virtual cow casts a realistic shadow on the grass in the same way as the surrounding real trees in the scene do. Table 1 shows changes in processing time due to the number of reference images. Here, the number of reference images is equal to the number of area light sources that approximate the entire illumination distribution of the scene.
In these examples, we approximated the real illumination distribution by using 400 light sources and selected those light sources according to a threshold value. In this case, in spite of the highly realistic shading and shadows achieved in the composite image, the virtual object was able to be superimposed onto the scene at the frame rate of approximately 3 to 4 frames per second by using the proposed method.

### 2.2.1 Discussion

The approaches based on the linearity of scene radiance have one limitation with respect to illumination radiance: an image of the scene under novel illumination conditions cannot be synthesized accurately unless the illumination condition lies in the linear subspace spanned by the basis lights under which basis images are rendered. Therefore, it is essential to provide basis lights and their associated weights properly so that images under desirable illumination conditions can be synthesized.

Previous studies investigated how to choose a set of basis lights so that these lights could efficiently represent specific lighting configurations. For instance, Nimeoff, et al. introduced a set of steerable area light sources as basis lights to approximate the illumination effect of daylight ${ }^{31)}$. Dobashi, et al. defined a set of basis lights to represent directionality of spotlights based on spherical harmonics ${ }^{5}$. Later, Teo et al. introduced a hybrid method for synthesizing illumination effects from both area sources and directional spotlight sources ${ }^{56)}$. This method
also included a strategy for reducing the number of basis images based on principal components analysis. In terms of generating cast shadows with distinct boundaries, Naemura et al. introduced a way of adjusting interpolated weights of predefined basis point sources ${ }^{30)}$.

Recently, the effect of defining a set of basis lights in the frequency domain based on spherical harmonics was demonstrated in Refs. 2), 37), 38). In Section 4, we will reconsider this issue of efficiently synthesizing images of a scene under arbitrary illumination conditions and present a novel method for obtaining a set of basis images of real objects.

## 3. Inverse Lighting for Estimating Real-world Illumination

Methods that deal with an inverse problem of traditional model-based rendering ${ }^{\star}$ are called inverse rendering. The image brightness of an object can be computed as the function of the shape of the object, its surface reflectance properties, and the illumination condition where the object is located ${ }^{16), 17)}$. The relationship among these provides three research areas in inverse rendering:

- Shape-from-brightness for recovering the shape of the object from its reflectance properties and the known illumination condition.
- Reflectance-from-brightness for recovering the surface reflectance properties of the object from its shape and the known illumination condition.
- Illumination-from-brightness for recovering unknown illumination conditions of the scene based on the knowledge of the shape and the surface reflectance properties of the object.
In inverse rendering, the first two kinds of analyses, shape-from-brightness and reflectance-from-brightness, have been intensively studied using the shape from shading method ${ }^{18) \sim 20), 36)}$, as well as through reflectance analysis research ${ }^{1), 15), 21), 23), 26), 32), 50), 59) . ~}$

In contrast, relatively limited amounts of research have been conducted in the third area, inverse lighting. In general, real scenes include both direct and indirect illumination distributed in a complex way, and this makes it

[^4]difficult to analyze characteristics of the illumination distribution of the scene from image brightness in inverse lighting. As a consequence, most of the previously proposed approaches were conducted under very specific illumination conditions, for example, there were several point light sources in the scene; those approaches were difficult to extend to more natural illumination conditions ${ }^{19), 21), 50), 58), 63)}$. Alternatively, multiple input images taken from different viewing angles were necessary ${ }^{24), 34)}$.

