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A study on Non-Correspondence in Spread between Objective Space and Design Variable Space and Application to Genetic Search

Tomohiro Yoshikawa, Toru Yoshida, Toshihiro Kishigami
Dept. of Computational Science and Engineering
Nagoya University
Furo-cho, Chikusa-ku, Nagoya 466-8603, Japan

Abstract— *Recently, a lot of studies on Multi-Objective Genetic Algorithm (MOGA), in which Genetic Algorithm is applied to Multi-objective Optimization Problems (MOPs), have been reported actively. MOGA has been also applied to engineering design fields, then it is important not only to obtain high-performance Pareto solutions but also to analyze the obtained Pareto solutions and extract some knowledge in the problem. In order to analyze Pareto solutions obtained by MOGA, it is required to consider both the objective space and the design variable space. In this paper, we define "Non-Correspondence in Spread" between the objective space and the design variable space. We also try to extract the Non-Correspondence area in Spread with the index defined in this paper. Moreover, we apply the defined index to genetic search to obtain Pareto solutions that have different design variables one another having similar fitness values. This paper applies the above index to the trajectory designing optimization problem and extracts Non-Correspondence area in Spread int the acquired Pareto solutions. This paper also shows that robust Pareto solutions can be acquired by the genetic search with the index.*

Keywords: Non-Correspondence, Objective Space, Design Variable Space, Distributed Area, Multi-objective Optimization Problem

1. Introduction

Genetic Algorithm (GA) is expected to be effective for solving Multi-objective Optimization Problems (MOPs), which maximizes or minimizes multiple objective functions at the same time. Recently, Multi-Objective Genetic Algorithm (MOGA), applying GA to MOPs, are getting much attention and a lot of studies have been reported[1]. Generally, it is difficult to obtain the optimized solution satisfying all objective functions because of their trade-offs. Then, it is necessary to obtain Pareto solutions which are not inferior to other solutions in at least one objective function.

In recent years, it is reported that MOGA is applied to engineering design problems in the real-world due to the improvement of computing performance[2][3][4]. In the engineering design problems, it is required not only to obtain high performance Pareto solutions using MOGA but also to analyze and extract design knowledge in the problem. And in order to analyze Pareto solutions obtained by MOGA, it is required to consider both the objective space and the design variable space.

Obayashi obtained Pareto solutions for aircraft configuration problem by MOGA and tried to analyze the obtained Pareto solutions through the visualization of the relationship between fitness values and design variables using Self Organizing Map (SOM)[2]. Kudo *et al*. proposed a visualization method that visualized the geometric distance between data in the design variable space based on their relationship in the objective space, and they analyzed the relationship between the fitness values and the design variables in the conceptual design optimization problem of hybrid rocket engine[5].

In this paper, we analyze obtained Pareto solutions considering the objective space and the design variable space, and we especially focus on "Non-Correspondence" between two spaces. In this study, we have introduced 3 patterns of Non-Correspondence between the objective space and the design variable space.

- Non-Correspondence in Sequence
- Non-Correspondence in Spread
- Non-Correspondence in Linear Relationship

We have already reported on the Non-Correspondence in Sequence[6]. In this paper, we define "Non-Correspondence in Spread" and propose the quantitative index to extract Non-Correspondence area in Spread. Non-Correspondence area in Spread is the area where solutions are distributed densely in the objective space but are distributed widely in the design variables space, and vice versa.

This paper applies the proposed method to the trajectory designing optimization problem known as DESTINY (Demonstration and Experiment of Space Technology for INterplanetary voYage)[7] provided by Japan Aerospace Exploration Agency (JAXA). We apply NSGA-II (Non-dominated Sorting Genetic Algorithm-II)[8] to this problem. We also analyze the extracted Non-Correspondence area in Spread in the obtained Pareto solutions. We also apply the defined index to genetic search to obtain Pareto solutions that have different design variables one another having similar fitness values.

