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測地座標系を用いた均等色空間の構築

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あらまし
均等色空間 (UCS) の構築は色彩工学，画像処理において最も重要な課題の一つである．現在まで，色空間から均等色空間への写像を近似的に求める手法が主に考えられてきた．その結果，大域均等性と局所均等性を同時に満たす均等色空間の構築は難しいとされている．そこで本論文では，この逆の手法を用いて，リーマン幾何により，均等化された色空間の座標の写像の測地座標として，色空間内において求めている．色空間での色差の弁別楕円である MacAdam 楕円を用いた大域的かつ局所的に均等な色空間の構築アルゴリズムを示している．
キーワード　均等色空間, 測地線,MacAdam 楕円, リーマン空間

CONSTRUCTION OF UNIFORM COLOR SPACE USING GEODESIC GRIDS

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Abstract
Construction of a uniform color space (UCS) is one of the most challenging task in color engineering and image processing. A major method until now is to find an appropriate map which deforms a color space into the UCS. This seemed very difficult since this map has to uniformize the target color space in both local and global sense. In this paper, we switch to a methodology in the opposite direction. We simly calculate the inverse image of the uniformization map to obtain a geodesic grid as a "polar coordinate system" of UCS. Based on recent theoretical result on UCS, an efficient algorithm for both local and global UCS is obtained giving perceptive discriminative data in a color space.
keyword　Color Space, Uniform Color Space, MacAdam Ellipses, Riemannian Space, Geodesic

1. Introduction

A uniform color space(UCS) is, according to various literature, characterized by two features, one global and the other local: (1) It is a space in which the perceptional difference between any pair of colors agrees with the Euclidean distance, or the length of the straightline between the two color vectors; (2) It is a color space where the local curveness is straighten up so that the discrimination elliptics or ellipsoids of color matching at every points are rectified into unit circles or unit spheres centered at these points.

In fact, the strategies for construction of UCS simply followed these two above-mentioned definitions of the UCS. i.e., either to force the color difference to agree with geometric or Euclidean distance, or force the the jnd or discrimination elliptics in color matching such as MacAdam elliptics into unit circles. The only way to do this was to certain nonlinear map to deform the original color space locally or globally.

As a result, a number of approximative UCS is available, including several versions of standard UCS recommended by the CIE(Commission International de l'Eclairage) [4] [12][2][7]. However, it seems that no methods were able reach a UCS satisfying both global and local definitions. E.g., the standard version of CIELAB space behaves better for global color difference comparing with the CIELUV space, but can not locally uniformize the MacAdam elliptics. On the other hand, the standard CIELUV space shows better local uniformation to rectify the MacAdam elliptics than the CIELAB space but worse in global uniformization.[12][7].

The reason for this is twofold. The first one is of conceptual, As shown in [5], the global and local UCS are in fact the same Riemannian space stated from different points of view. In particular, global geometry of a Riemannian space is uniquely determined by its local metric tensor. The second reason is mainly of implementational. Rather than starting from global fitting, it is natural and easier to start from local rectification of the discrimination ellipsoids. However, on the other hand, to reach a global UCS using the local metric is a nontrivial task. For instance, one needs global information such as the correspondence between sampling points in the color space and their images in the UCS, which is notoriously difficult as already known in estimation of the nonlinear maps.

In this paper we propose a new method to obtain UCS. Instead of trying to find a uniformation map which deforms a color space into the UCS, we will go in the opposite direction, i.e. to make use the inverse map of the above map and try to build the inverse images of the coordinate chart of the UCS. The major difference between these two approaches is that the latter is better understood and the inverse images of coordinate chart in the UCS can be easily obtained as geodesics of in the original color space as a Riemannian space. As results one obtains an inverse image of the polar coordinate chart of the UCS as a geodesic grid in the original color space so that the coordinate of any color vector can be read from this grid. In this way, we constructed a global UCS, which according to our recent theoretical results in [5] is also a local UCS. Finally, computer simulation is also shown for construction of UCS using discriminant elliptics data obtained by MacAdam in 1942[3].