Pioneering work in the field of inverse lighting for recovering natural illumination conditions of real scenes was proposed by Marschner and Greenberg ${ }^{28)}$. This work proposed to approximate the entire illumination with a set of basis lights located in a scene and estimated their radiance values from shadings of objects observed in that scene. Although this method had an advantage over the previous methods, as it did not require knowledge about the light locations of the scene, the estimation relied on the changes in appearance observed on an object surface assumed to be Lambertian, and therefore some restrictions were imposed on the shape of the object, for example, the object must have a large amount of curvature.
Later, Ramamoorthi and Hanrahan defined the conditions under which condition inverse rendering could be done robustly, based on their proposed signal-processing framework that described the reflected light field as a convolution of the lighting and the bidirectional reflectance distribution function (BRDF) ${ }^{37}$ ). Their analysis showed that changes in appearance observed on Lambertian surfaces were not necessarily suitable for estimating high frequency components of illumination distribution of a scene.
As one solution in the field of inverse lighting, we demonstrated the effectiveness of using occluding information of incoming light in estimating an illumination distribution of a scene in Refs. 42), 45). Shadows in a scene are caused by the occlusion of incoming light, and thus contain various pieces of information about the illumination of the scene. Nevertheless, shadows have been used for determining the 3D shape and orientation of an object that casts shadows onto the scene $\left.{ }^{7}, 22\right), 27$ ),51), while very few studies have focused on the illuminant information that shadows could provide. In our proposed method, image brightness inside shadows was effectively used for providing distinct clues to estimate an illumination distribution


Fig. 4 An inverse lighting approach for recovering an illumination distribution of a scene from image brightness inside shadows cast by an object of known shape in the scene.
(Fig. 4). We briefly explain our approach based on shadows in the next section.

In the following, we refer to the image with shadows as the shadow image, to the object that casts shadows onto the scene as the occluding object, and to the surface onto which the occluding object casts shadows as the shadow surface.

### 3.1 Illumination Recovery from Shadows

We first derive a formula that represents a relationship between the illumination distribution of a real scene and the irradiance at a surface point in the scene. Let $L(\theta, \phi)$ be the illumination radiance per unit solid angle coming from the direction $(\theta, \phi)$ defined by the polar coordinate system $\theta,(0 \leq \theta \leq \pi)$ in elevation and $\phi,(0 \leq \phi<2 \pi)$ in azimuth; then the total irradiance of a surface point on the shadow surface is ${ }^{16)}$

$$
\begin{equation*}
E=\int_{0}^{2 \pi} \int_{0}^{\frac{\pi}{2}} L(\theta, \phi) S(\theta, \phi) \cos \theta \sin \theta d \theta d \phi \tag{1}
\end{equation*}
$$

where $S(\theta, \phi)$ are occlusion coefficients; $S(\theta, \phi)$ $=0$ if $L(\theta, \phi)$ is occluded by the occluding object, and otherwise $S(\theta, \phi)=1$. The surface normal of the shadow surface is set to the direction $(\theta=0, \phi=0)$.

Some of the incoming light at this surface point is reflected toward the image plane as a secondary light source with the scene radiance of this point. The bidirectional reflectance distribution function (BRDF) denoted as $f\left(\theta, \phi ; \theta_{o}, \phi_{o}\right)$ characterizes the reflectance property of an object: $(\theta, \phi)$ and $\left(\theta_{o}, \phi_{o}\right)$ are incident and reflection directions with respect to the surface normal of the object surface.

Thus, by integrating the product of the BRDF and the illumination radiance over the entire hemisphere, the scene radiance $I\left(\theta_{o}, \phi_{o}\right)$ viewed from the direction $\left(\theta_{o}, \phi_{o}\right)$ is computed
as

$$
\begin{align*}
I\left(\theta_{o}, \phi_{o}\right)= & \int_{0}^{2 \pi} \int_{0}^{\frac{\pi}{2}} f\left(\theta, \phi ; \theta_{o}, \phi_{o}\right) L(\theta, \phi) \\
& S(\theta, \phi) \cos \theta \sin \theta d \theta d \phi \tag{2}
\end{align*}
$$

