A Study on Non-Correspondence in Spread between Objective Space and Design Variable Space in Pareto Solutions

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Abstract— *Recently, a lot of studies on Multi-Objective Genetic Algorithm (MOGA), in which Genetic Algorithm is applied to Multi-objective Optimization Problems (MOPs), have been reported actively. MOGA has been also applied to engineering design fields, then it is important not only to obtain Pareto solutions having high performance but also to analyze the obtained Pareto solutions and extract the knowledge in the designing problem. In order to analyze Pareto solutions obtained by MOGA, it is required to consider both the objective space and the design variable space. In this paper, we define "Non-Correspondence in Spread" between the objective space and the design variable space. We also try to extract Non-Correspondence area in Spread with the index defined in this paper. This paper applies the proposed method to the trajectory designing optimization problem and extracts Non-Correspondence area in Spread in the acquired Pareto solutions.*

Keywords: Non-Correspondence, Objective Space, Design Variable Space, Distributed Area, Multi-objective Optimization Problem

1. Introduction

Genetic Algorithm (GA) is expected to be effective for solving Multi-objective Optimization Problems (MOPs), which maximizes or minimizes multiple objective functions at the same time. Recently, Multi-Objective Genetic Algorithm (MOGA), applying GA to MOPs, are getting much attention and a lot of studies have been reported[1]. Generally, it is difficult to obtain the optimized solution satisfying all objective functions because of their trade-offs. Then, it is necessary to obtain Pareto solutions which are not inferior to other solutions in at least one objective function.

In recent years, it is reported that MOGA is applied to engineering design problems in the real-world due to the improvement of computing performance[2][3][4]. In the engineering design problems, it is required not only to obtain high performance Pareto solutions using MOGA but also to analyze and extract design knowledge in the problem. And in order to analyze Pareto solutions obtained by MOGA, it is required to consider both the objective space and the design variable space.

Obayashi obtained Pareto solutions for aircraft configuration problem by MOGA and tried to analyze the obtained Pareto solutions through the visualization of the relationship between fitness values and design variables using Self Organizing Map (SOM)[2]. Kudo *et al*. proposed a visualization method that visualized the geometric distance between data in the design variable space based on their relationship in the objective space and analyzed the relationship between the fitness values and the design variables in the conceptual design optimization problem of hybrid rocket engine[5].

In this paper, we analyze obtained Pareto solutions considering the objective space and the design variable space, and we especially focus on "Non-Correspondence" between two spaces. In this paper, we define "Non-Correspondence in Spread" and propose the quantitative index to extract Non-Correspondence area in Spread. Non-Correspondence area in Spread is the area where solutions are distributed densely in the objective space but are distributed widely in the design variables space, and vice versa. Moreover, this paper extends the index of non-correspondence to more practical index, which allows a designer to select the contributory design variables and fitness functions and to define the distance function.

This paper applies the proposed method to the trajectory designing optimization problem known as DESTINY (Demonstration and Experiment of Space Technology for INterplanetary voYage)[7] provided by Japan Aerospace Exploration Agency (JAXA). We apply NSGA-II (Non-dominated Sorting Genetic Algorithm-II)[8] to this problem and analyze the extracted Non-Correspondence area in Spread in the obtained Pareto solutions.

2. Non-Correspondence in Spread

In this paper, we focus on Non-Correspondence in Spread. The area with Non-Correspondence in Spread, called Non-Correspondence area in Spread, is defined as the area where solutions are distributed densely in the objective space but are distributed widely in the design variables space, and vice versa. (Hereinafter we call simply "Non-Correspondence area"). Figure 1 shows an example of Non-Correspondence area. In Fig. 1, data 5-6-7-8 are distributed widely in the design variable space compared to the distribution of the

objective space. It is important for designer to know this area in Pareto solutions because designer can select design variables from many design patterns in consideration of the cost of design or difficulty level of design.
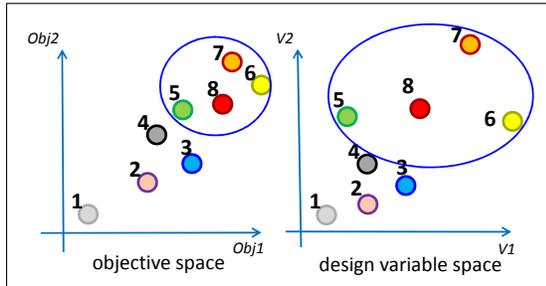
Fig. 1: Non-Correspondence are in Spread