2. Non-Correspondence in Spread

2.1 Definition of Non-Correspondence in Spread

In this paper, we focus on Non-Correspondence in Spread. The area with Non-Correspondence in Spread, called Non-Correspondence area in Spread, is defined as the area where solutions are distributed densely in the objective space but are distributed widely in the design variables space, and vice versa (Hereinafter we call simply "Non-Correspondence area."). The former means that there are a lot of design patterns with similar performance and the later means that the design variables are sensitive, i.e. the small change of design variables causes the large change of fitness values. For designers, the former is especially important because they can select design variables from some design patterns having similar performance in consideration of the cost of design or the difficulty level of design and the later is because they have to choose design variables very carefully. Figure 1 shows an example of Non-Correspondence area. In Fig. 1, data 5-6-7-8 are distributed widely in the design variable space compared to the distribution of the objective space.

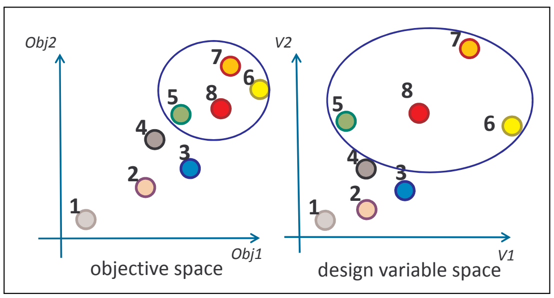


Fig. 1: Non-Correspondence area in Spread

2.2 Index for Non-Correspondence Area in Spread

Here, we define the quantitative index for Non-Correspondence in Spread to extract the Non-Correspondence area. The index is calculated in the following procedure.

1) Define the neighborhood radius ϵ (eq. (1)) in the objective space or the design variable space.
2) Extract the individuals as target individuals within radius ϵ from individual i.
3) Calculate the center of gravity of the target individuals.
4) Calculate the index for Non-Correspondence in Spread v_i according to eq. (2).

By the above procedure, the index v_i is calculated for each individual. The neighborhood radius ϵ is defined by eq. (1). In eq. (1), η denotes the parameter that defines the neighborhood radius, f_{lmax}, f_{lmin} mean the maximum and the minimum fitness values, which are normalized, in the Pareto solutions for objective function l, and M_f is the number of objective functions. x_{lmax}, x_{lmin} mean the maximum range and the minimum range of design variables l, and M_d is the number of design variables. If the neighborhood is defined in the objective space, the upper equations in eq. (1) and eq. (2) are employed and otherwise the lower equations are employed to calculate the value of index v_i. In eq. (2), d_{dcik} is the normalized Euclidean Distance between target individual k and the center of gravity in the design variable space, d_{fcik} is that in the objective space, N is the number of the target individuals and v_i is the index for individual i. Individuals with large indexes are distributed densely in the objective space / design variable space and distributed widely in the design variable space / objective space.

$$\epsilon = \begin{cases} \frac{\sqrt{\sum_{l=1}^{M_f}(f_{lmax}-f_{lmin})^2}}{\eta} \\ \text{(Neighborhood was defined in the objective space.)} \\ \frac{\sqrt{\sum_{l=1}^{M_d}(x_{lmax}-x_{lmin})^2}}{\eta} \\ \text{(Neighborhood was defined in the design variable space.)} \end{cases} \tag{1}$$

$$v_i = \begin{cases} \frac{1}{N}\sum_{k=1}^{N}(d_{dcik}) \\ \text{(Neighborhood was defined in the objective space.)} \\ \frac{1}{N}\sum_{k=1}^{N}(d_{fcik}) \\ \text{(Neighborhood was defined in the design variable space.)} \end{cases} \tag{2}$$

2.3 Application to Genetic Search

By applying the index defined above to genetic search, it is aimed to obtain solutions whose fitness values are robust to the change of design variables. In this paper, we apply the index value instead of Crowding Distance that is one of the distinguish mechanism in NSGA-II. We introduce the new search criterion R according to eq. (4) . In eq. (4), CD_i is Crowding Distance of individual i and v_i is the index value of Non-Correspondence in Spread defined by eq. (2). As the aim of 2.2 is to extract the Non-Correspondence area, v_i is calculated using the distance between target individuals and the center of gravity of them. On the other hand, v_i is calculated according to eq. (3) using the normalized Euclidean Distance d_{dik} between the individual i and the target individual k because the index value is introduced into each individual as search criterion. In addition, this paper aims to obtain solutions with the robustness to the change of fitness values in regard to the change of design variables, then the neighborhood radius is defined in the objective space.