2. Geometry of color spaces

The color spaces have been known to have nontrivial or non-Euclidean intrinsic geometry in both global and local scales. In the global scale, the disagreement between the perceptional color difference of two color stimuli and the Euclidean distance between the two color vectors in a color space has been a major problem in color matching. Researches have been reported on efforts to construct a global UCS, including building the Munsell color space [12][2]. However, global construction of such a UCS in a computational way turned out very nontrivial.

Quantitative study of local geometry of a color space began with the discovery of threshold phenomenon in color matching by Wright, MacAdam and Stiles. Specifically, they found the just-noticeable difference (jnd) or just-perceptible difference(jpd) thresholds near a center color are not constant but change distinguishably depending on the direction that the test color deviates from the center color[3].

These results strongly suggest that the color space is a Riemannian space rather than an Euclidean space.

A Riemannian space is a space M with a positive-definite symmetric matrix $G(\boldsymbol{x})$ smoothly defined on $\boldsymbol{x} \in M$ such that the infinitesimal distance near $\boldsymbol{x}$ is measured by $(d\boldsymbol{x}, d\boldsymbol{x})_G = d\boldsymbol{x}^T G(\boldsymbol{x}) d\boldsymbol{x}$. Let $\boldsymbol{x} = (x^1, ..., x^n)^T, G = [g_{ij}]$, then

$$(d\boldsymbol{x}, d\boldsymbol{x})_G = g_{ij} dx^i dx^j$$

(Here the Einstein symbol $a^i b_i = \sum_i a^i b_i$ is used).

The matrix G is called a Riemannian metric. The threshold elliptics or ellipsoids then induce Riemannian metric in the 2D or 3D color spaces. Hereafter we will consider an nD Euclidean space $\mathbb{R}^n$ where the metric G is smoothly defined on points in $\mathbb{R}^n$, and denote the Riemannian space as a pair $M = (\mathbb{R}^n, G)$.

Suppose that one had found a map from a UCS U to the color space M, $f : U \longrightarrow M, \boldsymbol{y} \longmapsto \boldsymbol{x}$. As before, we suppose this map is a global diffeomorphism or its Jacobian matrix Df is full rank everywhere. Thus its inverse $h := f^{-1} : M \longrightarrow U, \boldsymbol{x} \longmapsto \boldsymbol{y}$ is also a global diffeomorphism. Notice here, h is the uniformation map used most researches. In this paper, we will mainly use its inverse map for its theoretical clarity and more importantly, the accessibility of its images in the color space.

Suppose two color vectors $\boldsymbol{x}, \boldsymbol{x}' \in M$ are mapped to $\boldsymbol{y} := h(\boldsymbol{x}), \boldsymbol{y}' := h(\boldsymbol{x}')$ in the UCS U. Then the distance between these two colors is naturally the length of the straight line connecting images of these two color vectors $\overline{\boldsymbol{y}\boldsymbol{y}'}$ in U.

The global distance of any two colors $\boldsymbol{x}, \boldsymbol{x}'$ measured in the color space can be defined as the length of the inverse image of the straight line $\overline{\boldsymbol{y}\boldsymbol{y}'}$ under h. For the distance to be well defined, the h has to be distance preserving. i.e.

$$d_M(\boldsymbol{x}, \boldsymbol{x}') := \|f(\overline{\boldsymbol{y}\boldsymbol{y}'})\|_M = \|\overline{\boldsymbol{y}\boldsymbol{y}'}\|_U =: d_U(\boldsymbol{y}, \boldsymbol{y}')$$

As we show later, the inverse images of straight lines in an Euclidean space, or the "straight lines" in a Riemannian space is a most important and special class of curves called geodesics. They have nice properties such as be trajectories of mass points without acceleration.