Here, we define the quantitative index for Non-Correspondence in Spread to extract the Non-Correspondence area. The index is calculated in the following procedure.

1) Define the neighborhood radius ϵ (eq. (1)) in the objective space or the design variable space.
2) Extract the individuals as target individuals within radius ϵ from individual i.
3) Calculate the center of gravity of the target individuals.
4) Calculate the index for Non-Correspondence in Spread v_i according to eq. (2).

By the above procedure, the index v_i is calculated for each individual. The neighborhood radius ϵ is defined by eq. (1). In eq. (1), η denotes the parameter that defines the neighborhood radius, f_{lmax}, f_{lmin} mean the maximum and the minimum fitness values in the Pareto solutions for objective function l, and M_f is the number of objective functions. x_{lmax}, x_{lmin} mean the maximum range and the minimum range of design variables l, and M_d is the number of design variables. If the neighborhood is defined in the objective space, the upper equation in eq. (2) is employed and otherwise the lower equation is employed to calculate the value of index v_i. In eq. (2), d_{dik} is the normalized Euclidean Distance between target individual k and the center of gravity in the design variable space, d_{fik} is that in the objective space, N is the number of the target individuals and v_i is the index for individual i. Individuals with large indexes are distributed densely in the objective space / design variable space and distributed widely in the design variable space / objective space.

$$\epsilon = \begin{cases} \dfrac{\sqrt{\sum_{l=1}^{M_f}(f_{lmax}-f_{lmin})^2}}{\eta} \\ \text{(Neighborhood was defined in the objective space.)} \\ \dfrac{\sqrt{\sum_{l=1}^{M_d}(x_{lmax}-x_{lmin})^2}}{\eta} \\ \text{(Neighborhood was defined in the design variable space.)} \end{cases} \tag{1}$$

$$v_i = \begin{cases} \dfrac{1}{N}\sum_{k=1}^{N}(d_{dik})^2 \\ \text{(Neighborhood was defined in the objective space.)} \\ \dfrac{1}{N}\sum_{k=1}^{N}(d_{fik})^2 \\ \text{(Neighborhood was defined in the design variable space.)} \end{cases} \tag{2}$$

3. Experiment

In this paper, we applied the above calculation to the trajectory designing optimization problem "DESTINY" provided by JAXA and analyzed the obtained Pareto solutions.

3.1 Trajectory Designing Optimization Problem

The aim of this problem is to reach the moon as early as possible with less fuel and to reduce the degradation of the solar array panel of the spacecraft due to the damage by the radiation of the Van Allen belt. As shown in Fig. 2, the spacecraft is launched by Epsilon Rocket and put elliptical orbits around the earth. Once being put in orbit, the spacecraft is released and accelerates with Ion Engine until it reaches the moon. The spacecraft firstly aims to gain the altitude of perigee and switches to gain the altitude of apogee on the way, then it gradually moves closer to the moon.

This paper tries to optimize of trajectory designing of the spacecraft until it reaches the moon ((1),(2) in Fig. 2). The objective functions, the design variables, and the range of each design variable in this problem are shown in TABLE 1, TABLE 2, and TABLE 3, respectively. $V6$ is used in the case of optimization for 6 objective functions. As shown in TABLE 1, this problem can be expanded to six objective optimization problem. This paper deals with 5 objective functions $Obj1$, $Obj2$, $Obj3$, $Obj4$, $Obj5$ in TABLE 1.

