Regular Paper

Re-examination of Allen's Interval-based Temporal Logic

YOSHIFUMI MASUNAGA†

This paper re-examines Allen's interval-based temporal logic. First, it is shown that only three out of Allen's thirteen temporal relationships are topological invariants. Second, the topological and the ordering temporal relationship system are introduced to show that Allen's temporal relationship system is induced by the combination of these two systems. Third, it is shown that an arbitrary temporal relationship system such as beginning temporal relationship system is defined as a canonical surjection induced by a partition of the set of Allen's thirteen relationships. Fourth, a dependency structure among temporal relationship systems is shown. Combined reasoning for arbitrary temporal queries is also investigated based on those results. Last, it is shown that Allen's ***equals*** and ***meets*** relationships are necessary and sufficient to represent any one of Allen's thirteen relationships if null temporal intervals are used.

1. Introduction

Allen introduced the interval-based temporal logic[1] to represent temporal knowledge and temporal reasoning, where a temporal interval is defined as an ordered pair of time-points with the first point less than the second. It is shown that there are at most thirteen temporal relationships that could hold between two temporal intervals, which are referred to by *equals, before, after, during, contains, overlaps, overlapped_by, meets, met_by, starts, started_by, finishes,* and *finished_by*. Based on this result he developed a system which can infer new relationships which might hold when a new relationship is added to an existing network where the nodes represent individual intervals.

To represent temporal multimedia data such as audio and video, Allen's interval-based temporal logic is very powerful because such data are regarded as temporal intervals. Many investigations have been done based on this approach: Temporal intervals or time-lines have been used to represent and query historical databases[2], to represent multimedia playback processes with a modification of Petri net[3),4], to visualize temporal transformations of temporal data and to depict the temporal relationships of a composite temporal object[5),6], to graphically describe how media within a presentation are arranged over time[7], and to show a unified approach to representation, synchronization, and storage of temporal multimedia data based on an object-oriented approach[8]. Standardization activity on multimedia documents such as Hyper ODA is also based on this approach[9].

In addition, Allen's interval-based logic has been used intensively to represent spatial knowledge in the research field of geographical information systems (GIS). For example, two-dimensional spatial objects such as countries, states, towns, lakes are represented as two-dimensional spatial intervals, which are referred to minimum bounding rectangles (MBR)[10]. One-dimensional spatial objects such as roads and rivers are represented as a set of one-dimensional spatial intervals, i.e. lines. Similarly, three-dimensional spatial objects such as buildings can be represented as three-dimensional spatial intervals, i.e. blocks which are named minimum bounding blocks (MBB)[11]. Topological nature of spatial relationships are extensively investigated by Egenhofer, et al.[12]. Qualitative spatial reasoning about distance and direction is investigated by Frank[13]. Heterogeneous spatial reasoning is investigated by Sharma, et al.[14].

However, in spite of the important roles of Allen's interval-based temporal logic in spatio-temporal multimedia information system design and implementation, little investigation has been done to figure out its topological, algebraic, and semantic nature. Without knowing its topological nature, for example, we cannot answer even a very preliminary query like which one of the thirteen relationships is topological invariant. That is, the investigation is indispensable to design spatio-temporal query languages, and to show how to organize and manage spatio-temporal multimedia data.

† Department of Library and Information Science, University of Library and Information Science

In this paper, we re-examine Allen's interval-based temporal logic: In Section 2, it is shown that only three out of Allen's thirteen temporal relationships have topological nature; which are *equals, contains*, and *during*. We call a set of Allen's thirteen temporal relationships with transitivity table as Allen's relationship "system." We introduce the topological and the ordering temporal relationship system which consist of a set of eight topological temporal relationships and a set of three ordering temporal relationships, respectively. Then, it is shown that Allen's relationship system is derived from the composite combination of these two systems. In Section 3, we investigate canonical forms of temporal relationships. It is shown that temporal relationships are characterized as partitions of the set of Allen's thirteen relationships. Then, it is shown that an arbitrary temporal relationship is induced by a canonical surjection defined by an arbitrary partitioning, and that the total number of meaningful temporal relationships does not exceed 27,644,437. A dependency structure among temporal relationship systems is also shown. In Section 4, normal forms of Allen's thirteen temporal relationships are investigated. It is shown that any one of Allen's thirteen temporal relationships has its equivalent that is constructed using only *equals* and *meets* relationships if "null temporal intervals" are used. Based on this result, normal forms of composite temporal multimedia objects are investigated. Section 5 concludes this paper.

2. Re-examination of Allen's Interval-based Temporal Logic

2.1 Allen's Interval-based Temporal Logic

2.1.1 Definitions

Let R be the real line. We regard R as the time axis throughout this article. A temporal interval I is defined as an ordered pair of time-points stp and etp; $[\text{stp}, \text{etp}] (\in \text{R} \times \text{R})$ with the first point (= stp) less than the second (= etp). Start (end) time point of an interval I is denoted by I.stp (I.etp) using dot notation.

Allen[1] showed that there are at most thirteen temporal relationships that could hold between two temporal intervals, which are referred to by *equals, before, after, during, contains, overlaps, overlapped_by, meets, met_by, starts, started_by, finishes*, and *finished_by*. Note that they are mutually exclusive by definition. Note also that six out of them are the inverses of other six relationships. For example, relationship *after* is the inverse of *before*. Here we summarize the definition of thirteen temporal relationships, where I and J represent temporal intervals, and $\Longleftrightarrow$ is a shorthand notation of "if and only if."

