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Regular Paper

The Reconsideration of the Altruistic Decision Based on the Notion of the Bounded Rationality

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Abstract: This study proposes that there are two main problems which several works challenging the issue of cooperation have not assumed. Firstly, those works basically employ the best decision that every player knows all information regarding the payoff matrix, and selects the strategy of the highest payoff. Secondly, as Ohdaira and Terano also insist, when we represent specific tendering based on the model of game theory, we confront the restriction that a player can submit only a move in a match. Considering those issues, this paper enhances Ohdaira's previous discussion of the altruistic decision by newly introducing the notion of the bounded rationality which is essential for recognizing the decision with some compromise in limited information. Utilizing the model of match between two groups with the evolutionary process, this study shows that each group establishes cooperation of a high level in comparison with the previous study employing the second-best decision. In addition, showing the detailed sensitivity analysis regarding the probability of the rational decision and the probability of mutation in the evolutionary process, this paper also reveals that the small probabilistic rational decision (a little selection of the strategy of the first grade) has an effect on the rapid collapse of cooperation, while the growth of defection does not keep pace with the rate of that collapse. Moreover, this study exhibits that the change of the probability of mutation in the evolutionary process has a moderate effect on the speed of the collapse of cooperation.

Keywords: 2130601 Agent designs, architectures, and theories, 2130604 Agent-based simulation, 2130606 Agent-based system development, 2130704 Multi-agent games, 2130705 Imperfect information games

1. Introduction

Cooperation in the prisoner's dilemma game has been the most challenging theme for scientists dealing with social conflicts of interests. When the players engaged in the interactions of the prisoner's dilemma game are rational, they drop into the state of Nash equilibrium where each player cannot increase his/her payoff unless he/she changes his/her strategy. Therefore, an additional rule is necessary to escape from the state and to establish mutual cooperation. Nowak [1] presents five rules for the evolution of cooperation as follows; kin-selection [2], direct reciprocity [3], indirect reciprocity [4], [5], [6], [7], [8], network reciprocity [9], [10], [11], [12], [13], [14], [15], [16] and group (multilevel) selection [17]. Kin-selection indicates that natural selection can favor cooperation if the donor and the recipient of an altruistic move are genetic relatives. Direct reciprocity proposes that if the same two individuals repeatedly encounter, they might cooperate with each other. In indirect reciprocity, an individual helping someone establishes a good reputation, and he will be rewarded by others. Network reciprocity is based on the assumption that real populations are not well mixed, and implies that some individuals interact more often than others. Group selection has a mechanism of selection of two stages. Selection on the lower level (within groups) favors defectors, whereas selection on the higher level (between groups) favors cooperators.

For each rule, Nowak also provides simple rules for deciding whether natural selection leads to cooperation. Even if utilizing simple Markovian or one step conditional strategy (like Tit-For-Tat or Pavlov), a system with network reciprocity can escape from Nash equilibrium [16]. Of course, there are also some other rules; i.e., tag for the distinction [18], [19], [20], or costly punishment [21], [22]. The following is a brief explanation regarding those rules. Tag is the framework to distinguish players by each identifier. Players can check the identifiers of others, and know whether opponents are cooperative or not. Costly punishment means that players pay a certain cost to punish free riders, and hence it enables players to build mutual trust. Wang et al. [23], [24] have recently found that sparsity may become a rule resolving social dilemmas, especially when the density of population is close to the percolation threshold of the underlying network reciprocity.

In addition to the above arguments, many papers have revealed that coevolution between strategy and network reciprocity favors cooperation. The area of research regarding this topic expresses enormous progress in recent years [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37]. The fundamental papers on this subject [25], [26], [27], [28] consider that either a random or an intentional rewiring process contributes to maintaining cooperation. Pacheco et al. [29] discuss the case where individuals are different in their capability of searching new interactions. The review [30] surveys recent studies regarding coevolutionary

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games, and shows that coevolution has effects on the interaction network, the capability of reproducing players, their reputation, mobility or age.

