# A Research on the Self－optimizing Distributed Algorithm Constructing Rectilinear Steiner Tree in Virtual Grid Networks 

Yonghwan Kim $\dagger \quad$ Toshimitsu Masuzawa $\ddagger$<br>$\dagger$ Graduate School of Engineering，Nagoya Institute of Technology，Aichi，Japan<br>$\ddagger$ Graduate School of Information and Technology，Osaka University，Osaka，Japan

## 1 Introduction

Recently，wireless networks，such as MANETs or Sensor Networks，become popular and important in the distributed systems．In the wireless networks，each node can directly communicate only with nodes within its communication range．If the destination node is outside of the commu－ nication range of the source node，the message should be relayed to the destination node．The topology of wireless networks can be changed frequently because of the mobil－ ity of a node，node failures，or resource scarcity．There－ fore，the key issues of the wireless routing protocols con－ tain adaptability to the network dynamics and reduction of resource consumption．

We suppose one special node（named a home node）and several moving nodes（named target nodes）in virtual grid networks，which is obtained by virtually dividing a wireless network into a grid of geographical square regions called cells of the same size．A single node is selected as a router at each cell and inter－cell communication is realized by us－ ing the routers．And we assume that the set of paths，which consists of each path to each target node from a home node， are given．

In this paper，we consider maintenance of inter－cell com－ munication paths to each target node from the node．As we mentioned above，the power consumption is one of the important issue in wireless sensor networks，therefore，we consider that our goal is the construction of a rectilinear steiner tree（means the steiner tree on the grid topology， RST）which connects a home node to all target nodes．Steiner tree ensures that the total number of edges between routers from its definition．However，construction of RST in grid networks is known as $N P$－hard［2］problem．This implies that the designing of an distributed algorithm，which can find an optimal solution using local information only，is nearly impossible．Thus，in this paper，we firstly introduce the optimizing protocol with only three nodes，two target nodes and one home node，in a virtual grid network．More－ over，we briefly discuss an optimizing protocol which con－ structs RST for 4 or more nodes in virtual grid networks．

## 2 3－nodes RST Protocol

We adopt a routing protocol named Zigzag to make a short－ est path to each target node from a home node，and we propose some new protocols to combine some parts of two paths for reduction the number of edges contained in RST．


Figure 1：Three local updates to a path in Zigzag

## 2．1 A routing protocol Zigzag and its modification

Zigzag is a local－information－based self－optimizing routing protocol in virtual grid networks［1］．Protocol Zigzag find a shortest path between two nodes by repeatedly applying lo－ cal updates to the path until it converges to a shortest path． Zigzag detects a redundancy of the recent path locally with making zigzag－based path．Zigzag defines only three local updates on the node $p_{i}$ as Fig． 1.

We consider the combining of two paths which are trans－ formed（or been transforming）by Zigzag．However，the converged shortest path can be different depending on the initial path，even if the positional relation between two nodes is exactly same．Therefore，we modify the protocol Zigzag slightly．If a home node finds some specific starting direc－ tions，a home node updates those directions．Table 1 shows the detail rules of starting directions．

Table 1：Rule for fixing the starting directions

| 2－hop directions from home | modified directions |
| :---: | :---: |
| \｛UP，RIGHT\}(1st quadrant) | \｛RIGHT，UP\} |
| \｛LEFT，UP\}(2nd quadrant) | \｛UP，LEFT\} |
| \｛DOWN，LEFT\}(3rd quadrant) | \｛LEFT DOWN \} |
| \｛RIGHT，DOWN\}(4th quadrant) | \｛DOWN，RIGHT\} |

This modification makes it an open possibility to com－ bine two paths．We introduce our protocol in the Section 2．2．