NSGA-II was applied to the problem described above and 2000 Pareto solutions were obtained. We employed SBX[9] for the crossover and Polynomial Mutation[10]. Crossover rate was 1.0, mutation rate was 0.2, population size was 715, and generation was 100.

3.2 Extraction of Non-Correspondence Area in Spread

The result of the indexes for Non-Correspondence in Spread calculated by eq. (2) for obtained 2000 Pareto Solutions, in which the neighborhood was defined in the
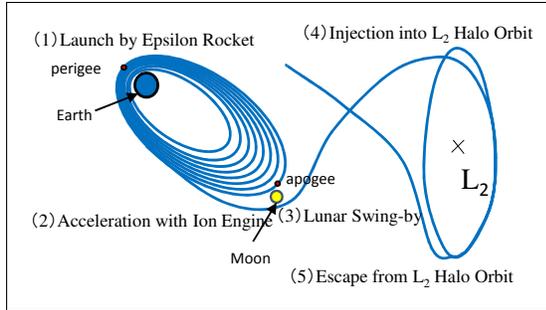
Fig. 2: Consept of DESTINY

Table 1: Objective Functions

$Obj1$	time to reach altitude of 20000km	Min
$Obj2$	IES (Ion Engine System) operation time	Min
$Obj3$	the time to reach the Moon	Min
$Obj4$	the maximum eclipse time	Min
$Obj5$	the time to reach an altitude of 5000km	Min
$Obj6$	Initial mass of the spacecraft	Max

objective space, are shown in Fig. 3. Neighborhood radius ϵ was set as $\eta = 8$ in eq. (1). The parameter of neighborhood radius ϵ was not sensitive and the results were not much changed by the difference of ϵ in the experiments of this paper. The individuals in Fig. 3 are sorted in descending order of the index v_i. The vertical axis shows the value of the index v_i and the horizontal axis shows the individual label.

We focused on the top 50 individuals with large indexes. Figure 4 shows the result of visualization of the distribution in which these 50 individuals are colored by red on the result of the objective space and the design variable space by Multi-Dimensional Scaling (MDS)[11]. As shown in Fig. 4, the individuals with red color are distributed widely in the design variable space compared to the distribution in the objective space. We extracted 2 individuals in these 50 individuals and the fitness values and design variables of them are shown in TABLE 4.

In TABLE 4, each fitness value in the second and the third rows is normalized by the maximum and the minimum fitness values of the obtained Pareto solutions into the range of [0,1], and each design variable is normalized by the feasible ranges shown in TABLE 3 into [0,1]. In TABLE 4, though A and B have similar fitness values each other, the design variables are widely different. For example, the launching dates are March and December, the launching times are 1 in the midnight and 8 in the morning, and $V3$ and $V5$ are also different. In this area, there were some individuals that design variables are widely different with similar fitness values.

TABLE 5 shows the extracted 2 individuals C and D in the same way with TABLE 4 by the result of the indexes in eq. (2), in which the neighborhood was defined in the design

Table 2: Design variables

$V1$	: Launching date
$V2$	: Launching time
$V3$	: Switching apogee-perigee date
$V4$	: Range of IES operation time in perigee rise phase
$V5$	: Range of IES operation time in apogee rise phase
$V6$	: Initial mass of spacecraft

Table 3: Ranges of design variables

$V1$	2017/1/1〜2018/1/1
$V2$	00:00:00〜24:00:00
$V3$	90〜365[days]
$V4$	0〜180[degrees]
$V5$	0〜180[degrees]
$V6$	350〜450[kg]
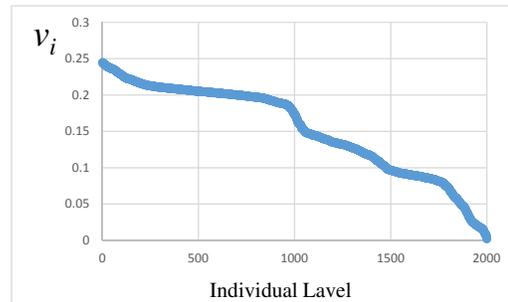
Fig. 3: Value of Index v_i in eq. (2) for each Individual (Neighborhood : Objective Space)