We now consider how to measure the length in M by geodesics. If a spatial curve $\boldsymbol{x}(t) = (x^1, \cdots, x^n)^T$ in a Riemannian space $M = (\mathbb{R}^n, G)$ is smooth for $a \leq t \leq b, \boldsymbol{x}(a) = \boldsymbol{x}, \boldsymbol{x}(b) = \boldsymbol{x}'$, i.e., if $\dot{x}^i$ exist and are continuous, then the integral

$$\theta = \int_a^b \sqrt{g_{ij}\dot{x}^i\dot{x}^j}dt \qquad G = (g_{ij}) \tag{1}$$

exists, and θ is the length between $\boldsymbol{x}(a) = \boldsymbol{x}$ and $\boldsymbol{x}(b) = \boldsymbol{x}'$.

Since the Gamut is metrically complete, it is also geometrically complete. Therefore, between any two points $\boldsymbol{x}$ and $\boldsymbol{x}'$ in M, there is a unique geodesic such that its length θ is minimal.

Thus the global distance between any two colors $\boldsymbol{x}$ and $\boldsymbol{x}'$ is equivalently defined as the geodesic distance or the arc length of the geodesic $\boldsymbol{x}(t)$ connecting $\boldsymbol{x} = \boldsymbol{x}(a), \boldsymbol{x}' = xx(b)$,

$$d_M(\boldsymbol{x}, \boldsymbol{x}') := \int_a^b \sqrt{g_{ij}\dot{x}^i\dot{x}^j}dt. \tag{2}$$

3. Local and global UCS

Before presentation of our approach to construct UCS, we need to show definitions and discuss their relationship.

Let $M = (\mathbb{R}^n, G(\boldsymbol{x})), n = 2, 3$ be a color space as a Riemannian space. Suppose a map from another Riemannian space $U = (\mathbb{R}^n, H(\boldsymbol{y}))$, i.e., a UCS of M to M itself. $f : U \longrightarrow M, \quad \boldsymbol{y} \longmapsto \boldsymbol{x}$. In fact a UCS is an Euclidean space $(\mathbb{R}^n, I)$.

Definition 1. *(Local UCS)*
A Riemannian space $U_l = (\mathbb{R}^n, I)$ is a local UCS of the color space $M = (\mathbb{R}^n, G)$ if there is an isometry (bijective local isometry) $\exists\ f_l\ :\ U_l \longrightarrow M$ s.t. $(f_l)_*(I) = G$, i.e.

$$(d\boldsymbol{y}, d\boldsymbol{y}')_I = (df_l(\boldsymbol{y}), df_l(\boldsymbol{y}'))_G \ \ \textit{or}\ \ \|d\boldsymbol{y}\|_I = \|df_l(\boldsymbol{y})\|_G$$

which implies $H(\boldsymbol{y}) = Df_l(\boldsymbol{y})^T G(\boldsymbol{x}) Df_l(\boldsymbol{y}) = I$.

Therefore a local UCS is a color space where the local curveness is straighten up. In particular, the metric G in M is transformed by f_l^{-1} to the identity matrix $H(\boldsymbol{y}) = I$, or locally the discrimination ellptics or ellipsoids in M are rectified into unit circles or unit spheres centered at $\boldsymbol{y}$ in U.

Definition 2. *(Global UCS)*
A Riemannian space $U_g = (\mathbb{R}^n, I)$ is a global UCS of $M = (\mathbb{R}^n, G)$ if there is a smooth map f_g,

$$\exists\ f_g\ :\ U_g \longrightarrow M$$

the image $f_g(\overline{\boldsymbol{y}\boldsymbol{y}'})$ of the straightline $\overline{\boldsymbol{y}\boldsymbol{y}'}$ between $\boldsymbol{y}$ to $\boldsymbol{y}'$ in U_g is a geodesic between $f_g(\boldsymbol{y})$ and $f_g(\boldsymbol{y}')$ in M and f_g preserves global distance. i.e.

$$\forall \boldsymbol{y}, \boldsymbol{y}' \in U_g, \qquad f_g(\boldsymbol{y}), f_g(\boldsymbol{y}') \in M$$
$$d_M(f_g(\boldsymbol{y}), f_g(\boldsymbol{y}')) = \sqrt{(\boldsymbol{y} - \boldsymbol{y}')^T(\boldsymbol{y} - \boldsymbol{y}')}. \tag{3}$$

Thus, in a global UCS, the perceptional difference between any pair of colors, or the geodesic distance between the two colors $f_g(\boldsymbol{y})$ and $f_g(\boldsymbol{y}')$ agrees with the Euclidean distance, or the length of the straight-line between the two color vectors.