In order to solve for the unknown radiance $L(\theta, \phi)$, which is continuously distributed on the unit sphere from the recorded pixel values of the shadow surface, the illumination distribution is first approximated by discrete sampling of radiance over the entire surface of the extended light source. As a result, the double integral in Eq. (2) is approximated as

$$
\begin{array}{r}
I\left(\theta_{o}, \phi_{o}\right)=\sum_{i=1}^{n} f\left(\theta_{i}, \phi_{i} ; \theta_{o}, \phi_{o}\right) L\left(\theta_{i}, \phi_{i}\right) \\
S\left(\theta_{i}, \phi_{i}\right) \cos \theta_{i} \omega_{i} \tag{3}
\end{array}
$$

where $n$ is the number of sampling directions, $L\left(\theta_{i}, \phi_{i}\right)$ becomes the illumination radiance per unit solid angle coming from the direction $\left(\theta_{i}, \phi_{i}\right)$, and $\omega_{i}$ is a solid angle for the sampling direction $\left(\theta_{i}, \phi_{i}\right)$.
In Eq. (3), the recorded pixel value $I\left(\theta_{o}, \phi_{o}\right)$ is computed as a function of the illumination radiance $L\left(\theta_{i}, \phi_{i}\right)$ and the $\operatorname{BRDF} f\left(\theta_{i}, \phi_{i} ; \theta_{o}, \phi_{o}\right)$. Accordingly, we take different approaches, depending on whether BRDF of the surface is given. For the sake of simplicity, we explain the case where the shadow surface is a Lambertian surface whose reflectance properties are given in the following.

BRDF for a Lambertian surface is known to be a constant. An equation for a Lambertian surface is obtained from Eq. (3) as

$$
\begin{equation*}
I\left(\theta_{o}, \phi_{o}\right)=\sum_{i=1}^{n} K_{d} L\left(\theta_{i}, \phi_{i}\right) S\left(\theta_{i}, \phi_{i}\right) \cos \theta_{i} \omega_{i} \tag{4}
\end{equation*}
$$

[^5]where $K_{d}$ is a diffuse reflection parameter of the surface.
From Eq. (4), the recorded pixel value $I$ for an image pixel is given:
\[

$$
\begin{equation*}
I=\sum_{i=1}^{n} a_{i} L_{i} \tag{5}
\end{equation*}
$$

\]

where $L_{i}(i=1,2, \cdots, n)$ are $n$ unknown illumination radiance values specified by $n$ node directions of a geodesic dome. The coefficients $a_{i}(i=1,2, \cdots, n)$ represent $K_{d} S\left(\theta_{i}, \phi_{i}\right) \cos \theta_{i} \omega_{i}$ in Eq. (4). We can compute these coefficients from the 3D geometry of a surface point, the occluding object and the illuminant direction. In our examples, we use a photo modeling tool called the 3D Builder from 3D Construction Company to recover the shape of an occluding object and also the camera parameters from a shadow image.

If we select a number of pixels, say $m$ pixels, a set of linear equations is obtained: $I_{j}=$ $\sum_{i=1}^{n} a_{j i} L_{i}(j=1,2, \cdots, m)$. Therefore, by selecting a sufficiently large number of image pixels, we are able to solve for a solution set of unknown $L_{i}$ 's. We solve the problem by using the linear least square algorithm with nonnegativity constraints (using a standard MATLAB function) to obtain an optimal solution with no negative components.

### 3.2 Issues in Inverse Lighting

We further addressed the following two issues in inverse lighting ${ }^{43)}$. First, the method combined the illumination analysis with an estimation of the reflectance properties of a shadow surface. This made the method applicable to the case in which reflectance properties of a surface were not known a priori, broadening the variety of images to which the method is applicable.

Second, an adaptive sampling framework for efficient estimation of illumination distribution was introduced. Using this framework, we were able to avoid unnecessarily dense sampling of the illumination and could estimate the entire illumination distribution more efficiently with a smaller number of sampling directions of the illumination distribution.

