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Controlling Dominance Area of Solutions in Multiobjective Evolutionary Algorithms and Performance Analysis on Multiobjective 0/1 Knapsack Problems

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This work proposes a method to control the dominance area of solutions in order to induce appropriate ranking of solutions for the problem at hand, enhance selection, and improve the performance of MOEAs on combinatorial optimization problems. The proposed method can control the degree of expansion or contraction of the dominance area of solutions using a user-defined parameter S. Modifying the dominance area of solutions changes their dominance relation inducing a ranking of solutions that is different to conventional dominance. In this work we use 0/1 multiobjective knapsack problems to analyze the effects on solutions ranking caused by contracting and expanding the dominance area of solutions and its impact on the search performance of a multi-objective optimizer when the number of objectives, the size of the search space, and the feasibility of the problems vary. We show that either convergence or diversity can be emphasized by contracting or expanding the dominance area. Also, we show that the optimal value of the area of dominance depends strongly on all factors analyzed here: number of objectives, size of the search space, and feasibility of the problems.

1. Introduction

Multiobjective evolutionary algorithms (MOEAs) [1),2)] are being increasingly investigated for solving multiobjective optimization problems. MOEAs are particularly suitable for this task because they evolve simultaneously a population of potential solutions to the problem at hand, which allows us to search a set of Pareto non-dominated solutions in a single run of the algorithm. Some important features of the latest generation MOEAs are that selection incorporates elitism and it is biased by Pareto dominance and a diversity preserving strategy in objective space. Pareto dominance based selection is thought to be effective for problems with convex and non-convex fronts and has been successfully applied, especially in two and three objective problems.

However, some current research reveals that ranking by Pareto dominance on problems with an increased number of objectives might not longer be effective [3)~5)]. It has been shown that the characteristics of multiobjective landscapes viewed in terms of non-dominated fronts (that are found in the process of non-domination sorting) can change drastically as the number of objectives increases, i.e., the number of fronts reduces substantially and become denser (more solutions per front) just by increasing the num-

ber of objectives [5)]. In this case, most sampled solutions at a given time turn to be non-dominated. That is, most solutions are assigned the same rank of non-dominance and Pareto selection weakens since it has to discriminate mostly based on diversity of solutions. Another factor that affects the density of the fronts is the complexity (ruggedness and number of local optima) of the individual single objective landscapes. It has been shown that the top non-dominated fronts become denser as the complexity of the landscapes reduces, and vice-versa [5)], which affects the behavior and effectiveness of conventional Pareto selection in two ways. First, in landscapes of low complexity the high density of the top non-dominated fronts combined with elitism makes the instantaneous elite-population to be mostly composed of individuals with the same non-domination rank since early generations, even in two objective problems. In this case, also, selection has to rely mostly on diversity rather than on Pareto dominance ranking. Second, on problems of increased complexity there could be too many but sparse fronts, where Pareto selection could become too strong increasing the likelihood that the algorithm gets trapped in local fronts. These studies suggest that for selection to be effective a more careful analysis of Pareto dominance relation is required when the number of objectives increase. In addition, for any number of objectives, the dominance relation

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should be appropriately revised according to the characteristics of the multiobjective problem.

Relaxed forms of Pareto dominance have been discussed within decision making in multiobjective optimization since long ago [6]~[8]. However, it is only recently that the evolutionary multiobjective optimization (EMO) community has turned its attention to relaxed forms of dominance for ranking solutions during the search process, and the effects of such dominance relaxations on the behavior and performance of the evolutionary optimizer are still not well understood. Within the EMO community, works on relaxed forms of Pareto dominance include ϵ-dominance [9] and α-domination [10]. ϵ-dominance acts as an archiving strategy and was proposed as a way of regulating convergence of a MOEA. The algorithm maintains a finite-size archive of non-dominated solutions, in which new points are only accepted if they are not ϵ-dominated by any other point of the current archive. ϵ-dominance strengthens selection during the archiving process. On the other hand, α-domination permits a solution $\boldsymbol{x}$ to dominate a solution $\boldsymbol{y}$ if $\boldsymbol{x}$ is slightly inferior to $\boldsymbol{y}$ in an objective but largely superior to $\boldsymbol{y}$ in some other objectives. α-domination was tried on an ad hoc continuous problem created specifically to illustrate a potential problem that Pareto selection could face. In addition, α-domination only introduces a method to strengthen selection and its effects have not been explained nor tested on standard test suit problems.

In this work, we propose a method to control the dominance area of solutions in order to induce appropriate ranking of solutions for the problem at hand, enhance selection, and improve the performance of MOEAs on combinatorial optimization problems. The proposed method can control the degree of expansion or contraction of the dominance area of solutions using a user-defined parameter S. Modifying the dominance area of solutions changes their dominance relation inducing a ranking of solutions that is different to conventional dominance. Contrary to ϵ-dominance and α-domination, the proposed method can strengthen or weaken selection by expanding or contracting the area of dominance and conceptually can be considered as a generalization of Pareto dominance. In addition, the motivation and method itself of the proposed approach

is different to ϵ-dominance and α-domination. See Sections 3 and 4 for a detailed explanation about ϵ-dominance, α-domination, and the proposed method.

In this work we analyze the effects on solutions ranking caused by contracting and expanding the dominance area of solutions and its impact on the search performance of a multiobjective optimizer when the number of objectives, the size of the search space, and the complexity of the problems vary. We chose NSGA-II as a representative elitist algorithm that uses dominance [11] and compare its performance with NSGA-II enhanced by the proposed method. We conduct our study on 0/1 multiobjective knapsack problems with $m = \{2, 3, 4, 5\}$ objectives varying the number of items n (size of search space is given by 2^n) and the feasibility ratio ϕ of the search space, which is a good indicator of the complexity of the landscapes in this kind of problems. This work clearly shows that either convergence or diversity can be emphasized by contracting or expanding the dominance area. Also, this work shows that the optimal value of S^* that controls the area of dominance depends strongly on all factors analyzed here: number of objectives, size of the search space, and feasibility of the problems.

2. Multiobjective Optimization Concepts and Definitions

A multiobjective optimization problem including m kinds of objective functions is defined as follows:

$$\begin{cases} \text{Maximize} & \boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x})) \\ \text{subject to} & \boldsymbol{x} \in \mathcal{F} \end{cases}$$

$$(1)$$

where, $\boldsymbol{x} \in \mathcal{F}$ is a feasible solution vector in the solution space $\mathcal{S}$ ($\mathcal{F} \subseteq \mathcal{S}$), and f_i ($i = 1, 2, \cdots, m$) are the m objectives to be maximized. That is, we try to find a feasible solution vector $\boldsymbol{x} \in \mathcal{F}$ in the solution space maximizing each objective function f_i ($i = 1, 2, \ldots, m$) in a vector fitness function $\boldsymbol{f}$. Important concepts used in determining a set of solutions for multiobjective optimization problems are dominance, Pareto optimality, Pareto set and Pareto front. Next we define *dominance* between solutions $\boldsymbol{x}, \boldsymbol{y} \in \mathcal{F}$ as follows: If

$$\forall i \in \{1, 2, \ldots, m\} : f_i(\boldsymbol{x}) \geq f_i(\boldsymbol{y}) \quad \wedge$$
$$\exists i \in \{1, 2, \ldots, m\} : f_i(\boldsymbol{x}) > f_i(\boldsymbol{y}).$$

$$(2)$$

are satisfied, $\boldsymbol{x}$ dominates $\boldsymbol{y}$. In the following, $\boldsymbol{x}$ dominates $\boldsymbol{y}$ is denoted by $\boldsymbol{f}(\boldsymbol{x}) \succeq \boldsymbol{f}(\boldsymbol{y})$. A solution vector $\boldsymbol{x}$ is said to be *Pareto optimal* with respect to $\mathcal{F}$ if it is not dominated by other solution vectors in $\mathcal{F}$. The presence of multiple objective functions, usually conflicting among them, gives rise to a set of optimal solutions. The set of Pareto optimal solutions (POS) is defined as

$$\mathcal{POS} = \{\boldsymbol{x} \in \mathcal{F} \,|\, \neg \exists \boldsymbol{y} \in \mathcal{F} : \boldsymbol{f}(\boldsymbol{x}) \succeq \boldsymbol{f}(\boldsymbol{y})\}, \tag{3}$$

and the Pareto front is defined as

$$\mathcal{F}ront = \{\boldsymbol{f}(\boldsymbol{x}) \mid \boldsymbol{x} \in \mathcal{POS}\}. \tag{4}$$

A convenient method to assign rank to solutions is by classifying them into non-dominated fronts [11]. Let us denote $\mathcal{Z}$ the set of solution we want to classify. The first front $\mathcal{F}ront_1$ is obtained from $\mathcal{Z}$ and corresponds to the set of POS in $\mathcal{Z}$. Let us denote this set as $\mathcal{POS}_1$. The subsequent fronts $\mathcal{F}ront_j; j > 1$, contain lower level non-dominated solutions and are obtained by disregarding solutions corresponding to the previous higher non-dominated fronts, i.e., $\mathcal{F}ront_j; j > 1$, is obtained from the set $\mathcal{Z} - \bigcup_{k=1}^{j-1} \mathcal{POS}_k$.