[Definition of Allen's Temporal Relationships]

(1) $(\forall \text{I}, \forall \text{J})(\mathit{equals}(\text{I}, \text{J}) \Longleftrightarrow \text{I.stp} = \text{J.stp and I.etp} = \text{J.etp})$

(2) $(\forall \text{I}, \forall \text{J})(\mathit{before}(\text{I}, \text{J}) \Longleftrightarrow \text{I.etp} < \text{J.stp})$

(3) $(\forall \text{I}, \forall \text{J})(\mathit{after}(\text{I}, \text{J}) \Longleftrightarrow \mathit{before}(\text{J}, \text{I}))$

(4) $(\forall \text{I}, \forall \text{J})(\mathit{during}(\text{I}, \text{J}) \Longleftrightarrow \text{I.stp} > \text{J.stp and I.etp} < \text{J.etp})$

(5) $(\forall \text{I}, \forall \text{J})(\mathit{contains}(\text{I}, \text{J}) \Longleftrightarrow \mathit{during}(\text{J}, \text{I}))$

(6) $(\forall \text{I}, \forall \text{J})(\mathit{overlaps}(\text{I}, \text{J}) \Longleftrightarrow \text{I.stp} < \text{J.stp and I.etp} > \text{J.stp and I.etp} < \text{J.etp})$

(7) $(\forall \text{I}, \forall \text{J})(\mathit{overlapped\_by}(\text{I}, \text{J}) \Longleftrightarrow \mathit{overlaps}(\text{J}, \text{I}))$

(8) $(\forall \text{I}, \forall \text{J})(\mathit{meets}(\text{I}, \text{J}) \Longleftrightarrow \text{I.etp} = \text{J.stp})$

(9) $(\forall \text{I}, \forall \text{J})(\mathit{met\_by}(\text{I}, \text{J}) \Longleftrightarrow \mathit{meets}(\text{J}, \text{I}))$

(10) $(\forall \text{I}, \forall \text{J})(\mathit{starts}(\text{I}, \text{J}) \Longleftrightarrow \text{I.stp} = \text{J.stp and I.etp} < \text{J.etp})$

(11) $(\forall \text{I}, \forall \text{J})(\mathit{started\_by}(\text{I}, \text{J}) \Longleftrightarrow \mathit{starts}(\text{J}, \text{I}))$

(12) $(\forall \text{I}, \forall \text{J})(\mathit{finishes}(\text{I}, \text{J}) \Longleftrightarrow \text{I.stp} > \text{J.stp and I.etp} = \text{J.etp})$

(13) $(\forall \text{I}, \forall \text{J})(\mathit{finished\_by}(\text{I}, \text{J}) \Longleftrightarrow \mathit{finishes}(\text{J}, \text{I}))$

2.1.2 Allen's Temporal Relationship System

The set of all Allen's thirteen temporal relationships, denoted by $\mathbf{R}_{\text{allen}}$, is "closed" for temporal intervals in the sense that one and only one of them holds between an arbitrary pair of two temporal intervals. Therefore, given a set of temporal intervals $\mathbf{I} = \{\text{I}_1, \text{I}_2, \ldots, \text{I}_n\}$, we can compute a set of instances of Allen's temporal relationships with respect to **I**; $\mathbf{R}_{\text{Allen}}|\mathbf{I}\times\mathbf{I} = \{\text{r}_{ij}|\text{r}(\text{I}_i, \text{I}_j) = \text{true}\}$. This set consists of n^2 elements. Using this set, a simple query like "Does I meet J?" can be answered directly by pattern matching method. However, this set of instances cannot answer a complex query like "Which interval is before K which is during J," where K is a new interval not in **I**. In order to answer this query, transitive relationships should be maintained. For example, if *during*(K, J) and *before*(I, J) hold, then we can infer that *before*(I, K) holds. The "transitivity table" is introduced by Allen to record inference rules between two temporal relationships[1]. We denote this table by $\mathbf{T}_{\text{Allen}}$. A similar work for spatial relationships is found in GIS research field, where the transitivity table is called the composition table[15]. To answer simple or complex queries, both $\mathbf{R}_{\text{Allen}}$ and $\mathbf{T}_{\text{Allen}}$ should be maintained. By $\boldsymbol{R}_{Allen} = (\mathbf{R}_{\text{Allen}}, \mathbf{T}_{\text{Allen}})$, we mean Allen's temporal relationship "system." Other relationship systems will be introduced in the following sections.

2.2 Topological and Ordering Temporal Relationships

2.2.1 Topological Temporal Relationships

Topology is a generalized concept of metric which is one of the most important concepts in spaces. A set M with a family U of its subsets is called a topological space if the following conditions are satisfied: (a) M and ϕ (empty set) are in U, (b) the intersection of any finite number of members of U is in U, and (c) the arbitrary union of members of U is in U. A topological space M with a topology U is denoted by (M, U). For any given set M, there are always two topologies on M. These are: (i) U consisting of all subsets of M, and (ii) U consisting of only M and ϕ. The real line R is a metric space with metric $\mathrm{d(x,y)} = |\mathrm{x} - \mathrm{y}|$ and therefore a topological space. The Euclidian n-dimensional space R^n is a topological space. Let $(\mathrm{M}_1, \mathrm{U}_1)$ and $(\mathrm{M}_2, \mathrm{U}_2)$ be two topological spaces. If there exists a continuous mapping f: $\mathrm{M}_1 \rightarrow \mathrm{M}_2$ such that f is a one-to-one and onto mapping, and f^{-1}: $\mathrm{Y} \rightarrow \mathrm{X}$ is continuous, then these two spaces are called homeomorphic, and f is called a homeomorphism. Properties of topological spaces that are preserved under homeomorphisms are called "topological invariants." For example, the property of connectedness is a topological invariant.