Theorem 1. *[5] The definition of a global UCS and the definition of a local UCS are equivalent.*

Therefore, if one could somehow found a UCS either local or global, then this UCS should be both locally and globally uniform. In the next section we will show how to construct such a UCS.

4. Algorithm to construct UCS

In order to construct a UCS, thus, one can follows either the local or the global definitions of the UCS. It seemed however, both approaches used the same straightforward way to construct a space: try to find a nonlinear map which deforms the color space locally or globally into UCS, respectively. Such a map $h : M \to U$ will be called the uniformation map.

To find such a uniformation map is quite different for local UCS and global UCS. In fact, it is easy to find a local uniformation h_l at a neighborhood of a point $\boldsymbol{x} \in M$ to a point $\boldsymbol{y} \in U$. One needs only to find a nonlinear map h_l s.t. $\boldsymbol{y} = h_l(\boldsymbol{x})$ and to satisfies ,

$$Dh_l^T(\boldsymbol{x}) Dh_l(\boldsymbol{x}) = G(\boldsymbol{x})$$

When $\boldsymbol{y} \in U$ is specified, this is only a linear algebra. However, it is hard to uniformize all neighborhoods of points $\forall \boldsymbol{x} \in M$ simultaneously, i.e. to find a global UCS. This is because to define or even to approximate such a global uniformation map needs to specify the image $\boldsymbol{y} \in U$ for arbitrary $\boldsymbol{x} \in M$. Such point- corresponding is hard and usually conducted by e.g. empirical color matching formula.

To avoid the difficulty for constructing the uniformization map h, we consider its inverse map $f : U \longrightarrow M, \boldsymbol{y} \longmapsto \boldsymbol{x}$. As mentioned before, this inverse map is better understood. In particular, the inverse images of the straightlines in U are geodesics in M. Moreover, these geodesics can also be easily calculated if the local discrimination data are available.

Define the Christoffel symbols Γ^i_{jk} as

$$\Gamma^i_{jk} = \frac{1}{2} g^{il} (\frac{\partial g_{lj}}{\partial x^k} + \frac{\partial g_{lk}}{\partial x^j} - \frac{\partial g_{jk}}{\partial x^l}) \tag{4}$$

where $G^{-1} = (g^{ij})$, the inverse matrix of G. The geodesics of the Levi-Civita connection are solutions of the second order ODE:

$$\ddot{x}^i + \Gamma^i_{jk}\dot{x}^j\dot{x}^k = 0. \tag{5}$$

Furthermore, we chose a particularly suitable coordinate chart, of the color space M, which is the inverse image of the polar coordinate system in the UCS U.

Denote $T_{\boldsymbol{x}}M$ the tangent space of $\boldsymbol{x} \in M$. Given a tangent vector $\boldsymbol{a} \in T_{\boldsymbol{x}}M$ at $\boldsymbol{x} \in M$, denote the geodesic starting from $\boldsymbol{x}$ with the initial vector $\boldsymbol{a}$ as $\gamma(t, \boldsymbol{a}), t$ the parameter, then an exponential map is defined as

$$\exp_{\boldsymbol{x}} : T_{\boldsymbol{x}}M \longrightarrow M \quad (6)$$
$$\boldsymbol{a} \longmapsto \exp_{\boldsymbol{x}}(\boldsymbol{a}) := \gamma(1, \boldsymbol{a}) \quad (7)$$

