A-022

Balanced (C_5, C_{20})-Foil Designs and Related Designs

Kazuhiko Ushio (Kinki University)

1. Balanced (C_5, C_{20})-Foil Designs

Let K_n denote *the complete graph* of n vertices. Let C_5 and C_{20} be *the* 5-*cycle* and *the* 20-*cycle*, respectively. *The* (C_5, C_{20})-2*t*-*foil* is a graph of t edge-disjoint C_5's and t edge-disjoint C_{20}'s with a common vertex. When K_n is decomposed into edge-disjoint sum of (C_5, C_{20})-2*t*-foils and every vertex of K_n appears in the same number of (C_5, C_{20})-2*t*-foils, we say that K_n *has a balanced* (C_5, C_{20})-2*t*-*foil decomposition*. This decomposition is to be known as *a balanced* (C_5, C_{20})-2*t*-*foil design*.

Theorem 1. K_n has a balanced (C_5, C_{20})-2*t*-foil design if and only if $n \equiv 1 \pmod{50t}$.

Example 1.1. Balanced (C_5, C_{20})-2-foil design of K_{51}**.** Starter : $\{(51, 1, 24, 48, 22), (51, 4, 6, 14, 21, 32, 44, 9, 28, 49, 13, 47, 38, 20, 10, 41, 35, 30, 16, 3)\}$.

Example 1.2. Balanced (C_5, C_{20})-4-foil design of K_{101}**.** Starter : $\{(101, 1, 46, 97, 44), (101, 7, 10, 26, 39, 61, 84, 15, 56, 94, 22, 5, 72, 37, 17, 79, 51, 42, 30, 25), (101, 2, 48, 95, 43), (101, 8, 12, 27, 41, 62, 86, 16, 58, 98, 24, 92, 74, 38, 19, 80, 69, 59, 32, 6)\}$.

Example 1.3. Balanced (C_5, C_{20})-6-foil design of K_{151}**.** Starter : $\{(151, 1, 68, 145, 66), (151, 10, 14, 38, 57, 90, 124, 21, 82, 139, 31, 6, 106, 54, 24, 117, 75, 62, 44, 37), (151, 2, 70, 141, 65), (151, 11, 16, 39, 59, 91, 126, 22, 84, 140, 33, 7, 108, 55, 26, 118, 77, 63, 46, 8), (151, 3, 72, 142, 64), (151, 12, 18, 40, 61, 92, 128, 23, 86, 131, 35, 137, 110, 56, 28, 119, 103, 88, 48, 9)\}$.

Example 1.4. Balanced (C_5, C_{20})-8-foil design of K_{201}**.** Starter : $\{(201, 1, 90, 193, 88), (201, 13, 18, 50, 75, 119, 164, 27, 108, 184, 40, 7, 140, 71, 31, 155, 99, 82, 58, 49), (201, 2, 92, 187, 87), (201, 14, 20, 51, 77, 120, 166, 28, 110, 185, 42, 8, 142, 72, 33, 156, 101, 83, 60, 10), (201, 3, 94, 188, 86), (201, 15, 22, 52, 79, 121, 168, 29, 112, 186, 44, 9, 144, 73, 35, 157, 103, 84, 62, 11), (201, 4, 96, 189, 85), (201, 16, 24, 53, 81, 122, 170, 30, 114, 174, 46, 182, 146, 74, 37, 158, 137, 117, 64, 12)\}$.

Department of Informatics, Faculty of Science and Technology, Kinki University, Osaka 577-8502, JAPAN. E-mail:ushio@info.kindai.ac.jp   Tel:+81-6-6721-2332 (ext.   5409) Fax:+81-6-6727-2024

2. Balanced C_{25}-Foil Designs

Theorem 2. K_n has a balanced C_{25}-*t*-foil design if and only if $n \equiv 1 \pmod{50t}$.

Example 2.1. Balanced C_{25} design of K_{51}. Starter : $\{(51, 1, 24, 48, 22, 26, 4, 6, 14, 21, 32, 44, 9, 28, 49, 13, 47, 38, 20, 10, 41, 35, 30, 16, 3)\}$.