Topological relationships between spatial objects have been investigated in the GIS research field. For example, Egenhofer, et al.[16] introduced "4-intersection," which is a 2×2 matrix defined by using the interior and the boundary of spatial regions to provide a framework for the description of topological spatial relationships. It is shown that there are eight topological relationships between two two-dimensional spatial regions, which are referred to by *t_disjoint, t_contains, t_inside, t_equals, t_meets, t_covers, t_coveredBy,* and *t_overlaps.* (To distinguish topological relationships from Allen's relationships, prefix "*t_*" is attached.) It is shown that topological relationships are topological invariants. Note that they are mutually exclusive and are closed for regions. Of course results are valid for the one-dimensional case except that the value of the intersection of the second row and the second column of the 4-intersection is 1 for two-dimensional case, while it is 0 for one-dimensional case because two boundaries have no common element in the latter case. The following shows the definition of eight topological relationships for two temporal intervals I and J. Borrowing the composition table for topological spatial relationships[15] we can define a topological temporal relationship system $\boldsymbol{R}_{topology} = (\mathbf{R}_{\text{topology}}, \mathbf{T}_{\text{topology}})$. Topological temporal relationship definition is given below:

[Definition of Topological Temporal Relationships]

(1) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_disjoint(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ I.etp < J.stp or I.stp > J.etp)

(2) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_contains(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ I.stp < J.stp and I.etp > J.etp)

(3) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_inside(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ $t\_contains(\mathrm{J}, \mathrm{I}))$

(4) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_equals(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ I.stp = J.stp and I.etp = J.etp)

(5) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_meets(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ I.etp = J.stp or I.stp = J.etp)

(6) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_covers(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ (I.stp = J.stp and I.etp > J.etp) or (I.stp < J.stp and I.etp = J.etp))

(7) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_coveredBy(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ $t\_covers(\mathrm{J}, \mathrm{I}))$

(8) $(\forall \mathrm{I}, \forall \mathrm{J})(t\_overlaps(\mathrm{I}, \mathrm{J}) \Longleftrightarrow$ (I.stp < J.stp and I.etp > J.stp and I.etp < J.etp) or (I.stp > J.stp and I.stp < J.etp and I.etp > J.etp))

2.2.2 Topological Invariants and Allen's Temporal Relationships

In this section we examine which temporal relationships are topological invariants among Allen's thirteen temporal relationships.

[Proposition 1] Allen's temporal relationship *equals* is a topological invariant.

(Proof) Suppose that there exists a topological space S being homeomorphic to the real line R under $\mathrm{f} : \mathrm{R} \rightarrow \mathrm{S}$, where the relationship *equals* does not hold in it. Then, there exist temporal intervals I = [stp1, etp1] and J = [stp2, etp2] on R under *equals* such that *equals*(f(I), f(J)) does not hold in S. Without loss of generality we assume that f(stp1) = f(stp2) does not hold. Since f is a homeomorphism, the inverse mapping f^{-1} exists so that $\mathrm{stp1} = \mathrm{f}^{-1}(\mathrm{f(stp1)}) = \mathrm{f}^{-1}(\mathrm{f(stp2)}) = \mathrm{stp2}$ does not hold in R. This is a contradiction. Q.E.D.

[Corollary 1] Allen's temporal relationships

contains and *during* are topological invariants.
(Proof) Proven similarly to Proposition 1.
Q.E.D.

[Proposition 2] Allen's temporal relationship *meets* is not a topological invariant.
(Proof) A mapping f defined by $f : R \to R$; x - -> $-x$ is a homeomorphism, because $g : R \to R$; $-x$ - -> x is an inverse mapping of f, and f and g are continuous. Suppose that $I = [stp1, etp1]$ and $J = [stp2, etp2]$ are under *meets*. Then, $stp1 < etp1 = stp2 < etp2$ holds. Now, f maps I and J to $[-etp1, -stp1](= f(I))$ and $[-etp2, -stp2](= f(J))$, respectively. However *meets*(f(I), f(J)) does not hold, which contradicts to the assumption that f is a homeomorphism. Q.E.D.

[Corollary 2] Allen's temporal relationships *before, after, overlaps, overlapped_by, met_by, starts, started_by, finishes,* and *finished_by* are not topological invariants.
(Proof) Proven similarly to Proposition 2.
Q.E.D.

[Theorem 1] Only *equals*, *contains* and *during* are topological invariants among Allen's thirteen temporal relationships.
(Proof) This is true by Proposition 1 and 2, and Corollary 1 and 2. Q.E.D.

Relationships which are topological invariants will be called topological relationships in short.

2.2.3 Ordering Temporal Relationships

In addition to topological spatial relationships, other spatial relationships such as direction relationships, distance relationships, ordering relationships have been introduced in GIS research field. For example, region A is west of region B if they are related under a direction relationship *West*(A, B). Ordering relationships are also important in temporal multimedia data modeling point of view. For example, we say that temporal interval I "precedes" (succeeds) temporal interval J if and only if $I.etp \le J.stp$ ($I.stp \ge J.etp$). Otherwise, I and J are "unordered."

Now let define three ordering temporal relationships; *o_precedes, o_succeeds* and *o_unordered* as follows:

[Definition of Ordering Temporal Relationships]

(1) $(\forall I, \forall J)(\textit{o\_precedes}(I, J) \Longleftrightarrow I.etp \le J.stp)$

(2) $(\forall I, \forall J)(\textit{o\_succeeds}(I, J) \Longleftrightarrow I.stp \ge J.etp)$

(3) $(\forall I, \forall J)(\textit{o\_unordered}(I, J) \Longleftrightarrow (\text{not}(\textit{o\_precedes}(I, J)) \text{ and not}(\textit{o\_succeeds}(I, J)))$

Note that these three relationships are mutually execlusive and are closed for temporal intervals. Therefore, we can introduce an ordering temporal relationship system $\boldsymbol{R}_{ordering} = (\mathbf{R}_{\text{ordering}}, \mathbf{T}_{\text{ordering}})$, where transitivity table $\mathbf{T}_{\text{ordering}}$ is defined as follows, and $\Longrightarrow$ is a shorthand notation of "if."