We also discussed the amount of information obtainable from a given image of a scene about the illumination distribution of the scene in Ref. 44). In general, the amount of information obtainable from an image is determined depending on how much of the shadow surfaces are blocked by objects in a scene and how much


Fig. 5 Synthesized appearance using the estimated illumination distribution.
of the scene is captured by the field of view of the camera taking the image of the scene. In particular, two main factors that control the stability of the illumination estimation from shadows were analyzed: blocked view of shadows and limited sampling resolution for radiance distribution inside shadows.
Based on the analysis, a robust method was presented. More specifically, for reliably estimating the illumination distribution of a scene by taking stability issues into consideration, we proposed changing the sampling density of the illumination distribution depending on the amount of information obtainable from a shadow image for a particular direction of the illumination distribution. In order to use radiance distribution inside a penumbra of shadows correctly, we introduced a super-sampling scheme for examining occlusion of incoming light from each light source. We also explained the optimal sampling of image pixels and the selection of illumination distribution samplings for more stable computation.

All of these extensions contributed to improving the stability and accuracy of illumination estimation from shadows; illumination distribution can be estimated in a reliable manner with these proposed improvements regardless of types of input images such as the shape of an occluding object or a camera position.
In the bottom row of Fig. 5, several synthetic objects were also superimposed onto the surface using the illumination distribution estimated from the input shadow image shown in this figure. It is worth noting that in this example, a relatively large area of the shadow surface is occluded by the occluding object, and it is often difficult to provide a correct estimate of the illumination distribution in such cases.

Even in this challenging case, our proposed approach was able to reliably estimate the il-
lumination distribution of the scene by taking stability issues into consideration. Shadows cast by those synthetic objects greatly resemble those cast by the real objects, and this shows that the estimated illumination distribution gives a good presentation of that of the real scene.

### 3.3 Discussion

Recently, based on the signal-processing framework proposed by Ramamoorthi and Hanrahan, it was shown that high frequency components of the appearance of an object surface could retain significant energy by taking the occlusion of incoming light as well as its BRDF into account in Ref. 35). This indicates that the use of shadows for the illumination estimation has the significant advantage of providing more clues to the high frequency components of illumination distribution of a scene. Later, Ramamoorthi, et al. provided more detailed frequency analysis of cast shadows using convolution and Fourier basis functions and showed a similar implication for inverse lighting from cast shadows in Ref. 40).

### 3.4 Acquiring Shading Model for Artistic Rendering

As an application for the proposed inverse lighting approach, we also presented a new technique for superimposing synthetic objects onto oil paintings with artistic shadings that were consistent with those originally painted by the artists in Ref. 48). In a colored medium such as oil painting, artists often use color shift techniques add artistic tones to their paintings as well as to enlarge their dynamic ranges.

We determined the mechanisms for color shifts performed by artists and automated their processes so that we could superimpose onto paintings synthetic objects that had shadings consistent with those in the paintings. In this work, we first studied characteristics of shadows observed both in real scenes and in paintings to discover how intrinsic color shifts had been performed by artists. In particular, we analyzed brightness distributions inside shadows observed in a painting. Then, we adapted the acquired mechanisms so that we could superimpose synthetic objects with consistent shadings onto oil paintings.

Synthesized results are shown in Fig. 6. Especially in the top-right image synthesized by our method, the synthetic object casts artistic shadows on the wooden table that are similar to those of the other objects originally painted by


Fig. 6 Superimposing synthetic objects into the paintings: "Still Life: Drawing Board, Pipe, Onions and Sealing-wax" by Vincent van Gogh and "Artist in Studio" by Rembrandt.
the artist, and this shows that the color modifications made by the artist are well approximated by the color modification functions obtained by our method.

