LA-001

A New Approach to Approximate the Collision Probability in an Automated Production Line

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Abstract

Flat Panel Displays (FPDs) are manufactured through many different processing equipments arranged sequentially in a line. Although the constant inter-arrival time (i.e., the tact time) of glass substrates in the line should be kept as short as possible, the collision probability between glass substrates increases as the time becomes shorter. Since the glass substrate is expensive and fragile, the collision should be avoided. In this paper, we give a simpler expression of the collision probability by a new approximation approach, which enables us to easily compute numerical values of the collision probability over a wider range of parameter values. We also carry out some simulations to evaluate the exact probabilities and confirm that our approximation approach yields reasonable results compared to the simulated results.

1 Introduction

Reflecting the increasing demand on Flat Panel Displays (FPDs) such as LCD, plasma display panel, etc., more effective method for their manufacturing is required. The production rate improves with technological advancements such as the rapid enlargement of glass substrates and the miniaturization of patterns. Accordingly, production line has to be modified to accommodate such advancements, and new optimization problems to be solved continue to arise. Lately an advanced system called a *Crystal Flow* [5] has been introduced in the production line of FPDs. It targets a higher level of line control in the next-generation production processes as well as in existing lines.

The main flow of FPD process is shown in Fig. 1 [7]. Each processing equipment in Fig. 1 is a specialized one such as cleaning, coater, proximity exposure, developer, etcher, resist remover, etc., and those equipments are connected in-line. Most of the production lines adopts a simple strategy to feed each glass substrate to the first equipment with a constant inter-arrival time, which is called the *tact time*. This strategy is simple and enables us to estimate the number of products precisely.

Due to solution foaming, chemicals, heat treating, etc., the processing time at each equipment is uncertain and may vary according to the condition at that time. If a substrate is sent to an equipment which is processing the previous substrate, the current substrate cannot be processed on the equipment. This phenomenon is called a *collision* between substrates. Since the glass substrate is expensive and fragile, the collision should be avoided as much as possible.

A collision-like phenomenon is called a *blocking* in scheduling theory, and is studied as an important factor to determine line efficiency [4]. However, depending on the rule how to process collisions (blocked calls cleared, blocked calls delayed, etc.), previous work mainly focused on performance measures in the steady state [3]. In this paper, given the number of jobs to be processed in the prescribed time span, our performance measure is the probability that there is at least one collision.

The tact time (i.e., the inter-arrival time of substrates at the first equipment) should be minimized to maximize the production rate, which however increases the collision probability. Thus there is a trade-off between the tact time and the collision probability. To consider this trade-off it is important to evaluate the collision probability under a given tact time.

The probability density function (pdf) of actual processing time is often represented by a bell-like curve such as Fig. 2 (e) . The pdf of the normal distribution is also bell-shaped, but it has a weakness that the pdf takes positive value in the negative domain, which is not true in the pdf of actual processing time. In this paper, we assume that the processing time follows an Erlang distribution.

The pdf of the Erlang distribution is defined as follows.

$$f(x;k,\lambda)=\frac{\lambda^k x^{k-1}e^{-\lambda x}}{(k-1)!} \quad \text{for } x\geq 0, \tag{1}$$

where two parameters k and λ are a positive integer and a positive real number, respectively. Its expectation and variance are given by k/λ and k/λ^2, respectively. Therefore, under the Erlang distribution, we can set the expectation and the variance independently by setting parameters λ and k appropriately. In Fig. 2, five different pdfs are plotted, where expectations of all cases are the same, but their variances decrease with cases of Fig. 2 (a) – (e). Some of these pdfs are bell-shaped, and the pdf of the Erlang distribution takes zero value for $x < 0$. Thus the Erlang distribution is flexible enough to represent actual processing times.

Based on a simplified model of the production line of FPDs, we derive the collision probability analytically as well as by numeric computing. It was shown in [2] that the collision probability can be approximately expressed by a closed form formula under the stated model. However, the formula proposed in [2] is not meant for numerical evaluation since it is quite complex. In this paper, we