[Transitivity Table for Ordering Temporal Relationship System]

(1) $(\forall I, \forall J, \forall K)(\textit{o\_precedes}(I, J) \text{ and } \textit{o\_precedes}(J, K) \Longrightarrow \textit{o\_precedes}(I, K))$

(2) $(\forall I, \forall J, \forall K)(\textit{o\_precedes}(I, J) \text{ and } \textit{o\_succeeds}(J, K) \Longrightarrow \textit{o\_succeeds}(I, K) \text{ or } \textit{o\_unordered}(I, K) \text{ or } \textit{o\_precedes}(I, K))$

(3) $(\forall I, \forall J, \forall K)(\textit{o\_precedes}(I, J) \text{ and } \textit{o\_unordered}(J, K) \Longrightarrow \textit{o\_unordered}(I, K) \text{ or } \textit{o\_precedes}(I, K))$

(4) $(\forall I, \forall J, \forall K)(\textit{o\_succeeds}(I, J) \text{ and } \textit{o\_precedes}(J, K) \Longrightarrow \textit{o\_precedes}(I, K)) \text{ or } \textit{o\_succeeds}(I, K)) \text{ or } \textit{o\_unordered}(I, K))$

(5) $(\forall I, \forall J, \forall K)(\textit{o\_succeeds}(I, J) \text{ and } \textit{o\_succeeds}(J, K) \Longrightarrow \textit{o\_succeeds}(I, K))$

(6) $(\forall I, \forall J, \forall K)(\textit{o\_succeeds}(I, J) \text{ and } \textit{o\_unordered}(J, K) \Longrightarrow \textit{o\_precedes}(I, K) \text{ or } \textit{o\_unordered}(I, K))$

(7) $(\forall I, \forall J, \forall K)(\textit{o\_unordered}(I, J) \text{ and } \textit{o\_precedes}(J, K) \Longrightarrow \textit{o\_precedes}(I, K) \text{ or } \textit{o\_unordered}(I, K))$

(8) $(\forall I, \forall J, \forall K)(\textit{o\_unordered}(I, J) \text{ and } \textit{o\_succeeds}(J, K) \Longrightarrow \textit{o\_succeeds}(I, K) \text{ or } \textit{o\_unordered}(I, K))$

(9) $(\forall I, \forall J, \forall K)(\textit{o\_unordered}(I, J) \text{ and } \textit{o\_unordered}(J, K) \Longrightarrow \textit{o\_succeeds}(I, K) \text{ or } \textit{o\_unordered}(I, K) \text{ or } \textit{o\_precedes}(I, K))$

2.3 Characterization of Allen's Temporal Relationships

As shown in Theorem 1, only three out of Allen's thirteen temporal relationships are topological invariants. How do we characterize other ten relationships? In order to answer this question, we note first that from topological point of view there is no significance of whether temporal interval I precedes temporal interval J or I succeeds J because the values of the 4-intersection matrixes coincide with each other. In other words, distinction between Allen's temporal relationship *before* and its inverse *after* is meaningless from topological point of view. Furthermore, they can be grouped together as a topological relationship *t_disjoint* because $(\forall I, \forall J)(\textit{t\_disjoint}(I, J)$

Table 1 Characterization of Allen's temporal relationships.

topological \ ordering	*o_precedes*	*o_unordered*	*o_succeeds*
t_disjoint	*before*	NA	*after*
t_contains	NA	*contains*	NA
t_inside	NA	*during*	NA
t_equals	NA	*equals*	NA
t_meets	*meets*	NA	*met_by*
t_covers	*starts*	NA	*finishes*
t_coveredBy	*started_by*	NA	*finished_by*
t_overlaps	*overlaps*	NA	*overlapped_by*

$\Longleftrightarrow$ *before*(I, J) and *after*(I, J)) holds. Distinctions between Allen's *meets* and its inverse *met_by, overlaps* and *overlapped_by, starts* and *finishes,* and *started_by* and *finished_by* are meaningless by the same reason. They should be grouped together as *t_meets, t_overlaps, t_coveredBy,* and *t_covers*, respectively.

Distinction among three groups; {*before, meets*}, {*after, met_by*}, {*equals, during, contains, overlaps, overlapped_by, starts, started_by, finishes, finished_by*} makes sense from "ordering" point of view. Three ordering temporal relationships are defined, which are referred to by *o_precedes, o_succeeds,* and *o_unordered.* This is the ordering temporal relationship system introduced in the previous section.

Now, we can prove that Allen's temporal relationships are the results of the combination of the topological and the ordering temporal relationship system. This result is stated in the next theorem and is illustrated in **Table 1**.

[Theorem 2] Allen's thirteen temporal relationships are obtained by the combined composition of eight topological temporal relationships and three ordering temporal relationships.

(Proof) Each one of Allen's thirteen temporal relationships is characterized as follows:

(1) $(\forall I, \forall J)$(*t_disjoint*(I, J) and *o_precedes*(I, J) $\Longleftrightarrow$ *before*(I, J))

(2) $(\forall I, \forall J)$(*t_disjoint*(I, J) and *o_succeeds*(I, J) $\Longleftrightarrow$ *after*(I, J))

(3) $(\forall I, \forall J)$(*t_contains*(I, J) and *o_unordered*(I, J) $\Longleftrightarrow$ *contains*(I, J))

(4) $(\forall I, \forall J)$(*t_inside*(I, J) and *o_unordered*(I, J) $\Longleftrightarrow$ *during*(I, J))

(5) $(\forall I, \forall J)$(*t_equals*(I, J) and *o_unordered*(I, J) $\Longleftrightarrow$ *equals*(I, J))

(6) $(\forall I, \forall J)$(*t_meets*(I, J) and *o_precedes*(I, J) $\Longleftrightarrow$ *meets*(I, J))