## 4. Image-based Rendering under Novel Lighting Conditions

Inverse rendering carries out the opposite procedures of model-based rendering to provide object and illumination models of a real scene from photographically available information of the scene. Once models of a scene are acquired, new images of the scene under novel lighting and/or viewing conditions can be synthesized by using conventional model-based rendering techniques.
In contrast, the approach called image-based rendering directly uses the original set of input images of a scene to produce new images of the scene under novel conditions ${ }^{53)}$. Depending on which scene conditions should be modified, image-based rendering techniques are classified into three categories: image-based rendering under novel viewing conditions, imagebased rendering under novel lighting conditions, and image-based rendering under novel viewing and novel lighting conditions. We examine the second category, image-based rendering under novel lighting conditions.
In contrast with model-based rendering techniques, image-based rendering techniques do not require full radiometric computation to synthesize the photo-realistic appearance of objects in a scene. This means that the cost to produce new images of the scene is inde-
pendent of the scene complexity. Also, imagebased rendering techniques normally do not require geometric and photometric models of a scene. ${ }^{\text {is }}$ Image-based rendering, however, has a tendency to require many input images of a scene to synthesize a reasonably realistic appearance of the scene. This results in the requirement of a large amount of both computer memory and data storage.

While a large variety of possible appearances may seem to exist for a given object, previous research has demonstrated that the changes in appearance of an object for varying illumination can be represented with a linear subspace spanned by a set of basis images of the object. For instance, in the case of a convex Lambertian object, its appearance seen under distant illumination without attached and cast shadows can be described with a 3D linear subspace spanned from three input images of the object taken under linearly independent lighting conditions ${ }^{29), 52), 62)}$. Even taking into account attached shadows, most of the image variation of a human face or other object under varying illumination was shown to be adequately represented by a low-dimensional linear subspace slightly higher than $3 \mathrm{D}^{10), 14), 60)}$. A similar observation was utilized for object recognition in Refs. 12), 13).

A set of basis images spanning such a linear subspace is often provided by applying principal-component analysis to the input images of an object taken under different lighting conditions. Since little is known about how to sample the appearance of an object in order to obtain its basis images correctly, a large number of input images taken by moving a point light source along a sphere surrounding the object are generally provided.

Recent investigations in frequency-space analysis of reflection have shown that the appearance of an object under varying complex illumination conditions can be well approximated with a linear subspace spanned by basis images of the object, called harmonic images, each of which corresponds to an image of the object illuminated under harmonic lights whose distributions are specified in terms of spherical harmonics ${ }^{2), 37), 38)}$. 4,

Hence, if harmonic lights can be physically

[^6]constructed in a real setting, harmonic images of a real object can be obtained simply as images of the object seen under these light sources. However, harmonic lights are complex diffuse light sources comprising both negative and positive radiance and are thus difficult to physically construct in a real setting. Therefore, most of the previously proposed techniques synthetically compute harmonic images from the knowledge of an object's 3D shape and reflectance properties.

### 4.1 Appearance Sampling for Obtaining a Set of Basis Images

This difficulty motivated us to develop a method for analytically obtaining a set of basis images of a convex object for arbitrary illumination from input images of the object taken under a point light source in Ref. 46).

The main contribution of our work is that we show that a set of lighting directions can be determined for sampling images of an object depending on the spectrum of the object's BRDF in the angular frequency domain such that a set of harmonic images can be obtained analytically based on the sampling theorem on spherical harmonics ${ }^{9)}$.

Using those sampling directions determined from the sampling theory, we are able to obtain harmonic images of an object as finite weighted sums of its appearance seen under point light sources located at these directions. Our proposed method thus requires a significantly smaller number of input images than other techniques that do not take into account a relationship between a spectrum of BRDFs and a sampling density of illumination directions.

In addition, unlike other methods based on spherical harmonics, our method does not require the shape and reflectance model of an object used for rendering harmonic images of the object synthetically. Thus, our method can be easily applied to determine a set of basis images for representing the appearance change of a real object under varying illumination conditions.

