A Note on the Circuit-Switched Fixed Routing in Networks *

4 C − 1

Toshinori Yamada and Shuichi Ueno †
Department of Physical Electronics, Tokyo Institute of Technology ‡

1 Circuit-Switched Fixed Routing

The circuit-switched fixed-routing model has been adopted for some parallel computer systems, such as iPSC-2 and iPSC-860 by Intel, NCUBE/10 by nCUBE, and Symult 2010 by Ametek [5]. In this model, a fixed path is dedicated to every source-destination pair and data is pipelined through the path. Once a fixed path is established for a source-destination pair, the path exclusively uses all the edges that it traverses and no other fixed path that uses one of those same edges can be established simultaneously. Therefore, if multiple source-destination pairs wish to communicate simultaneously, the fixed paths dedicated to those source-destination pairs must be edge-disjoint.

Let G be a graph representing a network, and let $V(G)$ and $E(G)$ denote the vertex set and edge set of G, respectively. A routing ρ on G is a mapping from the set of all ordered pairs of vertices in G to the set of all paths in G such that $\rho([u,v])$ is a path connecting u and v. A communication request on G is a set of ordered pairs of vertices in G. If $[u,v]$ is in a communication request, u is called the source and v the destination of the pair. A communication request on G is called a partial permutation if each vertex appears in the request at most once as a source and at most once as a destination. A permutation is a partial permutation with $|V(G)|$ source-destination pairs.

Let G be a graph with routing ρ. For a source-destination pair $[u,v]$, $\rho([u,v])$ is called a fixed path dedicated to $[u,v]$. A scheduling for a permutation π is a decomposition of π into partial permutations such that the fixed paths dedicated to the source-destination pairs in each partial permutation are edge-disjoint. The size of a scheduling is the number of partial permutations in the decomposition. Let $\sigma(\pi,\rho,G)$ be the minimum size of a scheduling for a permutation π on a graph G with routing ρ. Define that

$$\begin{aligned} \sigma(\rho,G) &= \max_{\pi} \sigma(\pi,\rho,G), \text{ and} \\ \sigma(G) &= \min_{\rho} \sigma(\rho,G). \end{aligned}$$

Since the impact of vertex conflict and path length is negligible in circuit-switched fixed-routing networks as mentioned by Bokhari[1], $\sigma(G)$ is the dominant factor for the communication overhead in circuit-switched fixed-routing network G. Therefore, designing a routing ρ that attains $\sigma(G)$ and finding a scheduling with size $\sigma(\rho,G)$ are fundamental problems to minimize the communication overhead when realizing a permutation on a circuit-switched fixed-routing network G. The problems were first considered by Youssef [5]. Among other results, it is shown in [5] that $\sigma(G) = O(\sqrt{N})$ if G is a 2-dimensional square mesh with N vertices.

2 Upper Bound for Product Graphs

The mesh is a typical example of product graphs, many of which have emerged as attractive interconnection graphs for large multiprocessor systems. The product of two graphs G and H, denoted by $G \times H$, is the graph defined as follows:

$$\begin{aligned} V(G\times H) &= V(G)\times V(H); \\ E(G\times H) &= \{([u,v],[u',v])|\ (u,u') \in E(G)\} \\ &\quad \cup\{([u,v],[u,v'])|\ (v,v') \in E(H)\}. \end{aligned}$$

We can show the following upper bound for product graphs.

Theorem 1 *Let G_1 and G_2 be N_1- and N_2-vertex graphs with p_1 and p_2 edge-disjoint spanning trees, respectively. Then,*

$$\sigma(G_1 \times G_2) \le \max\{\lceil N_1/p_1\rceil, \lceil N_2/p_2\rceil\}.$$

*ネットワークの回線交換固定ルーティングについて
†山田 敏規, 上野 修一
‡東京工業大学 電子物理工学科

The theorem is proved by exhibiting a routing ρ and an $O(N \log N)$ time scheduling algorithm that attains

$$\sigma(\rho, G_1 \times G_2) \leq \max\{\lceil N_1/p_1 \rceil, \lceil N_2/p_2 \rceil\},$$

where $N = |V(G_1 \times G_2)| = N_1 \times N_2$.

