Collaboration is Fun : How We Came to Analyze Snake Cube Puzzles

Zachary Abel　Erik D. Demaine　Martin L. Demaine
Sarah Eisenstat　Jayson Lynch　Tao B. Schardl
Massachusetts Institute of Technology

〔受賞論文〕
Finding a Hamiltonian Path in a Cube with Specified Turns is Hard
Zachary Abel, Erik D. Demaine, Martin L. Demaine, Sarah Eisenstat, Jayson Lynch and Tao B. Schardl
（Massachusetts Institute of Technology）
Journal of Information Processing Vol.21, No.3, pp.368-377 (2013)

This paper grew out of an MIT class (6.849) about Geometric Folding Algorithms taught by E. Demaine in 2010 and taken by several of the authors (among many other students). During the class and for a year afterward, we met roughly weekly to tackle unsolved problems in the field of geometric folding. This open problem session was a highly collaborative and fun experience for everyone, and is a great way to do research.

Often the problems we considered were inspired by geometric puzzles. Both Demaines had played with the Snake Cube puzzles for years, and often wondered about the computational complexity of solving the puzzle. The puzzle is made up of several unit cubes connected by a single elastic routed through the face centers of the cubes. The goal is to fold this chain of unit cubes into a larger cube. Given the connections between Snake Cubes and both fixed-angle linkages (often used to model protein folding) and Hamiltonian paths (a classic problem in computer science), the Demaines brought the puzzle to the open problem session. And so began this paper.

The natural complexity class for the Snake Cube puzzle is NP, because it is easy to describe and check a solution to the puzzle : there are just four possible rotation angles at each joint, and given those angles, it is easy to compute the resulting shape and see whether it is correct. In fact, we show that the puzzle is NP-complete, meaning that it is among the very hardest problems/puzzles that have checkable solutions, and thus likely cannot be solved by an efficient computer algorithm. This result gives us a formal mathematical sense in which the Snake Cube puzzles are challenging, which helps explain why they are so fun to solve.

We proved that the puzzle is hard using a sequence of reductions. First we studied a much more general problem (which is thus easier to show hard): given a chain of cubes and a target shape, fold the chain to fit inside the shape. We showed that this “packing” problem is as hard as a known NP-complete problem called 3-partition. Second we showed that all of the blank space in the packing could be filled up by extending the chain of cubes, which turns the packing problem into a shape-folding problem: can the given chain of cubes be folded into the target shape? Finally we showed how to use part of the chain to force the folding of a large obstacle, whose complement in a large cube is exactly the target shape for folding the rest of the chain. Together this proves that the original Snake Cube puzzle is NP-complete.

Figure : A commercially available Snake Cube puzzle consisting of 27 unit cubes.

（2014 年 5 月 31 日受付）

Zachary Abel is a Ph.D. candidate in the Department of Applied Mathematics at the Massachusetts Institute of Technology (MIT) working with Professor Erik D. Demaine in the Computer Science and Artificial Intelligence Lab (CSAIL). His primary research is in computational and discrete geometry, including such topics as computational origami and geometric rigidity theory. Mr. Abel is currently funded under the National Science Foundation (NSF) Graduate Research Fellowship Program, and he received his Bachelors of Arts in Mathematics and Computer Science from Harvard in 2010.

Erik D. Demaine is a professor in computer science at the Massachusetts Institute of Technology. Before that, he received a B.Sc. degree from Dalhousie University in 1995, and M.Math. and Ph.D. degrees from University of Waterloo in 1996 and 2001, respectively. His research interests range throughout algorithms, from data structures for improving web searches to the geometry of understanding how proteins fold to the computational difficulty of playing games.

Martin L. Demaine is an artist and mathematician. He started the first private hot glass studio in Canada and has been called the father of Canadian glass. Since 2005, he has been the Angelika and Barton Weller Artist-in-Residence at the Massachusetts Institute of Technology. Both Martin and Erik work together in paper, glass, and mathematics.

Sarah Eisenstat is a Ph. D. student in the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology, working with Professor Erik D. Demaine. Her current research interests include computational geometry and algorithms. She received her Sc.B. and Sc.M. degrees from Brown University in 2008 and 2009, respectively.

Jayson Lynch received his B.Sc. in Physics and Computer Science from MIT in 2013 and remains there as a Ph.D. candidate in the Department of Electrical Engineering and Computer Science working with Professor Erik D. Demaine. His research interests include algorithms, computational geometry, and the interplay between physics and computation.

Tao B. Schardl is a Ph.D. candidate in the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology. He works in the Supertech group with Professor Charles Leiserson. His primary research is on developing parallel algorithms and parallel data structures. He received his S.B. and M.Eng. degrees from MIT in 2009 and 2010, respectively.