An overview of our hardware set-up used for obtaining the input images of the objects is shown in Fig. 7; an array of light sources is mounted on a turntable. These light sources are equally spaced in elevation, and the set of

[^7]

Fig. 7 Hardware set-up and synthesized images of objects under natural illumination. The first row shows illumination maps. The second, third, and fourth rows show the synthesized appearance of objects under the corresponding illumination map.
light sources is rotated around the objects in an azimuthal direction based on the sampling theorem on spherical harmonics.

Figure 7 shows the appearance of those objects synthesized by our method under natural illumination conditions. In this figure, the synthesized appearance of the test objects changes significantly depending on the characteristics of the given illumination distributions, and this shows that the complex appearance of its structural colors are well represented by the set of basis images obtained by our method.

Furthermore, we carefully studied the issue of aliasing and further extended the method based on the sampling theorem for reducing the artifacts due to aliasing, by substituting extended light sources $(E L S)$ for a point light source to sample the appearance of a real object in Ref.47). The use of ELS for modeling the shape and reflectance of an object was originally introduced in Ref. 33). We extended their analysis further in the angular frequency domain so that the harmonic images of an object of arbitrary surface materials could be obtained without suffering from aliasing caused by insufficient sampling of its appearance.

## 5. Conclusion

This paper addressed two issues of synthesizing a photo-realistic appearance of an object under natural illumination conditions: capturing real-world illumination and modeling complex appearances of real objects for variable illumination. Regarding the first issue of cap-
turing and modeling real-world illumination, we used both image-based lighting and inverse lighting methods.

Concretely, we proposed an efficient imagebased lighting method for capturing the illumination distribution of a real scene automatically from a pair of omni-directional images based on the proposed omni-directional stereo algorithm. Additionally, we considered stability issues of the inverse lighting approach and showed the effectiveness of using occluding information of incoming light in estimating an illumination distribution of a scene. Our proposed method was able to estimate even complex illumination distribution of a natural scene from image brightness observed inside shadows cast by an object of known shape in the scene.
Regarding the second issue of modeling and synthesizing complex appearances of real objects for variable illumination, we carefully studied the issue of sampling appearance of real objects in the frequency domain. Based on our analysis, we introduced a novel method for analytically obtaining a set of basis images from input images of a real object taken under a point light source. The main contribution of our work was that we showed that a set of lighting directions was able to be determined for sampling images of an object depending on the spectrum of the object's bidirectional reflectance distribution function (BRDF) in the angular frequency domain such that a set of basis images could be obtained analytically based on the sampling theorem on spherical harmonics.

Once a set of basis images is obtained, the appearance of the object can be synthesized simply as a linear combination of these basis images under novel lighting conditions, which may be provided by either image-based lighting methods or inverse lighting methods. The future direction of this study includes extending our method for modeling the appearance of an object seen from arbitrary viewing directions and synthesizing its appearance under novel lighting and novel viewing conditions.

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[^0]:    $\dagger$ National Institute of Informatics
    $\dagger \dagger$ Interfaculty Initiative in Information Studies, The University of Tokyo

[^1]:    * It is worth noting that State et al. previously introduced the use of a steel ball to capture the reflections at a single point ${ }^{55}$ ).

[^2]:    * An image pixel with high gradient values in two orthogonal directions, e.g., a corner point, is extracted as a feature point.

[^3]:    \# A scene was assumed to consist of both real objects and synthetic objects with fixed scene geometry and the scene was viewed from a fixed viewing point under dynamically changing illumination.

[^4]:    * Model-based rendering techniques synthesize the appearance of objects based on empirically or analytically given reflection models.

[^5]:    4. Node directions of a geodesic dome can be used for uniform sampling of the illumination distribution.
[^6]:    $\star$ Some image-based rendering techniques make use of geometric models of a scene for better compression of its appearance.

[^7]:    4 4 Harmonic images have also been used for the purpose of efficient rendering of an object under complex illumination ${ }^{39), 54)}$.
    动的 Surface Reflectance Sampler, TechnoDream21 Corporation.