(7) $(\forall I, \forall J)$(*t_meets*(I, J) and *o_succeeds*(I, J) $\Longleftrightarrow$ *met_by*(I, J))

(8) $(\forall I, \forall J)$(*t_covers*(I, J) and *o_precedes*(I, J) $\Longleftrightarrow$ *starts*(I, J))

(9) $(\forall I, \forall J)$(*t_covers*(I, J) and *o_succeeds*(I, J) $\Longleftrightarrow$ *finishes*(I, J))

(10) $(\forall I, \forall J)$(*t_coveredBy*(I, J) and *o_precedes*(I, J) $\Longleftrightarrow$ *started_by*(I, J))

(11) $(\forall I, \forall J)$(*t_coveredBy*(I, J) and *o_succeeds*(I, J) $\Longleftrightarrow$ *finished_by*(I, J))

(12) $(\forall I, \forall J)$(*t_overlaps*(I, J) and *o_precedes*(I, J) $\Longleftrightarrow$ *overlaps*(I, J))

(13) $(\forall I, \forall J)$(*t_overlaps*(I, J) and *o_succeeds*(I, J) $\Longleftrightarrow$ *overlapped_by*(I, J))

Q.E.D.

3. Support of Arbitrary Temporal Relationships

3.1 Canonical Forms of Temporal Relationships

It is possible to define other temporal relationships in addition to topological, ordering, and Allen's temporal relationships. For example, we can define "beginning" temporal relationships as follows, where I and J are temporal intervals:

(1) $(\forall I, \forall J)$(*beginsEarlierThan*(I, J) $\Longleftrightarrow$ I.stp < J.stp)

(2) $(\forall I, \forall J)$(*beginsSameTime*(I, J) $\Longleftrightarrow$ I.stp = J.stp)

(3) $(\forall I, \forall J)$(*beginsLaterThan*(I, J) $\Longleftrightarrow$ I.stp > J.stp)

By definition, these three relationships are mutually exclusive and are closed for temporal intervals. Therefore we can define beginning temporal relationship system $\boldsymbol{R}_{beginning}$ as follows: (Of course, "ending" temporal relationships {*endsEarlierThan, endsLaterThan, endsSameTime*} can be defined in the same manner.)

Beginning temporal relationships are characterized by Allen's temporal relationships:

(1) $(\forall I, \forall J)$(*beginsEarlierThan*(I, J) $\Longleftrightarrow$ *before*(I, J) or meets(I, J) or *overlaps*(I, J) or *contains*(I, J) or *finished_by*(I, J))

(2) $(\forall I, \forall J)$(*beginsSameTime*(I, J) $\Longleftrightarrow$

equals(I, J) or *starts*(I, J) or *started_by*(I, J))

(3) $(\forall I, \forall J)(beginsLaterThan(I, J) \Longleftrightarrow$ *after*(I, J) or *met_by*(I, J) or *overlapped_by*(I, J) or *during*(I, J) or *finishes*(I, J))

Notice first that any Allen's temporal relationship appears only once in either one of three beginning temporal relationships, and second, only disjunctive operator "or" is used to join Allen's temporal relationships appeared in the right hand side of each beginning temporal relationship. That is, the set of three beginning temporal relationships induces a partition of the set of all Allen's thirteen relationships $\mathbf{R}_{\text{Allen}}$, where each class of this partition corresponds to a temporal meaning defined by the disjunction of Allen's temporal relationships belonging to that class.

In general let $\mathbf{R}_{\text{Allen}} = \{$*equals, before, after, during, contains, overlaps, overlapped_by, meets, met_by, starts, started_by, finishes, finished_by*$\}$ be the set of all Allen's thirteen temporal relationships. Suppose that π is a partition of $\mathbf{R}_{\text{Allen}}$ into i classes $(1 \le i \le 13)$; $c_1, c_2, \ldots, c_i$; where $c_i = \{r_{i_1}, r_{i_2}, \ldots, r_{i_{mi}}\}$. Then, the quotient set of $\mathbf{R}_{\text{Allen}}$ with respect to π is defined; $\mathbf{R}_{\text{Allen}}/\pi = \{c_1, c_2, \ldots, c_i\}$. Therefore, we can define a canonical surjection $\varphi_\pi : \mathbf{R}_{\text{Allen}} \rightarrow \mathbf{R}_{\text{Allen}}/\pi; r \dashrightarrow c$, where $r \in c$. Furthermore, let define a temporal relationship p_i of class c_i as; $p_i = r_{i_1}$ or r_{i_2} or ... or $r_{i_{mi}}$. Then, a temporal relationship system of $\boldsymbol{R}_{Allen}$ induced by π is defined as; $\boldsymbol{R}_\pi = \boldsymbol{R}_{Allen}/\pi = (\mathbf{R}_\pi, \mathbf{T}_\pi)$, where $\mathbf{R}_\pi = \{p_1, p_2, \ldots, p_i\}$ and $\mathbf{T}_\pi$ represents the transitivity table of this system which is calculated using Allen's transitivity table. $\boldsymbol{R}_\pi$ is well-defined because $\mathbf{R}_\pi$ is mutually exclusive and is closed for temporal intervals by definition. It is easy to show that all previously introduced temporal relationship systems are characterized by this approach:

[Characterization of $\boldsymbol{R}_{topology}$, $\boldsymbol{R}_{ordering}$, and $\boldsymbol{R}_{beginning}$]

(1) $\boldsymbol{R}_{topology}$ is induced by partition π_{topology} = $\{\{$*equals*$\}, \{$*before, after*$\}, \{$*during*$\}, \{$*contains*$\}, \{$*overlaps, overlapped_by*$\}, \{$*meets, met_by*$\}, \{$*starts, started_by*$\}, \{$*finishes, finished_by*$\}\}$.