3. Related Works

3.1 Domination Structures

Relaxed forms of Pareto dominance have been discussed within decision making in multiobjective optimization since long ago [6]~[8]. Given a decision space and a corresponding criterion (objective) space, a decision maker has to take a decision about what solution should be chosen. There have been two popular methods used to help making a decision. One of them is known as one dimensional comparison and the other one as Pareto optimality. Yu [6],[7] argued that the one dimensional comparison and Pareto optimality methods are two extreme cases in the entire domain of domination structures and showed that there are infinity valid methods lying between them, which suitability depends on how much information is known on the decision maker's preferences. Following Yu's notation, let us denote the objective space as Y, and a solutions in objective space as $y = (y_1, \cdots, y_m) \in Y$. Given two solutions y and $y^1 \in Y$. A nonzero vector $d \in R^m$ is a *domination factor* for $y \in Y$ iff $y^1 = y + \lambda d$, $\lambda > 0$, implies that y is preferred to y^1, assuming maximization in all objectives. The set of

all domination factors for y, together with the zero vector in R^m is denoted by $D(y)$. The family $\{D(y)|y \in Y\}$ is called the *structure of domination* of the decision problem [6],[7]. An important class of domination structures is $D(y) = \Lambda$ for all $y \in Y$, where Λ is a convex cone. In this case Λ is called the *domination cone*. With the above definitions, we say that $y^1 \in Y$ is *dominated* by $y \in Y$ if

$$y^1 \in y + D(y) = \{y + d|d \in D(y)\} \tag{5}$$

and we say that a point $y^0 \in Y$ is a *nondominated solution* iff

$$\neg \exists y \in Y | y^0 \in y + D(y) \tag{6}$$

A nondominated solution with respect to a domination cone Λ is also called a Λ-extreme point.

Now, it is easy to show that the concept of Pareto optimality is equivalent to using a dominance cone $\Lambda = \{d \in R^m | d \leq 0\}$. In the additive weight method for establishing preferences, one first find a suitable weight $\lambda = (\lambda_1, \cdots, \lambda_m)$ and then maximizes $\lambda \cdot y = \sum \lambda_j y_j$ over Y, $y = (y_1, \cdots, y_m)$. Given $\lambda > 0$, the solution concept implicitly uses the domination cone $\Lambda = \{d \in R^m | \lambda \cdot d \leq 0\}$. The weights establish tradeoffs among objectives, which determine the shape of the domination cone used by the solutions. To illustrate this, lets assume $m = 2$ objectives. The trade-off ratio of y_2 with respect to y_1 is defined by how many units of y_2 we want to sacrifice in order to increase a unit of y_1. Thus, the ratio λ_2/λ_1 gives the value of y_2 in terms of y_1 and the decision problem becomes one of maximizing $\lambda_1 y_1 + \lambda_2 y_2$ or equivalently $y_1 + \frac{\lambda_2}{\lambda_1} y_2$. To correctly predetermine the ratio λ_2/λ_1 is a difficult task. Thus, in practice lower and upper bounds are set for this ratio λ_2/λ_1 to have partial information on the decision maker's preferences, which determine the shape of the domination cone [6]. By varying the range defined by the lower and upper bound for tradeoffs we could achieve different domination cones. In one extreme case, if $0 < \lambda_2/\lambda_1 < \infty$ then we have $\Lambda = \{(d_1, d_2)|d_1 \leq 0, d_2 \leq 0\}$, which corresponds to Pareto optimality that renders the domination cone in the non-positive quadrant. Decreasing the range between lower and upper bound of λ_2/λ_1, the dominance cone extends until it forms a half-space $\Lambda = \{(d_1, d_2)|d_1 + \gamma d_2 \leq 0\}$ when the lower and upper bounds of λ_2/λ_1 are equal to $\gamma (> 0)$, which is the case of one dimensional comparison method.

Note that domination cones covering smaller areas than the minimum area covered by the Pareto optimality concept are not considered in Yu's extension. This is understandable because the motivation for Yu's approach relates to the process of decision making that assumes positive trade-off rates among objectives. On the other hand, in this work we propose to consider dominance areas smaller than the one given by the concept of Pareto optimality as well as larger ones, because the motivation for our work is to improve the search process of MOEAs either by weakening or selection compared to selection induced by conventional Pareto dominance.

3.2 Relaxed Forms of Dominance in MOEAs

Recently, some researchers have proposed the use of relaxed forms of Pareto dominance as a way of regulating convergence of a MOEA. Laummans, et al. [9] proposed a relaxed form of dominance for MOEAs called ϵ-dominance seeking to ensure both properties of convergence towards the Pareto-optimal set and properties of diversity among the solutions found. A solution $\boldsymbol{x}$ ϵ-dominates a solution $\boldsymbol{y}$ for some $\epsilon > 0$, assuming maximization in all objectives, if

$$\forall i \in \{1, 2, \ldots, m\} : (1 + \epsilon) \cdot f_i(\boldsymbol{x}) \geq f_i(\boldsymbol{y}). \tag{7}$$

ϵ-dominance acts as an archiving strategy, where new points are only accepted if they are not ϵ-dominated by any other point of the current archive. Thus, it strengthens Pareto selection during the archiving process. In addition, ϵ-dominance uses a set of boxes to cover the Pareto front, where the size of such boxes is set by the user-defined parameter ϵ. Within each box only one non-dominated solution is retained. Thus, by using a larger value of ϵ the user can accelerate convergence, while sacrificing the quality (preciseness) of the Pareto front obtained. In contrast, if a high quality of the front is required, a small value of ϵ must be adopted. The definition of ϵ is very important. However, it is not simple to find the most appropriate value of ϵ, especially if nothing is known in advance about the shape of the Pareto front. Also, to correlate the number of desired solutions with the value of ϵ chosen is not easy. In addition, ϵ-dominance eliminates the extreme points of the Pareto front, which may be undesirable in some cases.

Another strategy that relaxes Pareto dominance is α-domination proposed by Ikeda, et al. [10] to strengthen selection. The fundamental idea of α-domination is setting upper/lower bounds of trade-offs rates between two objectives [6),7)]. α-domination permits a solution $\boldsymbol{x}$ to dominate a solution $\boldsymbol{y}$ if $\boldsymbol{x}$ is slightly inferior to $\boldsymbol{y}$ in an objective but largely superior to $\boldsymbol{y}$ in some other objectives. To calculate α-dominance, first a relative fitness vector $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y})$ between two solutions must be established. The i-th component of $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y})$ is calculated by

$$g_i(\boldsymbol{x}, \boldsymbol{y}) = f_i(\boldsymbol{x}) - f_i(\boldsymbol{y}) + \sum_{j \neq i}^{m} \alpha_{ij}(f_j(\boldsymbol{x}) - f_j(\boldsymbol{y})) \tag{8}$$

where $f_i(\boldsymbol{x})$ is the fitness value of solution $\boldsymbol{x}$ on the i-th objective, and α_{ij} is the trade-off rate between the i-th and j-th objectives.