We can also list some latest papers regarding this topic as follows [31], [32], [33], [34], [35], [36], [37]. Poncela et al. [31] show that a coevolutionary preferential attachment (where a new player can make connections to either the randomly selected player or the player succeeded in the past) and growth scheme generate complex networks where cooperation survives. Szolnoki and Perc show that simple coevolutionary rules may lead to highly heterogeneous distributions of instructive behavior which contributes to cooperation [32]. Szolnoki et al. [33] present that there is an optimal maximal degree regarding the promotion of cooperation. Szolnoki and Perc also consider the dynamics of deletion and addition of links in interaction networks [34], and exhibit that the coevolutionary rule between the adoption of a new strategy and either the deletion or addition of links has a strong effect on the promotion of cooperation [35]. The studies [36], [37] introduce the diversity to adverse interactions of individuals, and suggest that swift reactions to adverse ties evolve in the situation of the prisoner's dilemma.

At this point, it should be noticed that interactions can be classified into two types regarding which rule described above effectively facilitates cooperation. That is, one is pairwise interactions in which two players engage (like the prisoner's dilemma game), and the other is referred to as group interactions that several players can attend at the same time (like the public goods game). For example, Nowak's five rules, especially direct reciprocity, indirect reciprocity and network reciprocity are effective mainly in pairwise interactions, while costly punishment is effective in group interactions and loses its effect of facilitating cooperation in pairwise interactions. Nowak describes that costly punishment promotes cooperation, however, it is not the mechanism for the evolution of cooperation because punishers cannot invade the group of defectors. However, Fowler [21] exhibits a possible solution for the emergence of punishers. He focuses on the fact that participation is not always compulsory in many interactions of public goods. Similarly, Rankin et al. [22] proposes that recent studies regarding the evolutionary significance of costly punishment [38], [39] are problematic, and that punishment still has the possibility of becoming the rule for the evolution of cooperation. Recent researches [40], [41] show that relaxing the fixed fine and cost of punishment can explain both the spontaneous emergence of punishment and its ability to prevent defectors.

Considering those discussions, this paper proposes that there are two major unuttered problems in previous studies. The first issue is that almost all the works dealing with cooperation employ the best decision that each player knows all information regarding payoff matrix, and selects the strategy of the highest payoff. In practice, however, the decision is not perfectly rational in every case, and includes some compromise. When making a decision, we also confront the situation of limited information which can be defined as a bounded rational condition. For example, some compromise on income and holding closed relationships in limited information cause fixed collusive tendering (cooperation of high level) in public works projects. Therefore, the inspection of the property of cooperation based on such a decision with compromise is greatly significant.

As a candidate of such a decision with compromise, Ohdaira and Terano have introduced the second-best decision [42], [43] or the corrected decision [44], and investigated the property of cooperation based on each decision. The second-best decision means that every group only selects the strategy of the second-grade as his/her representative strategy in his/her decision. The corrected decision is the mechanism that every group corrects the probability of selection for his/her representative strategy except the first grade when his/her strategy of the first grade has an extremely high payoff in comparison with the strategy of the second grade and lower. However, the author should point out here that the studies [42], [43], [44] do not mention the notion of the bounded rationality described above although these studies are actually based on it.

The second issue is the restriction that a player can submit its strategy of only one move in a match. Ohdaira and Terano also refer to the problem, and they propose that it is necessary to extend the strategy when modeling the specific tendering of pricing multiple bidding subjects (see Fig. 14 in Ref. [42] and Fig. 10 in Ref. [43]). They propose that the prisoner's dilemma game with sequential strategy is the best solution for the problem, and that this game can effectively describe the property of the evolution of strategy with a small number of players [42].