(2) $\boldsymbol{R}_{ordering}$ is induced by partition π_{ordering} = $\{\{$*before, meets*$\}, \{$*after, met_by*$\}, \{$*equals, during, contains, overlaps, overlapped_by, starts, started_by, finishes, finished_by*$\}\}$.

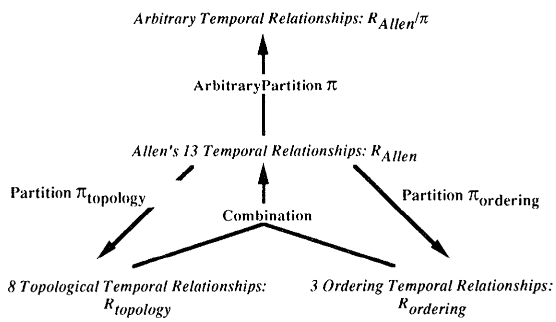

Fig. 1 Dependency structure among temporal relationships.

(3) $\boldsymbol{R}_{beginning}$ is induced by partition $\pi_{\text{beginning}} = \{\{$*before, meets, overlaps, contains, finished_by*$\}, \{$*equals , starts, started_by*$\}, \{$*after, met_by, overlapped_by, during, finishes*$\}\}$

Two particular temporal relationship systems are Allen's system itself $\boldsymbol{R}_{Allen}$, and a universal relationship system $\boldsymbol{R}_{universal}$ which are defined by partitions $\pi_{\text{Allen}} = \{\{$*equals*$\},\{$*before*$\},\{$*after*$\},\{$*during*$\},\{$*contains*$\}, \{$*overlaps*$\}, \{$*overlapped_by*$\}, \{$*meets*$\}, \{$*met_by*$\}, \{$*starts*$\}, \{$*started_by*$\}, \{$*finishes*$\}, \{$*finished_by*$\}\}$ and $\pi_{\text{universal}} = \{$*equals, before, after, during, contains, overlaps, overlapped_by, meets, met_by, starts, started_by, finishes, finished_by*$\}$, respectively. **Figure 1** depicts the dependency structure among temporal relationship definitions.

Formally any temporal relationship system can be defined according to an arbitrary partition of $\mathbf{R}_{\text{Allen}}$. Let $S_n(i)$ be the total number of partitions of a set of n distinct elements into i disjoint classes. The numbers $S_n(i)$ are often called the Stirling numbers of the second kind. Then $B(13) = S_{13}(1) + S_{13}(2) + \ldots + S_{13}(13)$ gives the maximum number of the different sets of temporal relationships which can be defined on it since the cardinality of $\mathbf{R}_{\text{Allen}}$ is 13. The numbers B(n) are often called Bell numbers, and B(13) is calculated at 27,644,437. However, it is not true that every arbitrarily defined temporal relationship has a practical meaning. For example, we can define a partition $\pi_{\text{nonsense}} = \{\{$*equals, before*$\}, \{$*finishes, overlapped_by*$\}, \{$*meets, contains*$\}, \{$*overlaps, met_by, starts, finished_by*$\}, \{$*started_by, during, after*$\}\}$, but it is hard to assign a practical meaning of this partitioning. That is, the total number of meaningful temporal relationships does not exceed

Table 2 Structure of combined transitivity table for Allen's temporal relationship system and "beginning" relationship system. **a**, **b**, **a**, **d**, **c**, **o**, **ob**, **m**, **mb**, **s**, **sb**, **f**, and **fb** represent Allen's temporal relationship *equals, before, after during, contains, overlaps, overlapped_by, meets, met_by, starts, started_by, finished,* and *finished_by*, respectively. **bET**, **bST**, and **bLT** represent *beginsEarlierThan, beginsSameTime,* and *beginsLaterThan*, respectively.

(I, K) / (I, J)	e	b	a	d	c	o	ob	m	mb	s	sb	f	fb	bET	bST	bLT
e																
b																
a																
d																
c																
o																
ob								T_{Allen}							$T_{Allen\text{->}beginning}$	
m																
mb																
s																
sb																
f																
fb																
bET																
bST								$T_{beginning\text{->}Allen}$							$T_{beginning}$	
bLT																

27,644,437.

3.2 Combined Reasoning for Temporal Queries

A single temporal relationship system is not capable enough of answering arbitrary temporal queries. For example, Allen's temporal relationship system is not able to answer a query like "which temporal intervals do precede temporal interval I?" because it doesn't support relationship *o_precedes*. Therefore, it is necessary to combine different types of relationship systems so that the combined system can answer a variety of queries.

For example, suppose that Allen's relationship system $\boldsymbol{R}_{Allen}$ and beginning temporal relationship system $\boldsymbol{R}_{beginning}$ are combined. Then, a combined transitivity table between two sets of relationships; $\mathbf{R}_{\mathrm{Allen}}$ and $\mathbf{R}_{\mathrm{beginning}}$ is defined in addition to two individual transitivity tables; $\mathbf{T}_{\mathrm{Allen}}$ and $\mathbf{T}_{\mathrm{beginning}}$. In this example the combined transitivity table of 16×16 is defined. **Table 2** depicts its global structure. Two essential subparts for combined reasoning are; $\mathbf{T}_{\mathrm{Allen}\rightarrow beginning}$ and $\mathbf{T}_{\mathrm{beginning}\rightarrow Allen}$. A couple of interesting components of $\mathrm{T}_{\mathrm{Allen}\rightarrow beginning}$ is shown below. The rest of the matrix components are computed similarly.