A solution $\boldsymbol{x}$ α-dominates a solution $\boldsymbol{y}$, assuming maximization in all objectives, if

$$\begin{aligned} \forall i \in \{1, 2, \ldots, m\} : g_i(\boldsymbol{x}, \boldsymbol{y}) \geq 0 \quad \wedge \\ \exists i \in \{1, 2, \ldots, m\} : g_i(\boldsymbol{x}, \boldsymbol{y}) > 0. \end{aligned} \tag{9}$$

To calculate α-domination, α_{ij} trade-off rates must be properly set for each pair of objectives. Assessing the appropriate trade-offs between objectives could be a difficult problem, especially if nothing is known in advance about the landscape and shape of Pareto front. Thus, α-domination assumes lower and upper bounds for the trade-off rates as explained above. In other words, α-domination strengthens selection because only dominance areas larger than the conventional Pareto dominance are considered.

To summarize, important differences between our work and the above related works are as follows. First, the proposed method can strengthen or weaken selection by expanding or contracting the area of dominance, and conceptually can be considered as a generalization of Pareto dominance. On the other hand, ϵ-dominance and α-domination only strengthen selection. Second, the proposed method focuses on selection at each generation, whereas ϵ-dominance focuses on the archiving process. α-domination focuses on selection, but the approach and its motivation are different. α-domination was tried on an ad hoc continuous problem created specifically to illustrate a potential problem that Pareto selection could face and its effects have not been explained nor tested on standard test suit problems. In ad-
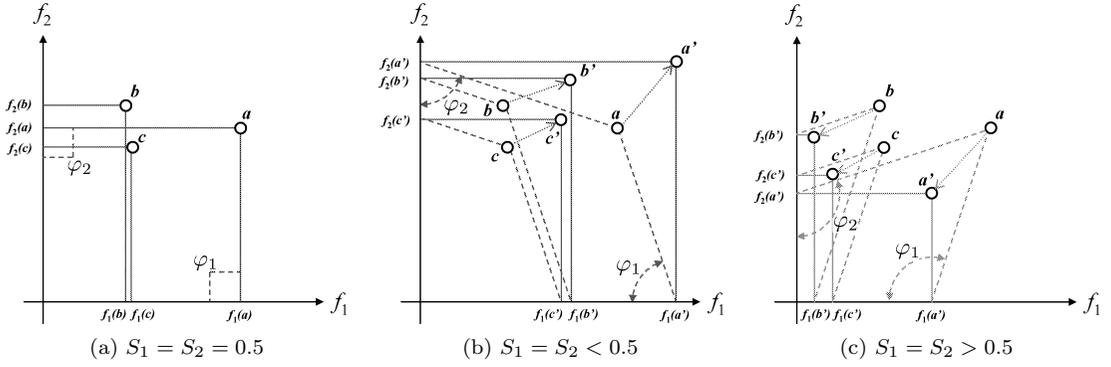
Fig. 2   Conventional dominance (a) and examples of expanding (b) and contracting (c) the dominance area of solutions.
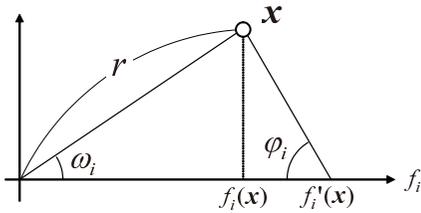
Fig. 1   Fitness modification to change the covered area of dominance.

dition, α-domination cannot weaken selection. Third, in this work we analyze the effects on ranking of solutions by expansion or contraction of the dominance area of solutions, which has not been done before. Moreover, we also analyze the impact on the performance of MOEAs in combinatorial optimization problems.

4. Proposed Method

4.1 Contraction and Expansion of Dominance Area

In this work, we try to control the covered area of dominance. Normally, the dominance area is uniquely determined with a fitness vector $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x}))$ in the objective space when a solution $\boldsymbol{x}$ is given. To contract and expand the dominance area of solutions, we modify fitness value for each objective function by changing the user defined parameter S_i in the following equation

$$f_i'(x) = \frac{r \cdot \sin(\omega_i + S_i \cdot \pi)}{\sin(S_i \cdot \pi)} \qquad (i = 1, 2, \cdots, m) \tag{10}$$

where $\varphi_i = S_i \cdot \pi$. This equation is derived from the Sine theorem. We illustrate the fitness modification in **Fig. 1**, where r is the norm of $\boldsymbol{f}(\boldsymbol{x})$, $f_i(\boldsymbol{x})$ is the fitness value in the i-th objective, and ω_i is the declination angle be-

tween $\boldsymbol{f}(\boldsymbol{x})$ and $f_i(\boldsymbol{x})$. In this example, the i-th fitness value $f_i(\boldsymbol{x})$ is projected (increased) to $f_i'(\boldsymbol{x}) > f_i(\boldsymbol{x})$ by using $\varphi_i < \pi/2$ ($S_i < 0.5$). In case of $\varphi_i = \pi/2$ ($S_i = 0.5$), $f_i(\boldsymbol{x})$ does not change and $f_i'(\boldsymbol{x}) = f_i(\boldsymbol{x})$. Thus, this case is equivalent to the conventional dominance. On the other hand, in case of $\varphi_i > \pi/2$ ($S_i > 0.5$), $f_i(\boldsymbol{x})$ is projected (decreased) to $f_i'(\boldsymbol{x}) < f_i(\boldsymbol{x})$. Such fitness modification changes the dominance area of solutions. We show an example in **Fig. 2** (a)–(c), where three solutions $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are distributed in 2-dimensional objective space. In Fig. 2 (a), $\boldsymbol{a}$ dominates $\boldsymbol{c}$, but $\boldsymbol{a}$ and $\boldsymbol{b}$, and $\boldsymbol{b}$ and $\boldsymbol{c}$ do not dominate each other. However, if we modify fitness values for each solution by using Eq. (10), the location of each solution moves in the objective space, and consequently the dominance relationship among solutions changes. For example, if we use $S_1 = S_2 < 0.5$ as shown in Fig. 2 (b), the dominance area of solutions $\boldsymbol{a}'$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ is expanded from the original one of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. This causes that $\boldsymbol{a}'$ dominates $\boldsymbol{b}'$ and $\boldsymbol{c}'$, and $\boldsymbol{b}'$ dominates $\boldsymbol{c}'$. That is, expansion of dominance area by smaller S_i (< 0.5) works to produce a more fine grained ranking of solutions and would strengthen selection. On the other hand, if we use $S_1 = S_2 > 0.5$ as shown in Fig. 2 (c), the dominance area of solutions $\boldsymbol{a}'$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ is contracted from the original one of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. This causes that $\boldsymbol{a}'$, $\boldsymbol{b}'$ and $\boldsymbol{c}'$ do not dominate each other. That is, contracting the area of dominance by larger S_i (> 0.5) works to produce a coarser ranking of solutions and would weaken selection.

4.2 Effects of Controlling Dominance Area

As indicated above, expanding or contracting the dominance area of solutions change the
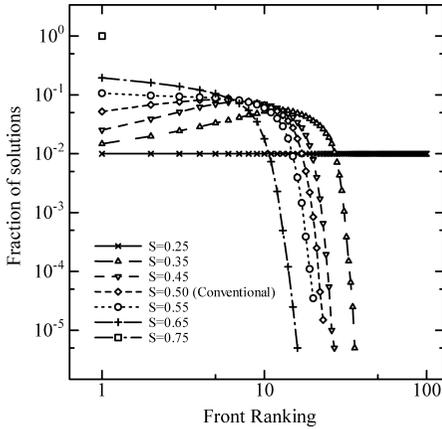
Fig. 3 Solutions per front varying the parameter S.

dominance relation of some solutions and therefore modify the distribution of the fronts (number of fronts and solutions per front). Since front distribution significantly relates to selection, we verify and illustrate the effect of expanding or contracting the dominance area on the distribution of the fronts changing the parameter S_i in Eq. (10). Here, we randomly generate 100 solutions in the 2-dimensional objective space of $[0, 1]^2$, calculate dominance among them after recalculating fitness with Eq. (10), and perform a non-domination sorting to obtain the fronts. We repeat the above steps a 1,000 times and calculate the average number of fronts and solutions per front, for each value of S_i. In this work, we use a common parameter $S = S_i$ $(i = 1, 2, \cdots, m)$ for all objective functions, because we assume that all objective functions are normalized. **Figure 3** shows the fraction of number of solutions per front varying S in the range $[0.25, 0.75]$ in intervals of 0.1 along with results for conventional dominance $(S = 0.5)$.