## 2．2 Path Combining Protocol

In this Section，we introduce our protocol to reduce the number of edges combining two paths．In the previous Section，we modified the protocol Zigzag thus we can eas－ ily find the positional relationship between two paths．We explain how to combine two paths in detail．To easy to understand，first of all，we consider the case when two tar－ get nodes are in the neighboring quadrants（e．g．，one target node is in the 1st quadrant，and another one is in the 2nd quadrant）．

Fig．2（a）shows an example case when two target nodes are in the neighboring quadrants， 1 st and 2 nd quadrants． In this case，we can find $U$－shaped path，from $(0,1)$ to $(1,1)$ ，on a home node（the origin）．From this U－shaped path，a home node recognizes that two target nodes are in the neighboring quadrants and two paths can be com－ bined．Note that，this recognition might be incorrect be－ cause Zigzag is not converged yet，but a home node can decide it at the current moment and our protocol can re－ solve this local miss．Fig．2（b）presents the situation after combining detected U－shaped path．Total number of edges is less than Fig．2（a），however we cannnot find more com－ bining points although this is not optimal solution．There－ fore，we introduce a new virtual node named virtual home node（ $v$ Home）．$v$ Home is located on the home node ini－ tially，and after combining two path on U－shaped path as Fig．2（b），vHome moves one hop along the combined path． In the case of Fig．2（b），vHome is located on（ 0,1 ）．vHome operates exactly same as a home node，this changes the Zigzag paths on Fig．2（b）．The path to left target changes its starting directions \｛UP，LEFT\} due to the modification of Zigzag（Section 2．1）．The zigzag part of the path to right target moves left by Zigzag protocol．

Fig．3（a）illustrates the path after vHome is located on $(0,1)$ ．Because of local updates of Zigzag protocol and its modification，our protocol can find the combining point again．A virtual home moves repeatedly and this protocol can be eventually converged as Fig．3（b）．

Our protocol using vHome basically updates the path，if it finds U－shaped path．However，we cannnot know whether a protocol Zigzag is converged or not using local informa－ tion only．As we mentioned，our protocol＇s local update （combining）is sometimes incorrect．However，our protocol allows not only moving forward but also moving backward of $v$ Home when it finds U－shaped path．

Finally we introduce one more simple rule to our pro－ tocol．By our protocol，the path to $v$ Home from a home node is created．When $v$ Home is not on the origin，if the direction just after $v$ Home is the reverse direction of one just before vHome，vHome moves backward with one hop


Figure 2：An example case of neighboring quadrants


Figure 3：Coordinating using a virtual home


Figure 4：Three rules of our protocol
because the last hop of common path is redundant trivially． Fig． 4 summarizes the local update＇s rule of our protocol．

## 3 N－nodes RST Protocol

We briefly introduce our distributed algorithm which con－ structs the RST with 3 nodes．Our proposed protocol can ensure the minimum RST construction using local informa－ tion only．Unfortunately，an improvement of our protocol which can construct the RST with 4 or more nodes is diffi－ cult．Our protocol uses vHome，which becomes the steiner point after convergence．A RST with N nodes has at most （N－2）steiner points，this implies the a RST with 3 nodes may have a single steiner point（ vHome ）．More vHomes are required for constructing the RST with 4 or more nodes．It has 3 or more path（from home to each target）and many U－shaped path can be created and collided．This means the generalized protocol for constructing the RST can deal with these problems．However，selective combining pro－ tocol using local information only is impossible，thus，we consider an approximation algorithms which can construct the RST with a reasonable number of edges．

## References

［1］S．Takatsu，F．Ooshita，H．Kakugawa，and T． Masuzawa，＇＂Zigzag：Local－information－based self－ optimizing routing in virtual grid networks，＂Proceed－ ings of the 33rd International Conference on Dis－ tributed Computing Systems（ICDCS），pp．358－368， 2013.
［2］F．K．Hwang，＂On Steiner Minimal Trees with Rec－ tilinear Distance，＂SIAM Journal on Applied Mathe－ matics，Vol．30，No．1，pp．104－114， 1976.