(a) $(\forall \mathrm{I}, \forall \mathrm{J}, \forall \mathrm{K})(\mathit{before}(\mathrm{I}, \mathrm{J})$ and $\mathit{beginsEarlierThan}(\mathrm{J}, \mathrm{K}) \Longrightarrow \mathit{before}(\mathrm{I}, \mathrm{K}) \Longrightarrow \mathit{beginsEarlierThan}(\mathrm{I}, \mathrm{K}))$

(b) $(\forall \mathrm{I}, \forall \mathrm{J}, \forall \mathrm{K})(\mathit{during}(\mathrm{I}, \mathrm{J})$ and $\mathit{beginsSameTime}(\mathrm{J}, \mathrm{K}) \Longrightarrow \mathit{during}(\mathrm{I}, \mathrm{K})$ or $\mathit{overlapped\_by}(\mathrm{I}, \mathrm{K})$ or $\mathit{after}(\mathrm{I}, \mathrm{K}) \Longrightarrow \mathit{beginsLaterThan}(\mathrm{I}, \mathrm{K}))$

Now, it is possible to answer a query like "retrieve a temporal interval I which meets J, and J begins earlier than K." A straightforward implementation of such query processing system is possible by using a logic programming language like Prolog.

4. Normal Form Issues of Allen's Interval-based Temporal Logic

4.1 Introduction of Null Temporal Intervals

Suppose that two temporal intervals I and J are related under Allen's temporal relationship *before*(I, J). Then there is a suspension time between them because I.etp < J.stp. To represent suspension, we have introduced "null temporal intervals"[17]. In this case, the null temporal interval N is defined as a time interval, provided that I.etp = N.stp and N.etp = J.stp. Therefore, a new composite temporal relationship *meets*(*meets*(I, N), J) is defined. We

Table 3 Equivalents of Allen's temporal relationships when null temporal intervals are used.

Allen's Seven Primitive Temporal Relationships	Equivalents
equals(I, J)	*equals*(I, J)
before(I, J)	(∃N)(*meets*(*meets*(I, N), J))
during(I, J)	(∃N1)(∃N2)(*equals*(*meets*(*meets*(N1, I), N2), J))
overlaps(I, J)	(∃N1)(∃N2))(*equals*(*meets*(I, N1), *meets*(N2, J)))
meets(I, J)	*meets*(I, J)
starts(I, J)	(∃N)(*equals*(*meets*(I, N), J))
finishes(I, J)	(∃N)(*equals*(*meets*(N, I), J))

say that relationship *before*(I, J) is equivalent to composite relationship *meets*(*meets*(I, N), J) because we can prove that $(\forall I, \forall J)(before(I,J) \Longleftrightarrow (\exists N)(meets(meets(I,N),J)))$. In general, we can show the next result.

[Theorem 3] Any one of Allen's thirteen temporal relationships has its equivalent obtained by using only the relationships *equals* and/or *meets* when null temporal intervals are used.

(Proof) The proof is trivial for *equals* and *meets*. Therefore, let us prove the theorem for relationships *before, after, during, contains, overlaps, overlapped_by, met_by, starts, started_by, finishes,* and *finished_by*. First, we examine *before* defined by $(\forall I, \forall J)(before(I,J) \Longleftrightarrow I.etp < J.stp)$. Now, let us introduce a null temporal interval N such that I.etp = N.stp and N.etp = J.stp. Construct a predicate $(\exists N)(meets(meets(I,N),J))$, then it is easy to see that $(\exists N)(meets(meets(I,N),J))$ holds if and only if *before*(I, J) holds for any I and J, i.e. they are equivalent. Proofs are similar for the other ten temporal relationships and are omitted here. Q.E.D.

Table 3 summarizes the equivalents proved in Theorem 3.

4.2 Normal Forms of Composite Temporal Multimedia Objects

Here we discuss an application of Theorem 3 to composite temporal multimedia object representation. Multimedia objects such as audio and video are represented by temporal intervals. Usually, multimedia objects are composed of several other component multimedia objects. For example, a composite video *before*(V1, V2) can be defined as a composition of two component videos V1 and V2, respectively, related under Allen's temporal relationship *before*. Notice that temporal relationships are considered as "composition operators" as well to combine component objects. Now, suppose that null temporal interval N represents the suspension time between V1 and V2. It is interpreted as a null video, which consists of a sequence of black frames – a frame totally filled with black. Therefore, no distinction can be seen when *before*(V1, V2) and *meets*(*meets*(V1, N), V2) are played back by a video player, i.e. two composite temporal videos are "equivalent." This is a practical meaning of null temporal intervals. The same argument holds for audio case.

In general, let **I** and **M** be a set of temporal intervals and a set of media, respectively. We associate a temporal interval with medium information defined by a function $\mu : \mathbf{I} \rightarrow \mathbf{M}$ in such a way that $\mu(I)$ = video if I is a (null) video interval. Suppose that *before*(I, J) holds. If $\mu(I) = \mu(J)$, then $(\forall I, \forall J)(before(I,J) \Longleftrightarrow (\exists N)(meets(meets(I,N),J)))$ holds. Otherwise, $(\forall I, \forall J)(before(I,J) \Longleftrightarrow (\exists N1, \exists N2)(equals(meets(I,N1), meets(N2,J))))$ holds. The reason why we distinguish these two cases is that if $\mu(I) = \mu(J)$ does not hold (denoted by $\mu(I) <> \mu(J)$), then the composite object *meets*(*meets*(I, N), J) cannot be played back correctly either by a video player or an audio player. To play it back correctly, *meets*(I, N1) and *meets*(N2, J) are fed to a video and an audio player, respectively, provided that their playbacks start at the same time, i.e. synchronously. **Figure 2** shows two different equivalents of *before*(I, J) corresponding to the difference of media.