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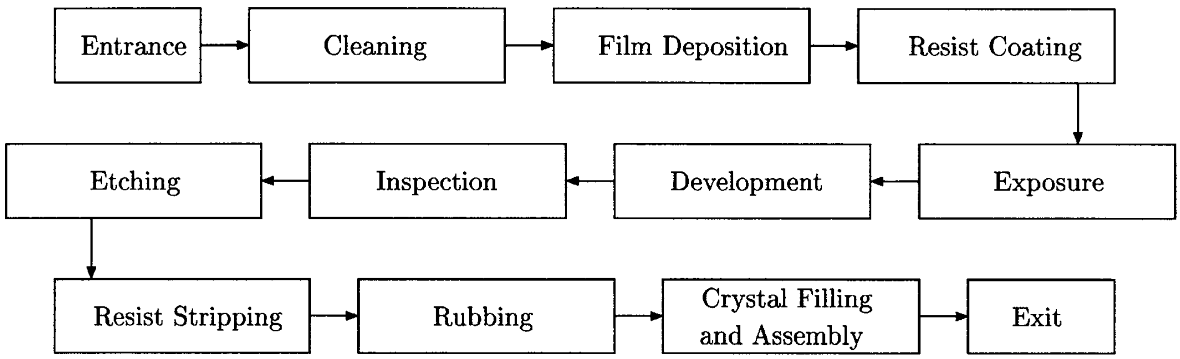

Fig. 1. FPD Process Flow.

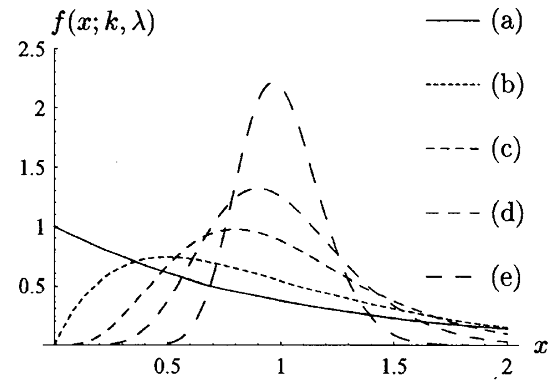

Fig. 2. The pdf of the Erlang distribution: (a) $k = 1$, $\lambda = 1$ (exponential distribution); (b) $k = 2$, $\lambda = 2$; (c) $k = 5$, $\lambda = 5$; (d) $k = 10$, $\lambda = 10$; (e) $k = 30$, $\lambda = 30$.

give a simpler expression of the collision probability by a new approximation approach, which enables us to easily compute numerical values of the collision probability over a wider range of parameter values. We also carry out simulations to evaluate the exact probabilities, and confirm that our approximation approach yields reasonable results in comparison with the simulated results.

2 Model

In this paper, the following notations will be used:

- $M_1, M_2, \ldots, M_m$: m different machines in the line.
- $J_1, J_2, \ldots, J_n$: n jobs to be processed.
- $T_i^{(j)}$ (> 0): Processing time of job J_i on machine M_j.
- T_{tact} (> 0): Tact time, i.e., the time difference between the start time instants of J_i and J_{i+1} for all $1 \le i \le n-1$ at the entrance to the line.

The production model is illustrated in Fig. 3. With the same time interval T_{tact}, jobs are successively fed to the line from the entrance. Every job is first processed on machine M_1. It is then automatically transported to the next machine M_2 after finished on M_1. It is assumed for simplicity that the transportation time between machines is nil. As soon as M_2 receives the job, it starts processing.

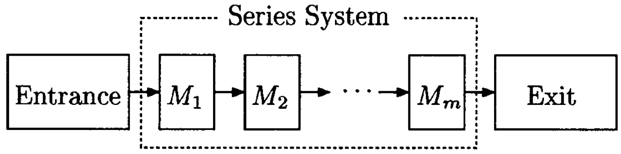

Fig. 3. The production model.

In this manner, every job is processed on machines in the order of $M_1, M_2, \ldots, M_m$, and then sent to the exit.

The collision occurs if the next job arrives at M_j while M_j is still processing the current job. The following lemma on the collision condition between jobs was given in [1].

Lemma 1 *Suppose that $T_i^{(j)} = t_i^{(j)}$ for all $1 \le i \le n$ and $1 \le j \le m$. For n jobs, $n \ge 2$, there is no collision in the above production line of m machines if and only if*

$$\sum_{j=1}^{l} t_i^{(j)} \le T_{tact} + \sum_{j=1}^{l-1} t_{i+1}^{(j)}$$

holds for all $1 \le i \le n-1$ and $1 \le l \le m$.

We assume that the processing time $T_i^{(j)}$ on M_j is a random variable that follows the Erlang distribution with parameters k_j and λ_j. Furthermore we assume that all $T_i^{(j)}$ ($1 \le i \le n, 1 \le j \le m$) are independent of each other.

3 Approximation of collision probability

We sketch a derivation of approximate collision probability. By Lemma 1, the probability that there is no collision is given by

$$\Pr\left(\sum_{j=1}^{l} T_i^{(j)} \le T_{tact} + \sum_{j=1}^{l-1} T_{i+1}^{(j)} : 1 \le i \le n-1, 1 \le l \le m\right). \tag{2}$$

Unfortunately, it does not seem that (2) can be simplified further. Therefore, we first approximate (2) by

$$\Pr\left(\sum_{j=1}^{m} T_i^{(j)} \le T_{tact} + \sum_{j=1}^{m-1} T_{i+1}^{(j)} : 1 \le i \le n-1\right), \tag{3}$$

i.e., the probability that there is no collision on the last machine M_m, since the collision is most likely to occur on M_m if it occurs at all. Note that (3) approximates (2) from above, since collisions on other machines are neglected. Moreover, we try to approximate (3) by considering only two consecutive jobs.