From this figure, note that if we gradually expand the area of dominance by decreasing S below 0.5, the number of fronts increases and the ranking of solutions by non-dominance can be fine grained. Note that for maximum expansion of the dominance area $S = 0.25$ there is one solution per front. On the other hand, if we gradually contract the area of dominance by increasing S above 0.5, the number of fronts decreases and ranking of solutions by non-dominance becomes coarser. Note that for maximum contraction of the dominance area $S = 0.75$ there is only one front that contains all solutions. Since different rankings can be produced, we can expect that the optimum parameter S^* that yields

maximum search performance exists for a given kind of problem.

5. Benchmark Problems, Metrics, and Parameters

In this paper we use multiobjective 0/1 knapsack problems [12] as benchmark problems to study and compare the effects on search performance of ranking solutions by expanding or contracting their dominance area. The problem (KPn-m) is formulated to maximize the function

$$f_j(\boldsymbol{x}) = \sum_{i=1}^{n} x_i \cdot p_{i,j} \tag{11}$$

subject to

$$g_j(\boldsymbol{x}) = \sum_{i=1}^{n} x_i \cdot w_{i,j} \le W_j \tag{12}$$

where $x_i \in \{0, 1\}$ $(i = 1, 2, \cdots, n)$ are elements of solution vector $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$, which gives a combination of items. Thus, we use binary representation in this work. Note that here we are interested in finding a set of non-dominated Pareto solutions. Also, $p_{i,j}$ and $w_{i,j}$ $(j = 1, 2, \cdots, m)$ denote profit and weight of item i according to knapsack (objective) j. W_j is the capacity of knapsack j, and solutions not satisfying this condition are considered as infeasible solutions $\bar{\mathcal{F}} = (\mathcal{S} - \mathcal{F})$. We can control the complexity (difficulty) of the problem by varying the number of objectives m, the number of items n, and the feasibility ratio of the search space ϕ. In this paper, we use benchmark problems with $m = \{2, 3, 4, 5\}$ objectives, $n = \{100, 250, 500, 750\}$ items and feasibility ratio $\phi = \{0.75, 0.5, 0.25\}$ downloaded from Ref. 13), for which we know the true Pareto non-dominated set only in case of two objectives $m = 2$. In these particular problems, we use a constant S for all objectives because the scale of each objective function is similar.

The hypervolume is used as a metric to evaluate sets of non-dominated solutions obtained by MOEAs. The hypervolume measures the m-dimensional volume of the region in objective space enclosed by the obtained non-dominated solutions and a dominated reference point [14]. Here we use $(f_1, f_2, \cdots, f_m) = (0, 0, \cdots, 0)$ as the reference point to calculate the hypervolume. A set of non-dominated solutions showing higher value of hypervolume can be considered as a better set of solutions from both convergence and diversity viewpoints. The hypervol-

ume metric is a reliable metric and it is among the few recommended metrics to compare non-dominated sets [15]. To provide additional information separately on convergence and diversity of the obtained solutions in this work we also use Inverse Generational Distance (IGD) [16] and Spread (SP) [1], respectively. IGD takes the average distance for all members in the true Pareto front to their nearest solutions in the obtained set of non-dominated solutions (exactly the inverse process followed by Generational Distance GD [17]). Spread measures the degree of dispersion on the distribution of the obtained solutions. A set of Pareto non-dominated solutions showing smaller Spread can be considered as better solutions satisfying diversity condition.

In our study we compare the performance of a conventional NSGA-II [11] with NSGA-II enhanced by the proposed method. We adopt two-point crossover with a crossover rate $p_c = 1.0$ for recombination, and apply bit-flipping mutation with a mutation rate $p_m = 1/n$. In the following experiments, we show the average performance with 30 runs, each of which spent 2,000 generations. Population size is set to $|P| = 200$ and the parent and offspring population sizes $|Q|$ and $|R|$ are set to half the population size $|P|$, i.e., $|Q| = |R| = 100$.

6. Experimental Results and Discussion

6.1 Performance Varying the Number of Objectives

In the following sections we observe the effects of varying the parameter S that controls the area of dominance of the solutions on the performance of the algorithm measured by the hypervolume. Recall that $S = 0.5$ indicates conventional dominance, values of $S > 0.5$ indicate contraction of the dominance area of the solutions, and values of $S < 0.5$ indicate expansion of the dominance area of the solutions.

First, we observe the effect of varying S on problems with different number of objectives. **Figure 4** shows the values of the hypervolume achieved varying S in the range $[0.25, 0.75]$ in intervals of 0.05 on problems with $m = \{2, 3, 4, 5\}$ objectives, $n = 500$ items, and feasibility ratio $\phi = 0.50$. Note that in the figure the values of the hypervolume are normalized so that the value achieved at $S = 0.5$ is always 1.0. From this figure important observations are as follow. First, there is an optimum value S^* for
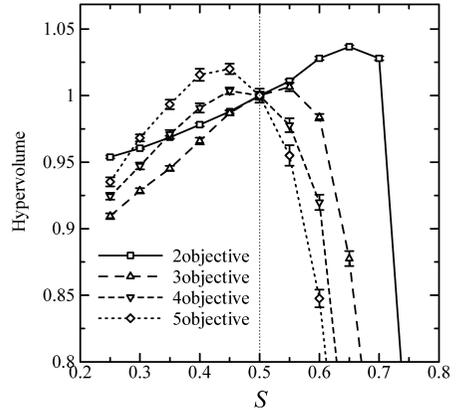
Fig. 4 Hypervolume as we increase the number of objectives m for problems with $n = 500$ items and $\phi = 0.5$ feasibility ratio.

each number of objectives that maximizes the hypervolume. Note however that the maximum value of hypervolume is not achieved by conventional dominance $(S = 0.5)$ for any number of objectives. Second, to achieve the maximum value of hypervolume, the degree of expansion or contraction of dominance area of solutions should be adjusted accordingly to the number of objectives. Note that maximum values of the hypervolume are achieved for two and three objectives by contracting the dominance area of the solutions $(S > 0.5)$, whereas for four and five objectives the maximum hypervolume values are achieved by expanding the dominance area of the solutions $(S < 0.5)$. Third, as a general trend in problems with $n = 500$ items and feasibility ratio $\phi = 0.50$, we observe that the optimum value S^* reduces as we increase the number of objectives. That is, increasing the number of objectives the area of dominance should be expanded by using smaller values of S^* to achieve maximum hypervolume.

Figure 5 (a) and (b) show the front distribution over generation by conventional dominance $(S = 0.5)$ and by contracting dominance with the optimum parameter $(S^* = 0.65)$, respectively, on $m = 2$ objectives, $n = 500$ items and feasibility ratio $\phi = 0.5$. Similarly, **Fig. 6** (a) and (b) show on $m = 4$ objectives the front distributions by conventional dominance $(S = 0.5)$ and by expanding dominance with the optimum parameter $(S^* = 0.45)$, respectively. Results are presented for the ten top fronts obtained from the combined population of parents and offspring before truncation. The horizontal line indicates the truncation point after front non-domination sorting. These figures illustrate and

(a) $S = 0.5$, conventional dominance

(b) $S^* = 0.65$, best by contracting dominance
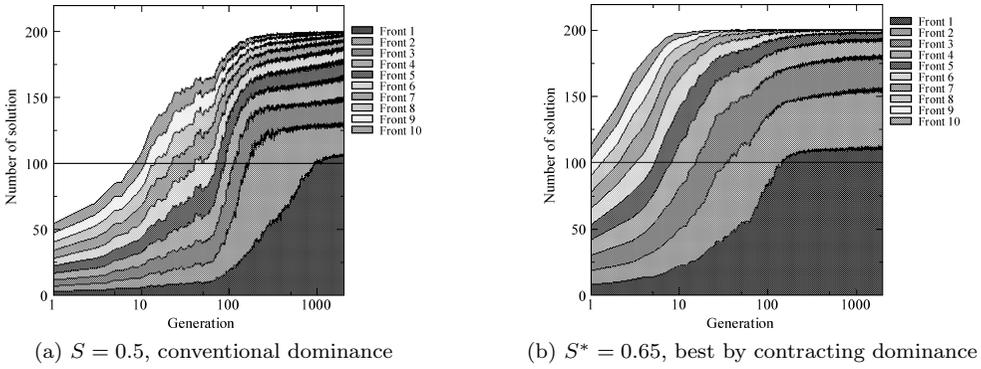
Fig. 5   Front distribution over generation $m = 2$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio.