If one draws all geodesics for all directions $\boldsymbol{a} \in T_{\boldsymbol{x}}M$ simultaneously, the image of the unit circle C in $T_{\boldsymbol{x}}M$ under $\exp_{\boldsymbol{x}}(\cdot)$ is an open neighborhood $N = \exp_{\boldsymbol{x}}(C)$ in M. This N is called a normal neighborhood of $\boldsymbol{x}$. Therefore, for any point $\boldsymbol{x} \in N$, there is a $t, \boldsymbol{a}$ and geodesic $\gamma(\cdot, \boldsymbol{a})$ such such that $\boldsymbol{x} = \gamma(t, \boldsymbol{a})$. i.e. $\boldsymbol{x}$ is determined uniquely by the length of the geodesic and the angle of $\boldsymbol{a}$. This "polar coordinate" of the normal neighborhood is called Riemannian or normal coordinate. which can be regarded as the inverse image of polar coordinate in UCS U. We assume the color space M can be defined as a normal neighborhood of a $\boldsymbol{x} \in M$.and build the normal coordinate chart as a geodesic grid.

Algorithm

Step 1 For a 2D color space, choose unit tangent vectors $\{\boldsymbol{a}_k, k = 1, ..., K\}$ centered at $\boldsymbol{x} \in M$, with angles $\{k\phi_0, \phi_0 = 2\pi/K$ for prechosen resolutions K; In 3D case, choose unit tangent vectors $\{\boldsymbol{a}_{ij}, i = 1, ..., K_1, j = 1, ..., K_2\}$ centered at $\boldsymbol{x} \in M$, with angles $(i\phi_0, j\psi_0), \phi_0 = 2\pi/K_1, \psi_0 = 2\pi/K_2$ for prechosen resolutions K_1, K_2.

Notice here the length of $\boldsymbol{a}, \|\boldsymbol{a}\| = (\boldsymbol{a}^T G \boldsymbol{a}) = 1$, the angle $\cos(\boldsymbol{a}, \boldsymbol{b}) = \boldsymbol{a}^T G \boldsymbol{b}$

Step 2 Draw geodesics $\{\gamma_{\boldsymbol{p}}(\theta, \boldsymbol{a}_k), k = 1, ..., K\}$ in the 2D case and geodesics $\gamma_{\boldsymbol{x}}(\theta, \boldsymbol{a}_{ij}), i = 1, ..., K_1, j = 1, ..., K_2$ in the 3D case;

Step 3 Output the geodesic grid. Denote the geodesic distance from $\boldsymbol{x}$ as l, the data $\{(l, k), n = 1, ..., N, k = 1, ..., K\}$ or $\{(l, i, j), n = 1, ..., N, i = 1, ..., K_1, j = 1, ..., K_2\}$ can be used as the 2D or 3D normal coordinates.

5. Simulation

The long and short axises of the discrimination elliptics in CIEYxy [3] are transformed into CIELUV and interpolated using Akima's algorithm [1] with interval 0.01. The geodesics are obtained using the third order Runge- Kutta method with resolution 0.1, started from the perfect reflecting diffuser. The first order partial derivatives in Christoffel symbol are calculated simply using the central difference. The resulting geodesics grid is shown in fig. 1.

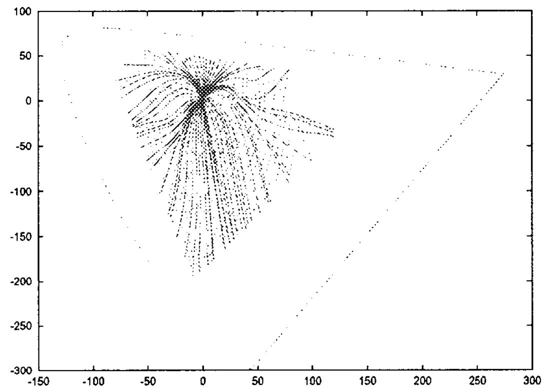

図 1: Geodesic grid

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