For this, we introduce the following event.

E_i : Event that, under the assumption that there are only two consecutive jobs J_i and J_{i+1}, there is no collision between them on the last machine M_m.

The probability of event E_i is given by

$$\Pr(E_i) = \Pr\left(\sum_{j=1}^{m} T_i^{(j)} \le T_{tact} + \sum_{j=1}^{m-1} T_{i+1}^{(j)}\right). \quad (4)$$

We introduce the following random variables:

$$X = \sum_{j=1}^{m} T_i^{(j)} \text{ and } Y = \sum_{j=1}^{m-1} T_{i+1}^{(j)}.$$

Since the probability distribution of the sum of two independent random variables is the convolution of their distributions, $f_X = f_{T_i^{(1)}} * f_{T_i^{(2)}} * \cdots * f_{T_i^{(m)}}$ holds for the pdf f_X of random variable X, where $f_{T_i^{(j)}}$ is the pdf of random variable $T_i^{(j)}$ and $*$ is the convolution operator.

The Fourier transform translates a convolution into a multiplication of functions. In this paper, we define the Fourier transform of a function $h(x)$ and inverse Fourier transform of $H(\omega)$ as follows:

$$\mathcal{F}[h(x)] = \int_{-\infty}^{\infty} h(x)e^{-i\omega x}dx,$$
$$\mathcal{F}^{-1}[H(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{i\omega x}d\omega.$$

Then, since the Fourier transform of the convolution is given by the product of the Fourier transforms,

$$\mathcal{F}\left[f_{T_i^{(1)}} * f_{T_i^{(2)}} * \cdots * f_{T_i^{(m)}}\right] = \mathcal{F}\left[f_{T_i^{(1)}}\right] \cdot \mathcal{F}\left[f_{T_i^{(2)}}\right] \cdot \cdots \cdot \mathcal{F}\left[f_{T_i^{(m)}}\right]$$

holds. Computing the inverse Fourier transform of the above expression, the convolution of the former functions is obtained, i.e.,

$$f_X = \mathcal{F}^{-1}\left[\mathcal{F}\left[f_{T_i^{(1)}}\right] \cdot \mathcal{F}\left[f_{T_i^{(2)}}\right] \cdot \cdots \cdot \mathcal{F}\left[f_{T_i^{(m)}}\right]\right].$$

In the same way, the pdf f_Y of random variable Y is also obtained.

Similarly, when X and Y are independently distributed, the pdf f_{X-Y} of the difference $X - Y$ is given by

$$\int_{-\infty}^{\infty} f_X(x+y)f_Y(y)dy.$$

This integral is known as the cross-correlation $f_Y \star f_X$. It satisfies $\mathcal{F}[f_Y \star f_X] = (\mathcal{F}[f_Y])^* \cdot \mathcal{F}[f_X]$, where the superscript asterisk denotes the complex conjugate. We then have

$$f_{X-Y} = f_Y \star f_X = \mathcal{F}^{-1}\left[(\mathcal{F}[f_Y])^* \cdot \mathcal{F}[f_X]\right].$$

Therefore, (4) can be rewritten as

$$\Pr(X - Y \le T_{tact}) = \int_{-\infty}^{T_{tact}} f_{X-Y}(x)dx. \quad (5)$$

Since we assume that all $T_i^{(j)}$ $(i = 1, 2, \ldots, n)$ have the same distribution function, $\Pr(E_1) = \Pr(E_2) = \cdots = \Pr(E_{n-1})$ holds. Although two events E_i and E_j $(i \ne j)$ are not independent, precisely speaking, we approximate the no-collision probability on the last machine M_m over all n jobs (i.e. (3)) by the $(n-1)$-th power of (5). In this case, the no-collision probability (i.e. (2)) is approximated by

$$\left\{\int_{-\infty}^{T_{tact}} f_{X-Y}(x)dx\right\}^{n-1}. \quad (6)$$

The approximate probability of collision is then given by subtracting (6) from 1.

Note that (6) is valid even if the processing time of each machine follows a general distribution. We can express (6) as a closed form formula if Erlang distribution is assumed. However, it may be quite complex. Therefore, it seems to be more effective to evaluate (6) numerically.