(a) $S = 0.5$, conventional dominance

(b) $S^* = 0.45$, best by expanding dominance
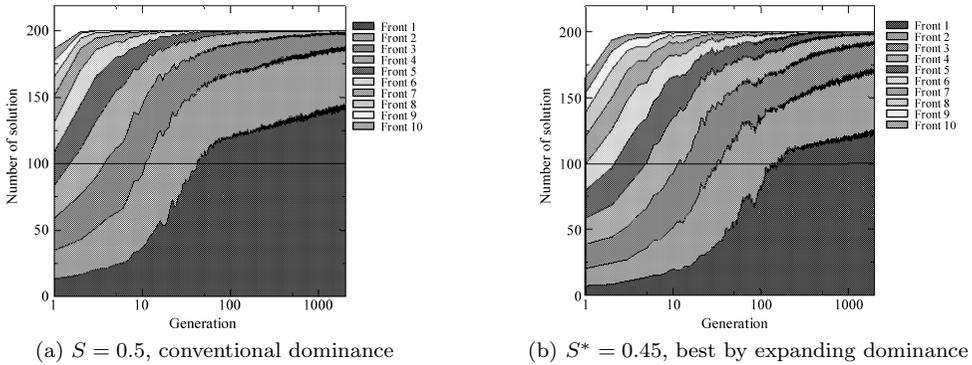
Fig. 6   Front distribution over generation $m = 4$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio.

corroborate our expectation that contraction or expansion of area of dominance changes the ranking of solutions. Remember that contraction of the area of dominance weakens selection and induces a coarse ranking of solution, as illustrated in Fig. 5 (a) and (b), which works better for two and three objectives. Also, remember that an expansion of the area of dominance strengthen selection and induces a fine grained ranking of solutions, as illustrated in Fig. 6 (a) and (b), which works better for four and five objectives.

6.2 Performance Varying the Size of the Search Space

Second, we observe the effects of varying S on problems with different number of items n. Note that the size of the search space is given by 2^n. **Figure 7** shows the hypervolume varying S on problems with $n = \{100, 250, 500, 750\}$ items and feasibility ratio $\phi = 0.5$ for $m = \{2, 3, 4, 5\}$ objectives. From Fig. 7 (a) we can see that in the case of $m = 2$ objectives the optimum S^* is similar for all n, around 0.65. However, from Fig. 7 (b), (c), and (d) we ob-

serve that increasing the number of items n produces a clear shift of the optimum S^* towards smaller values (greater expansion of area of dominance), especially in the case of $m = 4$ and $m = 5$ objectives. For example, for $m = 4$, note the optimal $S^* = \{0.55, 0.5, 0.45, 0.45\}$ on $n = \{100, 250, 500, 750\}$, respectively. In the previous section, fixing the number of items to $n = 500$, results suggested that the degree of expansion or contraction of dominance area of solutions should be adjusted according to the number of objectives. The results presented in this section suggest that the degree of expansion or contraction of dominance area of solutions should also be adjusted according to the size of the search space, especially for an increased number of objectives.

6.3 Performance Varying the Search Space Feasibility Ratio ϕ

Third, we observe the effects of varying S on problems with different feasibility ratio ϕ. **Figure 8** shows the hypervolume varying S on problems with feasibility ratio $\phi = \{0.75, 0.5, 0.25\}$ and $n = 500$ items for $m =$

(a) $m = 2$ objectives

(b) $m = 3$ objectives

(c) $m = 4$ objectives
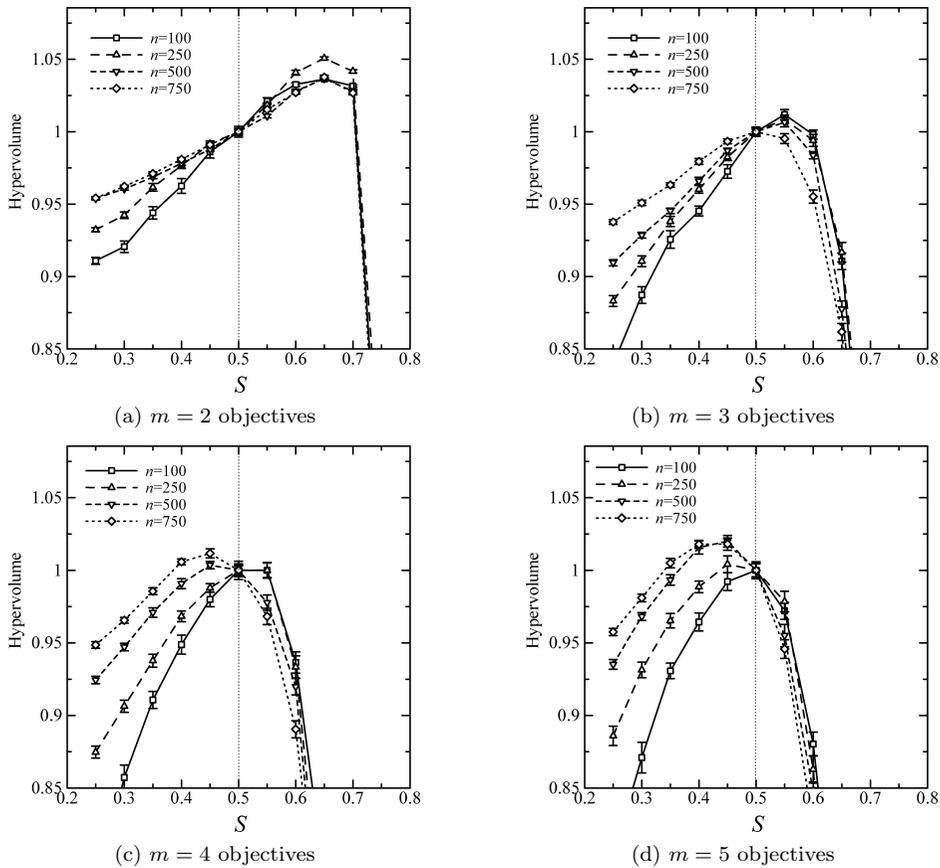
(d) $m = 5$ objectives

Fig. 7 Hypervolume as we increase the number of items n for problems with $m = \{2, 3, 4, 5\}$ objectives and $\phi = 0.5$ feasibility ratio.

$\{2, 3, 4, 5\}$ objectives. From Fig. 8 (a)–(d) note that the effects on problems with different feasibility ratio ϕ resemble those observed on problems with different number of items. That is, in $m = 2$ objectives the optimum S^* is the same for all ϕ. However, reducing the feasibility ratio ϕ from 0.75 to 0.25, there is a shift of the optimum S^* towards smaller values, which becomes more notorious for $m = 4$ and $m = 5$ objectives. For example, for $m = 4$, note the optimal $S^* = \{0.55, 0.45, 0.4\}$ on $\phi = \{0.75, 0.5, 0.25\}$, respectively. These results suggest that the optimum degree of expansion or contraction of dominance area of solutions also depends on the feasibility ratio of the search space (complexity of the landscapes), especially for an increased number of objectives.

Summarizing, the optimum degree of expansion or contraction of the dominance area depends on the three aspects investigated in this work; that is, number of objectives, size of the search space, and feasibility ratio of the search

space. For most real world combinatorial problems we can know in advance the number of objectives and size of the search space. Based on these information, we can use the results presented here as a good initial guidelines to properly set the degree of expansion or contraction of the area of dominance in order to achieve higher performance. However, the feasibility ratio (or complexity of the single objective landscapes) is usually unknown. It would be interesting to find adaptive ways to fine tune the parameter S for problems of different complexity.