$$v_i = \frac{1}{N}\sum_{k=1}^{N}(d_{dik}) \tag{3}$$

$$R_i = CD_i * v_i \tag{4}$$

The larger the value of CD or v is, the larger the value of R is. Thus it is expected that this method can consider the diversity in the objective space and the robustness of fitness values.

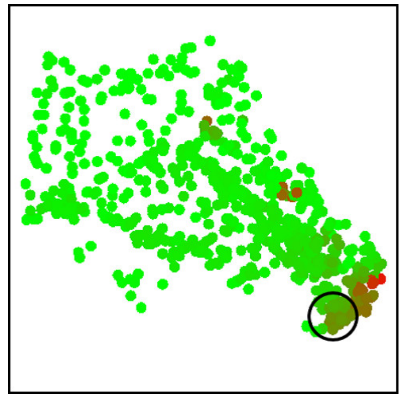
3. Experiment

In this paper, we applied the above calculation to the trajectory designing optimization problem "DESTINY" provided by JAXA[13] and analyzed the obtained Pareto solutions.

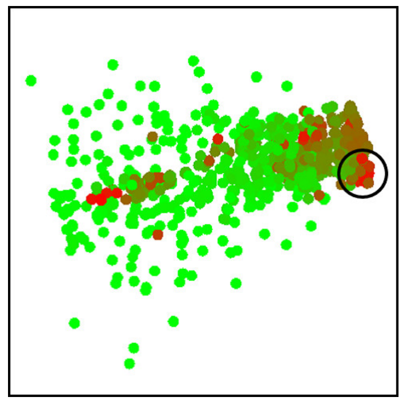
In the experimental condition here, population size was 100, and generation was 100. We compared the proposed method described in 2.3 and NSGA-II as the conventional method. Note that the difference between the proposed method and the conventional method was only the difference between Crowding Distance (CD_i) and R_i by eq. (4). Figure 8 shows the visualization result of the distribution of obtained Pareto solutions in the objective space by MDS. In Fig. 2, the color of solutions indicates the value of index v, in which red color means large value and green color means small value.

As shown in Fig. 2, Pareto solutions by the proposed method have many solutions with large value of v compared to the solutions by the conventional method. We extracted the solutions with large value of v (in the circle in Fig. 2(b)) from the obtained Pareto solutions by the proposed method. Next, we extracted the solutions having same fitness values in the conventional method and compared the values of design variables. The maximum and minimum fitness values in the extracted solutions are shown in TABLE 1.

We focused on Start Date and Time ($V1$ and $V2$), and showed the result of comparison in terms of Start Date and Time in Fig. 3. In Fig. 3, the horizontal axis shows the label of each solution and the vertical axis shows the value of each design variable. Red plots indicate Start Date and Time of solutions by the proposed method with the range of fitness values in TABLE 1 and blue plots indicate those of solutions by the conventional method. As shown in Fig. 3, the proposed method could obtain the solutions with Start Date and Time which the conventional method could not do (from December to February, from 23 o'clock to 2 o'clock). This result shows that the proposed method could obtain a lot of solutions having same fitness values and different Start Date and Time.