Example 2.2. Balanced C_{25}-2-foil design of K_{101}. Starter : $\{(101, 2, 48, 95, 43, 50, 7, 10, 26, 39, 61, 84, 15, 56, 94, 22, 5, 72, 37, 17, 79, 51, 42, 30, 25), (101, 1, 46, 97, 44, 52, 8, 12, 27, 41, 62, 86, 16, 58, 98, 24, 92, 74, 38, 19, 80, 69, 59, 32, 6)\}$.

Example 2.3. Balanced C_{25}-3-foil design of K_{151}. Starter : $\{(151, 3, 72, 142, 64, 74, 10, 14, 38, 57, 90, 124, 21, 82, 139, 31, 6, 106, 54, 24, 117, 75, 62, 44, 37), (151, 2, 70, 141, 65, 76, 11, 16, 39, 59, 91, 126, 22, 84, 140, 33, 7, 108, 55, 26, 118, 77, 63, 46, 8), (151, 1, 68, 145, 66, 78, 12, 18, 40, 61, 92, 128, 23, 86, 131, 35, 137, 110, 56, 28, 119, 103, 88, 48, 9)\}$.

Example 2.4. Balanced C_{25}-4-foil design of K_{201}. Starter : $\{(201, 4, 96, 189, 85, 98, 13, 18, 50, 75, 119, 164, 27, 108, 184, 40, 7, 140, 71, 31, 155, 99, 82, 58, 49), (201, 3, 94, 188, 86, 100, 14, 20, 51, 77, 120, 166, 28, 110, 185, 42, 8, 142, 72, 33, 156, 101, 83, 60, 10), (201, 2, 92, 187, 87, 102, 15, 22, 52, 79, 121, 168, 29, 112, 186, 44, 9, 144, 73, 35, 157, 103, 84, 62, 11), (201, 1, 90, 193, 88, 104, 16, 24, 53, 81, 122, 170, 30, 114, 174, 46, 182, 146, 74, 37, 158, 137, 117, 64, 12)\}$.

3. Balanced (C_{10}, C_{40})-Foil Designs

Theorem 3. K_n has a balanced (C_{10}, C_{40})-2*t*-foil design if and only if $n \equiv 1 \pmod{100t}$.

Example 3.1. Balanced (C_{10}, C_{40})-2-foil design of K_{101}**.** Starter : $\{(101, 1, 46, 97, 44, 87, 43, 95, 48, 2), (101, 7, 10, 26, 39, 61, 84, 15, 56, 94, 22, 5, 72, 37, 17, 79, 51, 42, 30, 25, 31, 6, 32, 59, 69, 80, 19, 38, 74, 92, 24, 98, 58, 16, 86, 62, 41, 27, 12, 8)\}$.

Example 3.2. Balanced (C_{10}, C_{40})-4-foil design of K_{201}**.** Starter : $\{(201, 1, 90, 193, 88, 175, 87, 187, 92, 2), (201, 3, 94, 188, 86, 171, 85, 189, 96, 4),$

(201, 13, 18, 50, 75, 119, 164, 27, 108, 184, 40, 7, 140, 71,
31, 155, 99, 82, 58, 49, 59, 10, 60, 83, 101, 156, 33, 72,
142, 8, 42, 185, 110, 28, 166, 120, 77, 51, 20, 14),
(201, 15, 22, 52, 79, 121, 168, 29, 112, 186, 44, 9, 144, 73,
35, 157, 103, 84, 62, 11, 23, 12, 64, 117, 137, 158, 37, 74,
146, 182, 46, 174, 114, 30, 170, 122, 81, 53, 24, 16)}.