6.4 Results on Complementary Metrics and Obtained Solutions

Figure 9 (a) and (b) show the Inverse Generational Distance IGD and Spread SP, respectively, varying the number of items on problems with $m = 2$ objectives and feasibility ratio $\phi = 0.5$. From these figures note that optimum IGD and SP (smaller values) are achieved when the dominance area of solutions
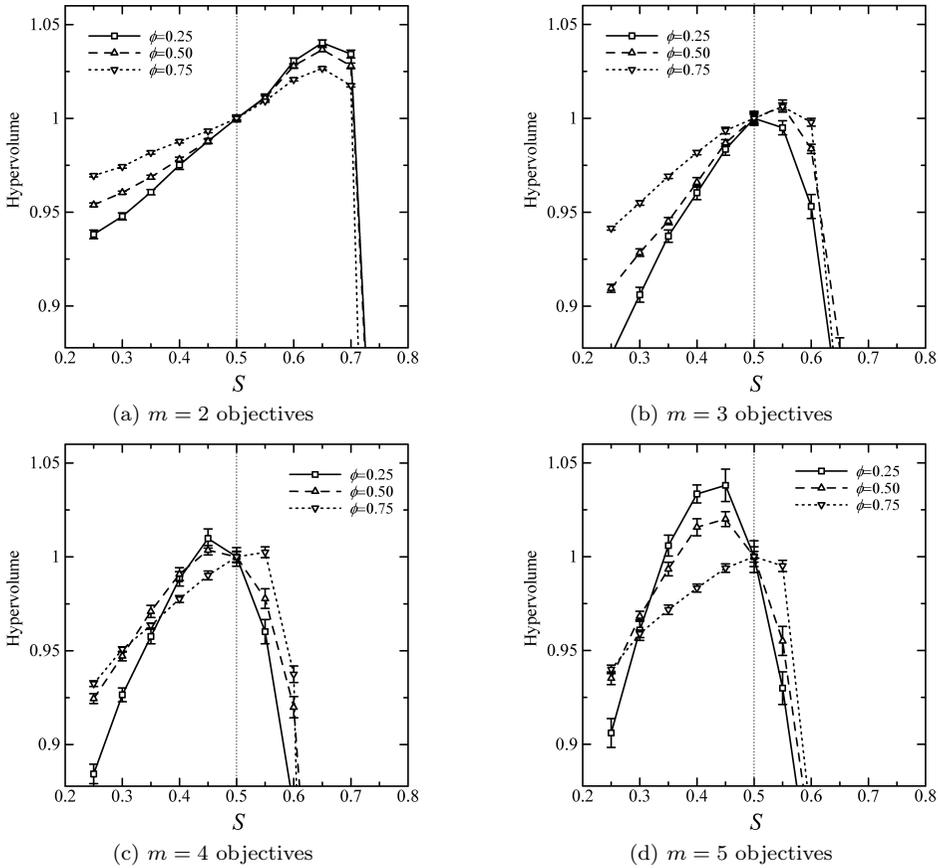
Fig. 8 Hypervolume as we decrease feasibility ratio for problems with $m = \{2, 3, 4, 5\}$ objectives and $n = 500$ items.
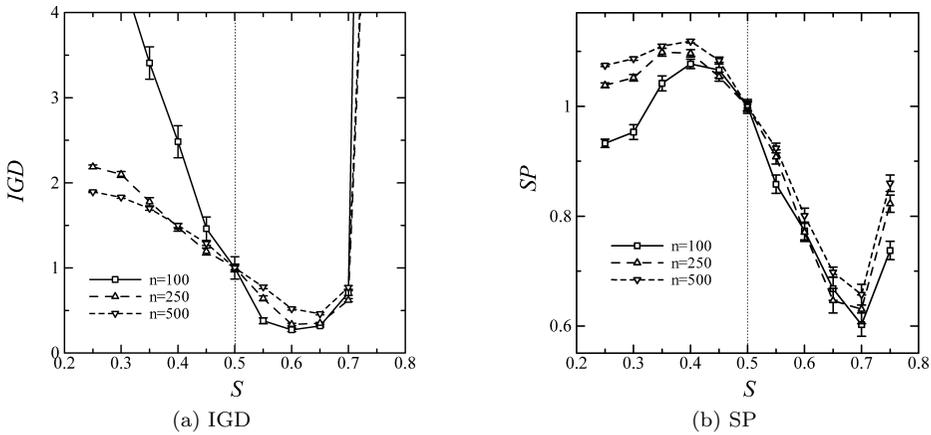
Fig. 9 Inverse generational distance IGD and Spread SP varying the number of items n on problems with $m = 2$ objectives and $\phi = 0.5$ feasibility ratio.

is contracted ($S > 0.5$) rather than by conventional dominance ($S = 0.5$), similar to the results shown in Fig. 7 (a). The values S achieving minimum IGD are almost coincident with

$S^* = 0.65$ achieving maximum hypervolume. However, in case of SP the values S are slightly shifted towards larger values. Also, note that the graph of SP shows a maximum peak in the

(a) Conventional dominance
($S = 0.5$)

(b) Contracting dominance
($S^* = 0.65$)

(c) Expanding dominance
($S = 0.40$)
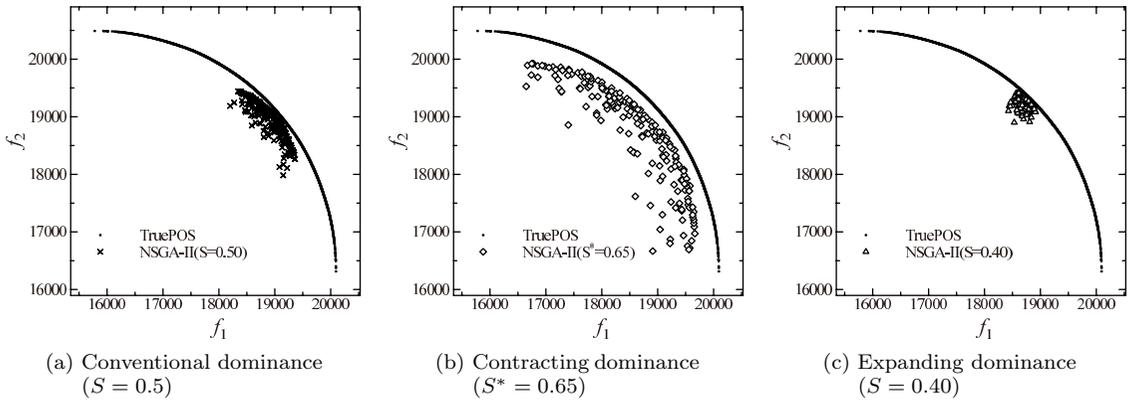
Fig. 10 Obtained solutions by conventional dominance $S = 0.5$, contracting dominance $S^* = 0.65$, and expanding dominance $S = 0.4$ for $m = 2$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio.

area of $S < 0.5$ and a minimum peak in the area of $S > 0.5$.

To analyze the above observations further, **Fig. 10** illustrates the obtained solutions in the final generation ($t = 2{,}000$) for all 30 simulations by conventional dominance $S = 0.5$, contracting dominance $S^* = 0.65$, and expanding dominance $S = 0.4$ for $m = 2$ objectives, $n = 500$ items, and $\phi = 0.5$ feasibility ratio. Note that solutions obtained by conventional dominance are close to the true Pareto front but are clustered in a limited region of objective space. By contracting dominance with the optimum parameter $S^* = 0.65$, we can spread the obtained solutions showing the maximum hypervolume, although convergence of some of them seems to deteriorate. On the other hand, by expanding dominance with $S = 0.4$ showing the maximum SP (worst spread), we can focus on convergence of solutions within a narrower region of objective space.