(a) Result of conventional method



(b) Result of proposed method

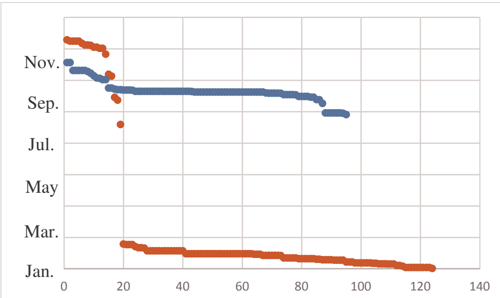
Fig. 2: Distribution of Pareto Solutions (Objective Space)

Table 1: Maximum and Minimum of Fitness Values

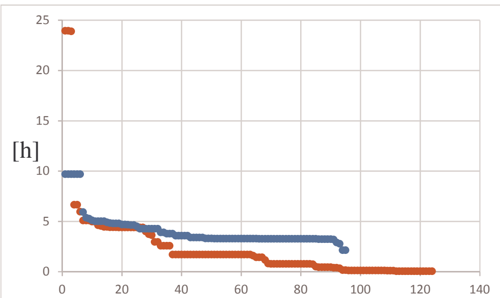
	Max	Min
$Obj1$	1544.51	1442.90
$Obj2$	8364.64	7999.78
$Obj3$	438.76	424.75
$Obj4$	1.893	1.144
$Obj5$	223.87	218.014

4. Conclusion

In this paper, we defined Non-Correspondence in Spread between the objective space and the design variable space. We proposed the quantitative index to extract Non-Correspondence area in Spread. This paper applied the proposed method to the trajectory designing optimization problem known as DESTINY provided by JAXA and analyzed the extracted Non-Correspondence area in Spread in the obtained Pareto solutions. This paper showed that the Pareto solutions having widely different design variables with similar fitness values could be extracted. This paper also applied the defined index to genetic search and obtained a lot of Pareto solutions having similar fitness values but different Start Date and Time. For the future work, we will apply the proposed method to other problems with more objective functions or higher dimensional design variables and study Non-Correspondence in Linear Relationship.



(a) Start Date（$V1$）



(b) Start Time（$V2$）

Fig. 3: Start Date and Time

5. Acknowledgment

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References

[1] K. Deb, *Multi-objective optimization using evolutionary algorithms*. Wiley, 2001.
[2] S. Obayashi, “Multiobjective design optimization of aircraft configuration (in japanese),” *The Japanese Society for Artificial Intelligence*, vol. 18, pp. 495–501, 2003.
[3] K. Deb, “Unveiling innovative design principles by means of multiple conflicting objectives,” *Engineering Optimization*, vol. 35, no. 5, pp. 445–470, 2003.
[4] K. H. Akira Oyama, Yasuhiro Kawakatsu, “Application of multiobjective design exploration to trajectory design of the next-generation solar physics satellite,” *Japanese Society for Evolutionary Computation*, 2010.
[5] F. Kudo and T. Yoshikawa, “Knowledge extraction in multi-objective optimization problem based on visualization of pareto solutions,” *WCCI 2012 IEEE World Congress on Computational Intelligence*, pp. 860–865, 2012.
[6] T. Yoshida and T. Yoshikawa, “An extraction of non-correspondence area between objective space and design variable space based on order correlation of distance relation（in japanese）,” *The Japanese Society for Artificial Intelligence*, 2013.
[7] A. L´ opez, A. Oyama, and K. Fujii, “Evaluating two evolutionary approaches to solve a many-objective space trajectory design problem,” *The Japanese Society for Evolutionary Computation*, 2012.
[8] K. Deb, *A Fast and Elitist Multiobjective Genetic Algorithm : NSGA-II*. 2002.
[9] T. Yoshida and T. Yoshikawa, “A study on Non-Correspondence in Spread between Objective Space and Design Variable Space for Trajectory Designing Optimization Problem,” *2014 IEEE Congress on Evolutionary Computation (CEC)*, pp. 2444–2450, 2014.
[10] K. Deb and R. Agrawal, “Simulated binary crossover for continuous search space,” *Complex Systems*, vol. 1, no. 9, pp. 115–148, 1994.
[11] K. Deb and M. Goyal, “A combined genetic adaptive search (geneas) for engineering design,” *Computer Science and Informatics*, vol. 26, pp. 30–45, 1996.
[12] J. W. Sammon, “A nonlinear mapping for data structure analysis,” *IEEE Transactions on computers*, vol. 18, no. 5, pp. 401–409, 1969.
[13] http://www.ciss.iis.u-tokyo.ac.jp/supercomputer/about/.