We focus on a special case in which $\lambda_1 = \lambda_2 = \cdots = \lambda$ holds. Due to the property of the Erlang distribution, the pdfs of X and Y are obtained as

$$f_X(x) = \frac{\lambda^{K_m} x^{K_m - 1} e^{-\lambda x}}{(K_m - 1)!},$$
$$f_Y(x) = \frac{\lambda^{K_{m-1}} x^{K_{m-1} - 1} e^{-\lambda x}}{(K_{m-1} - 1)!}$$

for $x \ge 0$, where $K_m = \sum_{j=1}^{m} k_j$. Then $(\mathcal{F}[f_Y])^* \cdot \mathcal{F}[f_X]$ can be written as

$$\frac{\lambda^{K_m + K_{m-1}}(\lambda - i\omega)^{k_m}}{(\lambda^2 + \omega^2)^{K_m}}.$$

4 Numerical Results

Based on the above formulae, we obtain some numerical results by using MATHEMATICA [8]. For our computation in this section, the number of jobs is set to $n = 1,000$, and parameters of the Erlang distributions are set so that the expectation and the variance of the processing time on each machine become equal to 1 and 0.001, respectively (i.e., $k = 1000$, $\lambda = 1000$ in (1)). The numerical results are shown in Table 1.

We also carried out the following simulations to evaluate the exact probabilities. The procedure is stated as follows: Given the number of jobs n, the number of machines m, the tact time T_{tact}, parameters of the Erlang distribution, and a positive integer c (specifying the number of iterations, which is related to the accuracy), derive the collision probability by the following algorithm.

Simulation Algorithm

Step 1: *loop* := 1.

Step 2: Generate the processing time $t_i^{(j)}$ $(1 \le i \le n, 1 \le j \le m)$ randomly from the Erlang distribution.

Table 1: Collision probability evaluated by approximate formula. [%]

		T_{tact}										
		1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70
m	3	90.86	19.18	1.19	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4	99.98	76.00	15.90	1.50	0.09	0.00	0.00	0.00	0.00	0.00	0.00
	5	100.00	98.56	54.77	10.86	1.28	0.11	0.01	0.00	0.00	0.00	0.00
	6	100.00	99.98	88.14	34.84	6.73	0.91	0.10	0.01	0.00	0.00	0.00
	7	100.00	100.00	98.61	66.01	20.39	3.95	0.59	0.07	0.01	0.00	0.00
	8	100.00	100.00	99.92	88.28	42.26	11.36	2.24	0.36	0.05	0.01	0.00
	9	100.00	100.00	100	97.40	66.15	24.48	6.15	1.24	0.21	0.03	0.00

Table 2: Collision probability evaluated by simulation. [%]

		T_{tact}										
		1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70
m	3	91.39	19.27	1.21	0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	4	99.99	77.83	16.32	1.52	0.09	0.00	0.00	0.00	0.00	0.00	0.00
	5	100.00	99.16	57.61	11.35	1.29	0.11	0.01	0.00	0.00	0.00	0.00
	6	100.00	100.00	91.23	37.52	7.19	0.98	0.10	0.01	0.00	0.00	0.00
	7	100.00	100.00	99.40	71.11	22.36	4.26	0.63	0.07	0.01	0.00	0.00
	8	100.00	100.00	99.99	92.39	47.06	12.72	2.46	0.39	0.05	0.01	0.00
	9	100.00	100.00	100.00	98.95	72.81	27.97	6.95	1.37	0.23	0.03	0.00

Step 3: Based on the condition in Lemma 1, check whether a collision occurs. Let $loop := loop + 1$. If $loop \le c$, return to Step 2; otherwise go to Step 4.

Step 4: Output the collision probability (the number of collisions observed in Step 3)/c.

The computation time is $\Theta(cmn)$. Through all simulations, we use Mersenne Twister [6] as the pseudorandom generator, and the number of iterations is set to $c = 1,000,000$. The simulation results are shown in Table 2.

Those Tables shows that, as the tact time becomes larger, the collision probability decreases, clearly exhibiting the trade-off between the tact time and the collision probability. We also confirmed that the collision probability increases with m. We may conclude that the numerical and simulation results are reasonably close in most cases.

5 Conclusions

We have presented a new approach to approximate the collision probability. Our approximate formula can evaluate the collision probability much faster than simulation, and has good accuracy as experimentally confirmed.

Although the production model in this paper doesn't have any buffer space between machines, such space may be very useful to avoid collisions between jobs. Analysis of the collision probability in the production model with buffer space is one of our future topics.

Acknowledgments

The authors would like to express gratitude to Dainippon Screen MFG. Co., Ltd., and FEBACS Co., Ltd., where informative discussions were done on the FPD process flow. This work was partially supported by the Scientific Grant-in-Aid from Ministry of Education, Culture, Sports, Science and Technology of Japan.

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