Based on this equivalence result, we can show normal forms of seven primitive temporal multimedia objects, which are defined by *equals*(I, J), *before*(I, J), *during*(I, J), *overlaps*(I, J), *meets*(I, J), *starts*(I, J), and *finishes*(I, J) corresponding to Allen's seven temporal relationships, where I and J represent temporal multimedia intervals such as video and audio. Normal forms of other six primitive objects corresponding to the inverse relationships; *after, contains, overlapped_by, met_by, started_by,* and *finished_by* are defined similarly and are omitted here. **Figure 3** shows the normal forms of seven primitive composite

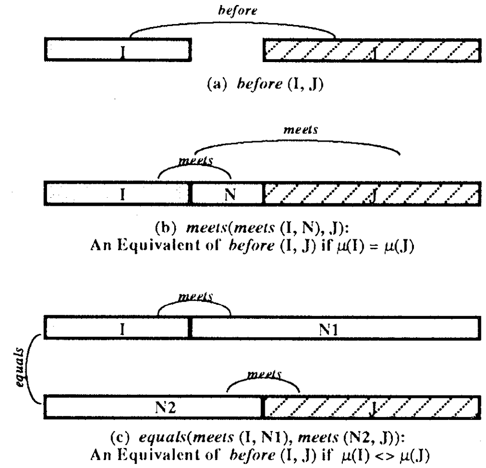

Fig. 2 Equivalent forms of *before*(I, J).

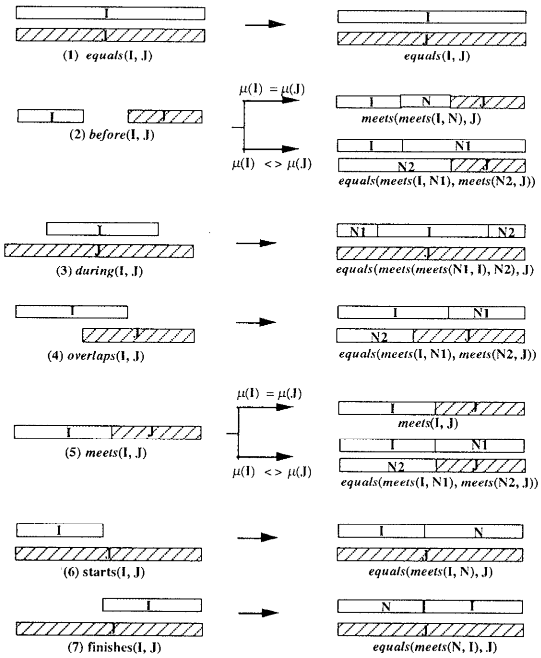

Fig. 3 Normal forms of seven primitive composite temporal multimedia objects.

temporal multimedia objects.

Normal forms of more complex composite temporal relationships can be calculated by applying the normal form translation rules depicted in Fig. 3 recursively. For example, the normal form of *meets*(*meets*(I, J), K) is *equals*(*meets*(*meets*(I, N1), K), *meets*(*meets* (N2, J), N3)), provided that $\mu(\mathrm{I}) = \mu(\mathrm{K}) <> \mu(\mathrm{J})$ for some N1, N2, and N3.

5. Conclusions

Allen's temporal logic was re-examined in this paper. We first investigated its topological nature, and showed that only three out of Allen's thirteen temporal relationships are topological invariants. The topological and the ordering temporal relationship were introduced. It is shown that Allen's temporal relationship system is induced by the combination of these two relationship systems. We have shown that any temporal relationship system such as beginning temporal relationship system is defined as a canonical surjection induced by a partition of the set of Allen's thirteen temporal relationships. It is shown that the total number of meaningful temporal relationships does not exceed 27,644,437. Combined reasoning for arbitrary temporal queries is also shown.

Normal forms of Allen's temporal relationships are investigated by introducing null temporal intervals. It is shown that any one of them has its equivalent which is defined by using only two Allen's temporal relationships, which are *equals* and *meets*. Based on this result, normal forms of composite temporal multimedia objects were shown so that they can be played back correctly.

Future works include an expansion of our work to spatio-temporal paradigm so that an entire framework for a multimedia data modeling and implementation can be achieved.

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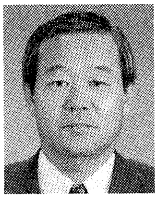

Yoshifumi Masunaga is a professor of University of Library and Information Science. He graduated in electrical and communication engineering from Post Graduate School of Tohoku University and received a Doctor of Engineering Degree in 1970. His research interest covers multimedia and object-oriented database system technology. He has been conducting a multimedia database project named OMEGA since 1984. He is a leader of a research group supported by the Ministry of Education, Science, Sports and Culture, Japan by the grant of Scientific Research on Priority Areas for "Research and Development of Advanced Database Systems for Integration of Media and User Environments" since 1996. He was a scientist of the Computer Science Department of the International Institute of Applied Systems Analysis (IIASA), Austria from 1975 to 1977. He was a visiting scientist of IBM San Jose Research Laboratory, USA from 1982 to 1983 where he belonged to System R* Project. He was a visiting scientist of University of Texas at Austin, USA and Technical University of Munich, Germany both in 1988. He was the chairperson of the Special Interest Group on Database Systems of the Information Processing Society of Japan (IPSJ) from 1991 to 1995. He was the chairperson of the Computer Science Area of IPSJ from 1995 to 1997. He has been the chairperson of the ACM SIGMOD Kyoto/Japan Chapter since 1995. He has been conducting many of international and domestic conferences including DASFAA international conferences, WWCA international conferences, and the 54th National Convention of IPSJ where the ACM track was created and ACM President Chuck House gave a talk. He is the authors of the books entitled "Foundations of Relational Databases" (OHM Publishing) and "Introduction to Relational Databases" (Science Publishing) both are written in Japanese, and the chief translator of the English books entitled "Introduction to Object-Oriented Databases" by Won Kim (MIT Press) and "Object-Oriented Technology: A Manager's Guide" by David Taylor (Addison-Wesley). He is the member of IPSJ, IEICE of Japan, ACM, and IEEE Computer Society.