Table 4: fitness values and design variables of selected individuals (A, B)

	Normalized Value		Actual Value	
	A	B	A	B
$Obj1$	0.006	0.011	1434.70	1437.75
$Obj2$	0.846	0.910	8545.60	8713.77
$Obj3$	0.035	0.0005	401.08	395.65
$Obj4$	0.097	0.167	1.524	2.009
$Obj5$	0.018	0.085	217.71	221.07
$V1$	0.201	0.916	2017/3/15	2017/12/1
$V2$	0.051	0.336	01:13:47	08:4:14
$V3$	0.977	0.313	358	175
$V4$	0.999	1.000	179.94	180.00
$V5$	0.818	1.000	147.99	180.00

variable space while neighborhood radius ϵ was set as $\eta = 8$ in eq. (1). In TABLE 5, though C and D have similar design variables each other, the fitness values are widely different. In this area, there were some individuals that fitness values were very sensitive to the change of design variables. Thus it is required for the designer to choose or design very carefully a Pareto solution in this area.

(a) objective space

(b) design variable space
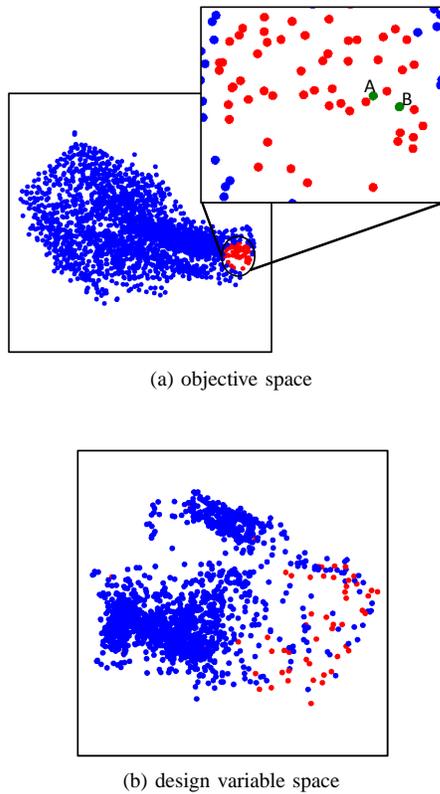
Fig. 4: Distribution of Pareto Solutions for Non-Correspondence Area (Neighborhood : Objective Space)

Table 5: fitness values and design variables of selected individuals (C, D)

	Normalized Value		Actual Value	
	C	D	C	D
$Obj1$	0.660	0.857	1801.06	1911.56
$Obj2$	0.226	0.061	6891.16	6452.99
$Obj3$	0.660	0.890	497.49	533.10
$Obj4$	0.156	0.694	1.938	5.689
$Obj5$	0.290	0.385	231.23	236.01
$V1$	0.750	0.750	2017/10/1	2017/10/1
$V2$	0.382	0.385	09:10:59	09:14:16
$V3$	0.038	0.038	100	100
$V4$	0.999	0.985	179.95	177.37
$V5$	0.864	0.841	155.53	151.43

4. Conclusion

In this paper, we defined Non-Correspondence in Spread between the objective space and the design variable space. We proposed the quantitative index to extract Non-Correspondence area in Spread. Moreover, this paper extended the index of non-correspondence to more practical index, which allowed a designer to select the contributory design variables or fitness functions and to define the dis-

tance function as the desirable difference. This paper applied the proposed method to the trajectory designing optimization problem known as DESTINY provided by JAXA and analyzed the extracted Non-Correspondence area in Spread in the obtained Pareto solutions. For the future work, we will apply to other problems with more objective functions and feedback the defined index and the extracted knowledge into the search and study Non-Correspondence in Linear Relationship.

5. ACKNOWLEDGMENT

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