6.5 Transition of Current Non-dominated Solutions

Figure 11 illustrates the transition of the current non-dominated solutions and the non-dominated solutions on the projected objective space (after contraction/expansion of dominance area) used for the selection process as explained in Section 4. Results are shown for $t = \{1, 10, 30, 100, 300, 2{,}000\}$ generations. Looking at solutions in the conventional (non-projected) objective space, from Fig. 11 (b.1) and (c.1), we can observe that since early generations ($t \geq 100$) diversity is favored at the expense of convergence by contraction of area of dominance with $S^* = 0.65$, while convergence is favored at the expense of diversity by expan-

sion of area of dominance with $S = 0.40$. Next, we look at solutions in the projected space. If we contract the dominance area of solutions ($S^* = 0.65$), we can see from Fig. 11 (b.2) that solutions approach closely to the projected true Pareto front. Also, note that from generation $t \geq 300$ the diversity of solutions increases remarkably. On the other hand, if we expand dominance area of solutions ($S = 0.40$), we can see from Fig. 11 (c.2) that the algorithm directs its search towards a limited region of the true Pareto front, within which high convergence is achieved.

To summarize, by controlling the dominance area of solutions changing parameter S we induce different projected objectives spaces in which the algorithm emphasizes different functionality (diversity/convergence). It would be an interesting subject of research to investigate the balance between diversity and convergence within a single run by adapting properly the parameter S.

7. Comparison with α-domination

In this section we compare results by the proposed method and α-domination in terms of hypervolume and computational time along with highlighting some important differences between both approaches.

7.1 Hypervolume

Figure 12 (b), (d) show the obtained hypervolume on problems with $m = \{2, 5\}$ objectives by α-domination varying α from 0.0 to 1.0 in intervals of 0.1. Here, we use a common parameter $\alpha = \alpha_{ij}$ ($i \neq j; i, j = 1, 2, \cdots, m$) for all tradeoff ratios. Note that $\alpha = 0.0$ is equivalent to conventional Pareto dominance
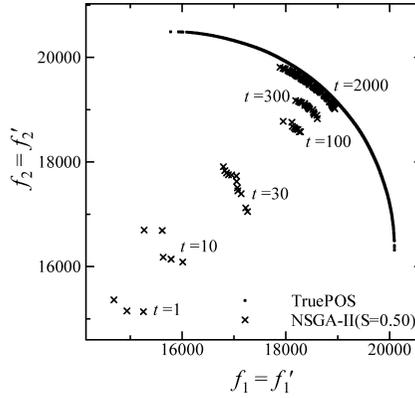
(a) Conventional dominance ($S = 0.5$)
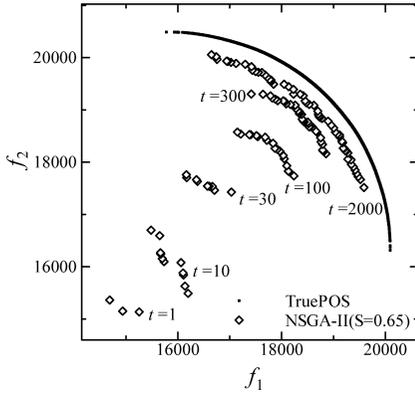
(b.1) Conventional objective space
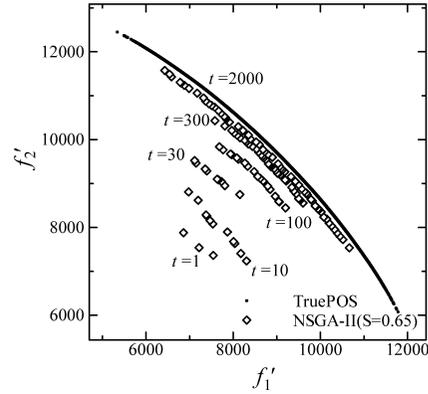
(b.2) Projected objective space

(b) Contracting dominance ($S^* = 0.65$)
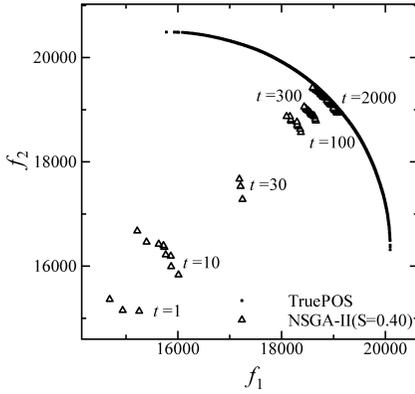
(c.1) Conventional objective space
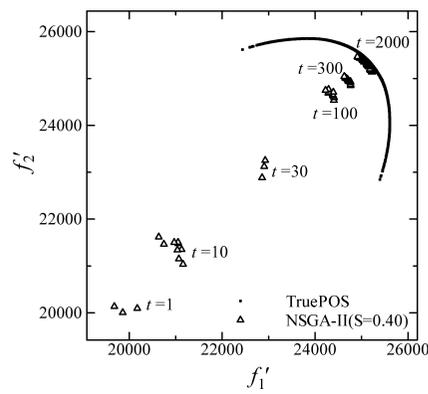
(c.2) Projected objective space

(c) Expanding dominance ($S = 0.40$)

Fig. 11 Current non-dominated solutions at $t = \{1, 10, 30, 100, 300, 2,000\}$.

and that selection is strengthened (dominance area increased) by increasing the value of α. Similarly, Fig. 12 (a), (c) show the obtained hypervolume by the proposed method varying S from 0.25 to 0.75. Remember that in the proposed method $S = 0.5$ is equivalent to conventional Pareto dominance, selection is strengthened (dominance area increased) by reducing S below 0.5 ($S < 0.5$), and selection is weakened (dominance area reduced) by increasing S above 0.5 ($S > 0.5$).

Looking at results by α-domination for $0.0 \leq$

(a) Proposed method, $m = 2$

(b) α-domination, $m = 2$

(c) Proposed method, $m = 5$
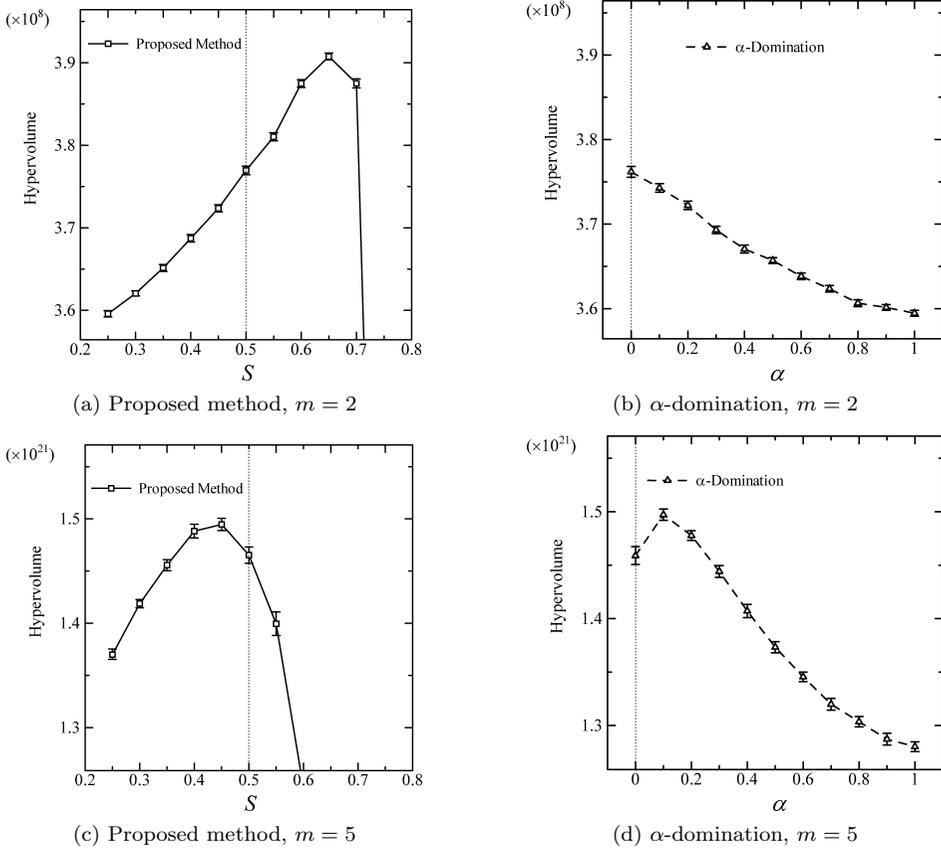
(d) α-domination, $m = 5$

Fig. 12 Hypervolume by proposed method and α-domination on problems with $m = \{2, 5\}$ objectives, $n = 500$ items and $\phi = 0.5$ feasibility ratio.