3 General Lower Bound

On the other hand, we have the following general lower bound.

Theorem 2 *For any N-vertex graph G,*

$$\sigma(G) = \Omega(\sqrt{N}/\Delta(G)),$$

where $\Delta(G)$ is the maximum vertex degree of G. ∎

Kaklamanis, Krizanc, and Tsantilas [3] showed that for any N-vertex graph G and any packet-switched oblivious routing ρ on G, there exists a permutation π such that ρ requires $\Omega(\sqrt{N}/\Delta(G))$ steps to realize π. Since the lower bound is derived from an estimate of the edge congestion, the same lower bound can be derived for the circuit-switched fixed routing by a slight modification of argument.

4 Tight Bounds for Some Product Graphs

From the theorems above, we can derive tight bounds for some product graphs. We denote the N-vertex path and cycle by P_N and C_N, respectively, and let $\prod_{i=1}^{d} G_i = G_1 \times G_2 \times \cdots \times G_d$. $R_d(k) = \prod_{i=1}^{d} P_k$ is the d-dimensional k-sided mesh, $D_d(k) = \prod_{i=1}^{d} C_k$ is the d-dimensional k-sided torus, and $Q_n = \prod_{i=1}^{n} P_2$ is the n-dimensional cube. We can show the following tight bounds.

Theorem 3

$$\sigma(Q_n) = \Theta(\sqrt{N}/\log N)$$

where $N = |V(Q_n)| = 2^n$;

$$\sigma(D_d(k)) = \Theta(\sqrt{N}/d)$$

if d is even where $N = |V(D_d(k))| = k^d$;

$$\sigma(R_d(k)) = \Theta(\sqrt{N}/d)$$

if d is even where $N = |V(R_d(k))| = k^d$. ∎

The lower bounds can be derived from Theorem 2 and the fact that $\Delta(Q_n) = n$ and $\Delta(D_d(k)) = \Delta(R_d(k)) = 2d$.

The upper bounds can be derived from Theorem 1 as follows. We first observe that

$$\begin{aligned} Q_n &= Q_{\lceil n/2 \rceil} \times Q_{\lfloor n/2 \rfloor}, \\ D_d(k) &= D_{d/2}(k) \times D_{d/2}(k), \text{ and} \\ R_d(k) &= R_{d/2}(k) \times R_{d/2}(k). \end{aligned}$$

We also observe that Q_n is n-edge-connected, $D_d(k)$ is $2d$-edge-connected, and $R_d(k)$ is d-edge-connected. Since it is well-known that an m-edge-connected graph has $\lceil (m-1)/2 \rceil$ edge-disjoint spanning trees [2, 4], we have the desired upper bounds.

Acknowledgements: The authors are grateful to an anonymous referee of JSPP'98 whose suggestions and comments on earlier manuscript motivated to improve the results as well as the presentation. They are also grateful to Prof. Y. Kajitani for his encouragement. The research is a part of CAD21 Project at TIT.

References

[1] S. H. Bokhari. Communication overheads on the intel iPSC-2 hypercube. ICASE Interim Report 10, May 1990.

[2] D. Gusfield. Connectivity and edge-disjoint spanning trees. *Information Processing Letters*, Vol. 16, pp. 87–89, 1983.

[3] C. Kaklamanis, D. Krizanc, and T. Tsantilas. Tight bounds for oblivious routing in the hypercube. *Mathematical Systems Theory*, Vol. 24, pp. 223–232, 1991.

[4] S. Kundu. Bounds on the number of disjoint spanning trees. *Journal of Combinatorial Theory (B)*, Vol. 17, pp. 199–203, 1974.

[5] A. Youssef. Off-line permutation routing on circuit-switched fixed-routing networks. *Networks*, Vol. 23, pp. 441–448, 1993.