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ON THE COMPLEXITY OF THREE-DIMENSIONAL CHANNEL ROUTING

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1 INTRODUCTION

The 3-D channel routing is a fundamental problem on the physical design of 3-D integrated circuits. Many results on the problem can be found in the literature [1],[3],[6],[7].

The 3-D channel is a 3-D rectilinear grid G consisting of columns, rows, and layers which are rectilinear grid planes defined by fixing x-, y-, and z-coordinates at integers, respectively. The numbers of columns, rows, and layers are called the width, depth, and height of G, respectively. (See Fig. 1.) G is called a (W, D, H)-channel if the width is W, depth is D, and height is H. A vertex of G is a grid point with integer coordinates. We assume without loss of generality that the vertex set of a (W, D, H)-channel is $\{(x, y, z)|1 \le x \le W, 1 \le y \le D, 1 \le z \le H\}$. A terminal is a vertex of G located in the top or bottom layer. A net is a set of terminals to be connected. The object of the 3-D channel routing problem is to connect the terminals in each net with a tree in G using as few layers as possible in such a way that trees spanning distinct nets are vertex-disjoint. A set of nets is said to be routable in G if G has vertex-disjoint trees spanning the nets.

This paper considers the complexity of the following decision problem.

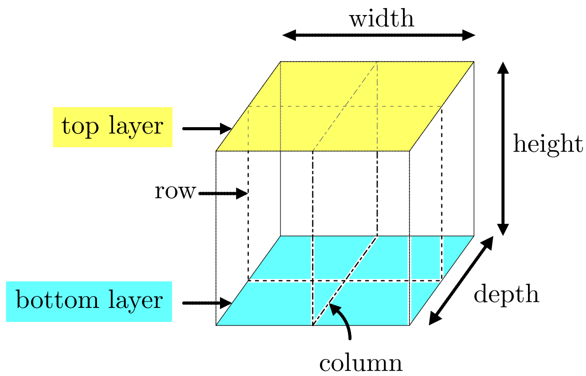

Fig. 1 Three-dimensional channel.

3-D CHANNEL ROUTING

INSTANCE: Positive integers W, D, H, a set of terminals

$$T = \{(a_i, b_i, H)|1 \le a_i \le W, 1 \le b_i \le D, 1 \le i \le p\} \cup \{(c_j, d_j, 1)|1 \le c_j \le W, 1 \le d_j \le D, 1 \le j \le q\},$$

and a partition of T into nets $N_1, \ldots, N_m$.

QUESTION: Is a set of nets $\{N_1, \ldots, N_m\}$ routable in a (W, D, H)-channel?

We have two well-known problems as subproblems of 3-D CHANNEL ROUTING, namely, PLANAR CHANNEL ROUTING and TWO-ROW CHANNEL ROUTING. These problems can be stated as follows.

PLANAR CHANNEL ROUTING

INSTANCE: Positive integers W, H, a set of terminals

$$T = \{(a_i, 1, H)|1 \le a_i \le W, 1 \le i \le p\} \cup \{(c_j, 1, 1)|1 \le c_j \le W, 1 \le j \le q\},$$

and a partition of T into nets $N_1, \ldots, N_m$.

QUESTION: Is a set of nets $\{N_1, \ldots, N_m\}$ routable in a $(W, 1, H)$-channel?

TWO-ROW CHANNEL ROUTING

INSTANCE: Positive integers W, H, a set of terminals

$$T = \{(a_i, 1, H) | 1 \leq a_i \leq W, 1 \leq i \leq p\} \cup \{(c_j, 1, 1) | 1 \leq c_j \leq W, 1 \leq j \leq q\},$$

and a partition of T into nets $N_1, \ldots, N_m$.

QUESTION: Is a set of nets $\{N_1, \ldots, N_m\}$ routable in a $(W, 2, H)$-channel?

It should be noted that TWO-ROW CHANNEL ROUTING has been called "UNRESTRICTED" TWO-LAYER CHANNEL ROUTING in the literature. The complexity of TWO-ROW CHANNEL ROUTING is a longstanding open question posed by Johnson[4], while PLANAR CHANNEL ROUTING can be solved in polynomial time as shown by Dolev, Karplus, Siegel, Strong, and Ullman[2].

The purpose of this paper is to show the following.

THEOREM: 3-D CHANNEL ROUTING *is* NP-*complete.*□

The complexity of TWO-ROW CHANNEL ROUTING is still open. Also, the complexity of the following problem is open for any fixed integer $k \geq 2$.

2.5-D CHANNEL ROUTING

INSTANCE: Positive integers W, H, a set of terminals

$$T = \{(a_i, b_i, H) | 1 \leq a_i \leq W, 1 \leq b_i \leq k, 1 \leq i \leq p\} \cup \{(c_j, d_j, 1) | 1 \leq c_j \leq W, 1 \leq d_j \leq k, 1 \leq j \leq q\},$$

and a partition of T into nets $N_1, \ldots, N_m$, where $k \geq 2$ is a fixed integer.

QUESTION: Is a set of nets $\{N_1, \ldots, N_m\}$ routable in a (W, k, H)-channel?

2 PROOF OF THE THEOREM (SKETCH)

It is easy to see that 3-D CHANNEL ROUTING is in NP. We show a polynomial time reduction from 3SAT, a well-known NP-complete problem, to 3-D CHANNEL ROUTING. Let

$$\phi(x_1, \ldots, x_n) = \bigwedge_{i=1}^{r} C_i$$

be a Boolean function in conjunctive normal form in which each clause C_i has three literals for $1 \leq i \leq r$. We employ a natural extension of Szymanski's reduction used to prove the NP-completeness of MANHATTAN CHANNEL ROUTING[5]. We first construct a $(13, 2, 6n + 2)$-channel, called a clause block, for each clause C_i. We next construct $r + 1$ copies of a $(5n, 24n + 2, 6n + 2)$-channel, called an enforcing block, which are introduced at the both sides of each clause block to avoid interactions between clause blocks. We finally construct two copies of a $(6n + 2, 2, 6n + 2)$-channel, called an end block, which are introduced at the both ends of a chain of the blocks above. Combining all the blocks together, we obtain a $(5rn + 13r + 17n + 4, 24n + 2, 6n + 2)$-channel with $84n^2r + 372n^2 + 322nr + 112n - 2$ nets for ϕ. We can prove that ϕ is satisfiable if and only if the nets are routable in the 3-D channel. Since the channel and nets can be constructed in polynomial time, we obtain the theorem.

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