$\alpha \leq 1.0$ and the proposed method for $0.25 \leq S \leq 0.5$, i.e., where selection is strengthened, we can see that the performance trend in terms of hypervolume is similar by both approaches. For example, in Fig. 12 (c) and (d) for $m = 5$ objectives, note that the highest hypervolume can be achieved by using a somewhat stronger selection than conventional Pareto dominance. In the case of α-domination this is achieved by increasing α from 0.0 to 0.1, whereas in the case of the proposed method this is achieved by reducing S from 0.5 to 0.45. Also, note that strengthening selection further by either method ($\alpha > 0.1$ or $S < 0.45$) the hypervolume deteriorates.

Thus, on problems where stronger selection needs to be used (compared to conventional Pareto dominance), either α-domination or the proposed method could be used to achieve a higher hypervolume. However, as it has been discussed throughout this paper, there could

be many problems where selection actually needs to be weakened (compared to conventional Pareto dominance) either because of the number of objectives, size of the search space, or feasibility ratio of the search space. In these cases, the proposed method can achieve higher values of hypervolume because selection can be weakened by simple increasing S above 0.5. This can clearly be seen between Fig. 12 (a) and (b) for $m = 2$ objectives, where the proposed method by weakening selection ($S = 0.65$) can achieve much higher hypervolume than α-domination.

7.2 Conceptual Difference and Computational Cost

An important conceptual difference between the proposed method and α-domination is the projected space used to perform calculations. In the proposed method we project efficiently all solutions to another objective space, where we can manipulate objective values to perform
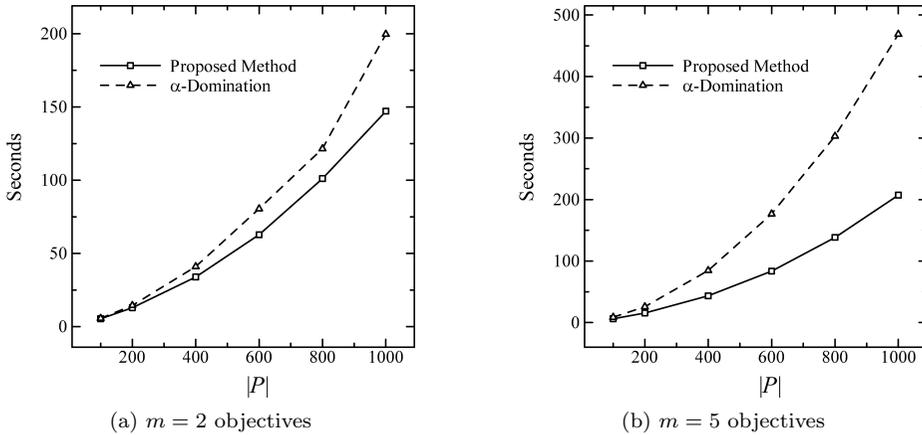
Fig. 13 Computational time by proposed method and α-domination vary-
ing the population size $|P|$ on problems with $m = 2$ and $m = 5$
objectives, $n = 500$ items and $\phi = 0.5$ feasibility ratio.

MOEAs operations on sets of solutions, such
as calculation of dominance, crowding distance,
and so on. On the other hand, α-dominations
maps the relationship between each pair of so-
lutions to a two-region separable space, where
dominant solutions clearly fall in one region and
non-dominated solutions in the other one. In
this two-region space no information about the
objectives is preserved and other operations in-
volving objective values, either between the two
solutions or over sets of solutions, cannot be di-
rectly performed.

One important implication of this concep-
tual difference is the computational cost needed
to calculate dominance by both methods. To
calculate α-domination we must first calculate
the g_i values of the two-region separable space
for pairs of individuals as indicated in Eq. (8).
Since to calculate each g_i value we need to
compute differences between m objectives, and
since there are m pieces of g_i for each pair of
individuals in the population, it requires a com-
putational order of $\mathcal{O}(m^2 N^2)$, where the num-
ber of objectives is m and the size of the popu-
lation is $|P| = N$. Once these values have been
computed, we can calculate dominance among
all individuals by using Eq. (9), which requires
a computational order of $\mathcal{O}(mN^2)$. Thus, the
overall computational order to calculate domi-
nance with α-domination is $\mathcal{O}(m^2 N^2 + mN^2)$.
On the other hand, in the proposed method we
compute new objective values as indicated by
Eq. (10), projecting the solutions to another ob-
jective space. This process only requires a com-
putational order of $\mathcal{O}(mN)$. Then, we can work
in the projected space where the calculation of

dominance requires a computational order of
$\mathcal{O}(mN^2)$. Thus, the overall computational or-
der to calculate dominance with the proposed
method is $\mathcal{O}(mN + mN^2)$, which is less than
$\mathcal{O}(m^2 N^2 + mN^2)$ required by α-domination.

Figure 13 shows the actual computational
time (CPU time) by α-domination and the pro-
posed method varying the size of the population
$|P|$ on a problem with $m = 2$ and $m = 5$ ob-
jectives, $n = 500$ items and $\phi = 0.5$ feasibility
ratio. Both algorithms run for 2,000 genera-
tions in the same computer, configured with
CPU Pentium IV 3.4 GHz. For α-domination
we set $\alpha = 0.0$ and for the proposed method
$S = 0.5$, so both methods render results similar
to conventional Pareto dominance in terms of
hypervolume. From the figure we can see that
the proposed method is computationally more
efficient than α-domination as the number of
objectives increases or the population size $|P|$
increases, which is in accordance with the above
computational orders.

α-domination is inspired from works on de-
cision making [6],[7] and included within the se-
lection process of a MOEA without paying at-
tention to its efficiency. During decision mak-
ing, the decision maker analyzes already found
solutions in order to choose one or more so-
lutions that satisfy his aspirations. For each
run (or probably multiple runs) of an opti-
mizer, the decision process would be usually
run once, so an inefficient calculation of pref-
erences might go unnoticed. However, in the
case of α-domination, dominance must be cal-
culated at each generation and its inefficiency
becomes quite apparent as shown in Fig. 13. On

the contrary, the proposed method is designed to work efficiently as a part of the instantaneous selection process of a MOEA that could evolve a population of solutions for a large number of generations.

8. Conclusions

In this work we have proposed a method that can control dominance area of solutions by a user defined parameter S. We showed that contracting or expanding the dominance area of solutions changes their dominance relation, modifying the distribution of solutions (number of fronts and number of solutions per front) in the multiobjective landscape. Since front distribution significantly relates to selection, we analyzed the effects on solutions ranking caused by contracting and expanding the dominance area of solutions and its impact on the search performance of a multi-objective optimizer. We used 0/1 multiobjective knapsack problems as benchmark problems and showed that the optimum value of S^* depends strongly on number of objectives, size of the search space, and feasibility ratio of the search space (complexity). In addition, we showed that significantly better performance can be achieved either on convergence or diversity of obtained solutions by contracting or expanding the dominance area rather than by using conventional dominance. Furthermore, we compared results by the proposed method with α-domination.

In this work, we have assumed a constant parameter $S_i = S$ on all objectives ($i = 1, 2, \cdots, m$) to control the expansion or contraction of dominance area. It would be interesting in the future to investigate the effect of varying S_i for each objective and control S adaptively, especially for problems of unknown characteristics. For example, a possible way to include adaptation could be to adjust instantaneously the value of S according to the number of obtained fronts at each generation. Another way would be changing S adaptively by observing the balance between convergence and diversity in the population. However, in both ways the determination of an appropriate change (increase or decrease) of S is a critical issue that must be carefully investigated in detail.

In addition, it could be valuable to combine this approach with the inclusion of preferences to guide the search towards a particular region of objective space. With the proposed method, we can improve either convergence or diversity of solutions but not simultaneously both. Therefore, we would like to combine the proposed method with other selection methods to achieve higher convergence while covering the whole true Pareto front. Furthermore, we should try our method on other kind of problems with more objectives and compare our method with other approaches that aim to solve many objective optimization problems.

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