Example 3.3. Balanced (C_{10}, C_{40})-6-foil design of K_{301}.   Starter :

{(301, 1, 134, 289, 132, 263, 131, 279, 136, 2),
(301, 3, 138, 280, 130, 259, 129, 281, 140, 4),
(301, 5, 142, 282, 128, 255, 127, 283, 144, 6),
(301, 19, 26, 74, 111, 177, 244, 39, 160, 274, 58, 9, 208,
105, 45, 231, 147, 122, 86, 73, 87, 14, 88, 123, 149, 232, 47,
106, 210, 10, 60, 275, 162, 40, 246, 178, 113, 75, 28, 20),
(301, 21, 30, 76, 115, 179, 248, 41, 164, 276, 62, 11, 212,
107, 49, 233, 151, 124, 90, 15, 31, 16, 92, 125, 153, 234, 51,
108, 214, 12, 64, 277, 166, 42, 250, 180, 117, 77, 32, 22),
(301, 23, 34, 78, 119, 181, 252, 43, 168, 278, 66, 13, 216,
109, 53, 235, 155, 126, 94, 17, 35, 18, 96, 175, 205, 236, 55,
110, 218, 272, 68, 260, 170, 44, 254, 182, 121, 79, 36, 24)}.

Example 3.4. Balanced (C_{10}, C_{40})-8-foil design of K_{401}.   Starter :

{(401, 1, 178, 385, 176, 351, 175, 371, 180, 2),
(401, 3, 182, 372, 174, 347, 173, 373, 184, 4),
(401, 5, 186, 374, 172, 343, 171, 375, 188, 6),
(401, 7, 190, 376, 170, 339, 169, 377, 192, 8),
(401, 25, 34, 98, 147, 235, 324, 51, 212, 364, 76, 11, 276,
139, 59, 307, 195, 162, 114, 97, 115, 18, 116, 163, 197, 308,
61, 140, 278, 12, 78, 365, 214, 52, 326, 236, 149, 99, 36, 26),
(401, 27, 38, 100, 151, 237, 328, 53, 216, 366, 80, 13, 280,
141, 63, 309, 199, 164, 118, 19, 39, 20, 120, 165, 201, 310,
65, 142, 282, 14, 82, 367, 218, 54, 330, 238, 153, 101, 40, 28),
(401, 29, 42, 102, 155, 239, 332, 55, 220, 368, 84, 15, 284,
143, 67, 311, 203, 166, 122, 21, 43, 22, 124, 167, 205, 312,
69, 144, 286, 16, 86, 369, 222, 56, 334, 240, 157, 103, 44, 30),
(401, 31, 46, 104, 159, 241, 336, 57, 224, 370, 88, 17, 288,
145, 71, 313, 207, 168, 126, 23, 47, 24, 128, 233, 273, 314,
73, 146, 290, 362, 90, 346, 226, 58, 338, 242, 161, 105, 48, 32)}.

4. Balanced C_{50}-Foil Designs

Theorem 4. K_n has a balanced C_{50}-t-foil design if and only if $n \equiv 1 \pmod{100t}$.

Example 4.1. Balanced C_{50} design of K_{101}.   Starter :

{(101, 2, 48, 95, 43, 50, 7, 10, 26, 39, 61, 84, 15, 56, 94, 22,
5, 72, 37, 17, 79, 51, 42, 30, 25, 31, 6, 32, 59, 69, 80, 19, 38,
74, 92, 24, 98, 58, 16, 86, 62, 41, 27, 12, 8, 52, 44, 97, 46, 1)}.

Example 4.2. Balanced C_{50}-2-foil design of K_{201}. Starter :

{(201, 4, 96, 189, 85, 98, 13, 18, 50, 75, 119, 164, 27, 108,
184, 40, 7, 140, 71, 31, 155, 99, 82, 58, 49, 59, 10, 60, 83,
101, 156, 33, 72, 142, 8, 42, 185, 110, 28, 166, 120, 77, 51,

20, 14, 100, 86, 188, 94, 3),
(201, 2, 92, 187, 87, 102, 15, 22, 52, 79, 121, 168, 29, 112,
186, 44, 9, 144, 73, 35, 157, 103, 84, 62, 11, 23, 12, 64, 117,
137, 158, 37, 74, 146, 182, 46, 174, 114, 30, 170, 122, 81, 53,
24, 16, 104, 88, 193, 90, 1)}.

Example 4.3. Balanced C_{50}-3-foil design of K_{301}.

Starter :
{(301, 6, 144, 283, 127, 146, 19, 26, 74, 111, 177, 244, 39,
160, 274, 58, 9, 208, 105, 45, 231, 147, 122, 86, 73, 87, 14,
88, 123, 149, 232, 47, 106, 210, 10, 60, 275, 162, 40, 246,
178, 113, 75, 28, 20, 148, 128, 282, 142, 5),
(301, 4, 140, 281, 129, 150, 21, 30, 76, 115, 179, 248, 41,
164, 276, 62, 11, 212, 107, 49, 233, 151, 124, 90, 15, 31, 16,
92, 125, 153, 234, 51, 108, 214, 12, 64, 277, 166, 42, 250,
180, 117, 77, 32, 22, 152, 130, 280, 138, 3),
(301, 2, 136, 279, 131, 154, 23, 34, 78, 119, 181, 252, 43,
168, 278, 66, 13, 216, 109, 53, 235, 155, 126, 94, 17, 35, 18,
96, 175, 205, 236, 55, 110, 218, 272, 68, 260, 170, 44, 254,
182, 121, 79, 36, 24, 156, 132, 289, 134, 1)}.

Example 4.4. Balanced C_{50}-4-foil design of K_{401}.

Starter :
{(401, 8, 192, 377, 169, 194, 25, 34, 98, 147, 235, 324, 51,
212, 364, 76, 11, 276, 139, 59, 307, 195, 162, 114, 97, 115,
18, 116, 163, 197, 308, 61, 140, 278, 12, 78, 365, 214, 52,
326, 236, 149, 99, 36, 26, 196, 170, 376, 190, 7),
(401, 6, 188, 375, 171, 198, 27, 38, 100, 151, 237, 328, 53,
216, 366, 80, 13, 280, 141, 63, 309, 199, 164, 118, 19, 39,
20, 120, 165, 201, 310, 65, 142, 282, 14, 82, 367, 218, 54,
330, 238, 153, 101, 40, 28, 200, 172, 374, 186, 5),
(401, 4, 184, 373, 173, 202, 29, 42, 102, 155, 239, 332, 55,
220, 368, 84, 15, 284, 143, 67, 311, 203, 166, 122, 21, 43,
22, 124, 167, 205, 312, 69, 144, 286, 16, 86, 369, 222, 56,
334, 240, 157, 103, 44, 30, 204, 174, 372, 182, 3),
(401, 2, 180, 371, 175, 206, 31, 46, 104, 159, 241, 336, 57,
224, 370, 88, 17, 288, 145, 71, 313, 207, 168, 126, 23, 47,
24, 128, 233, 273, 314, 73, 146, 290, 362, 90, 346, 226, 58,
338, 242, 161, 105, 48, 32, 208, 176, 385, 178, 1)}.

References   [1] K. Ushio and H. Fujimoto, Balanced
bowtie and trefoil decomposition of complete tripartite
multigraphs, *IEICE Trans. Fundamentals*, E84-A, 839–
844, 2001.   [2] ——, Balanced foil decomposition of com-
plete graphs, *IEICE Trans. Fundamentals*, E84-A, 3132–
3137, 2001.   [3] ——, Balanced bowtie decomposition of
complete multigraphs, *IEICE Trans. Fundamentals*, E86-
A, 2360–2365, 2003.   [4] ——, Balanced bowtie decomposi-
tion of symmetric complete multi-digraphs, *IEICE Trans.
Fundamentals*, E87-A, 2769–2773, 2004.   [5] ——, Bal-
anced quatrefoil decomposition of complete multigraphs,
IEICE Trans. Information and Systems, E88-D, 19–22,
2005.   [6] ——, Balanced C_4-bowtie decomposition of com-
plete multigraphs, *IEICE Trans. Fundamentals*, E88-A,
1148–1154, 2005.   [7] ——, Balanced C_4-trefoil decomposi-
tion of complete multigraphs, *IEICE Trans. Fundamen-
tals*, E89-A, 1173–1180, 2006.