On the basis of the above notion, this paper reconsiders the altruistic decision in Ohdaira's previous works [45], [46] as a new type of decision with compromise in a bounded rational condition, and also employs the framework of the prisoner's dilemma game with sequential strategy. The altruistic decision indicates that each player selects his/her one representative strategy in the same probability from his/her candidates of that strategy except the best one. Those candidates surely include the second-best strategy. Ohdaira's previous works [45], [46] show that the altruistic decision also facilitates cooperation in the prisoner's dilemma game with sequential strategy as the second-best decision does. However, the previous works [45], [46] do not mention the notion of the bounded rationality and do not discuss the distinction between the altruistic decision and the second-best decision. Therefore, the detail of the altruistic decision has not been resolved yet. As noted in the following section of the model, in order to recognize the altruistic decision (and also the second-best decision and the corrected decision) properly, we should introduce the notion of the bounded rationality because in the model, every group has a limited number of sequences of strategy and does not know the payoff matrix of the game.

This paper enhances the previous works [45], [46] with the evolutionary process by introducing the notion of the bounded rationality and showing the establishment of cooperation of a high level in the prisoner's dilemma game with sequential strategy in comparison with the case employing the second-best decision. In addition, the author shows the detailed sensitivity analysis regarding the probability of the rational decision and the probability of mutation in the evolutionary process described later. It shows that the change of mutual cooperation (cooperation between two groups) induced by the small probabilistic rational decision (a little selection of the strategy of the first grade) is different from the change of mutual defection (defection between them). This further experiment also exhibits that the value of the probability of mutation in the evolutionary process moderately influences those changes. In the following, first the author constructs the model of match between two groups utilizing the prisoner's dilemma game with sequential strategy, and then shows that the altruistic decision introducing the notion of the bounded rationality universally has an effect on the promotion of mutual cooperation and the suppression of mutual defection.

2. The Model

2.1 Prisoner's Dilemma Game

The prisoner's dilemma game (PDG) and the iterated prisoner's dilemma game (IPDG) are very popular, and often utilized for modeling social conflicts of interests. The basic framework of PDG is as follows. There are two players and they choose their strategy (Defection or Cooperation). They are mutually separated, so that each of them cannot know the strategy of the opponent. After choosing their strategy, they get their payoff according to the payoff matrix (see **Table 1**). The meaning of each capital letter is as follows; **T**: Temptation to defect, **R**: Reward for mutual cooperation, **P**: Punishment, **S**: Sucker's payoff. PDG requires the condition $\mathbf{T} > \mathbf{R} > \mathbf{P} > \mathbf{S}$, and the additional condition $2\mathbf{R} > \mathbf{T} + \mathbf{S}$ should be satisfied regarding IPDG. As noted in the introduction, mutual cooperation does not develop without an additional rule in PDG. When all players are completely rational, they reach the state of Nash equilibrium. That state is not desirable for players because they can get a higher payoff by mutual cooperation.

In IPDG, there is no undefeated strategy, and then the effect of strategy depends on the strategy of opponent. The pioneering work of IPDG is Axelrod's tournament. He reveals the essential rules for the emergence of cooperation as follows [47], [48]. The first is memory, that is, players remember the previous strategy of their opponents. The second is reciprocity. Players must give profit to each other; i.e. they must take the same action as the past move of their opponents. The popular Tit-for-Tat (TFT) strategy follows that rule. Players with TFT cooperate at the first iteration, and then cooperate when their opponents cooperated, otherwise they defect. Because players do not know how long the game lasts, they tend to make allowances for their opponents (the effect of the 'shadow of the future'). The study regarding IPDG generally deals with a reaction pattern (ex. TFT) as strategy, and discusses what strategy is successful. Many players will utilize the successful strategy, and then it becomes the majority within all reaction patterns.

2.2 Introduction of the PDG with Sequential Strategy and Reconsideration of the Altruistic Decision

As noted in the introduction, the studies based on indirect reciprocity or other rules employ the best decision. However, in the real world, there are some situations like fixed collusive tendering which does not seem to follow the best decision. Based on that notion, this study introduces the PDG with sequential strategy derived from the specific tendering [42], [43] and reconsiders

Table 1 Payoff matrix of the prisoner's dilemma game.

Player B / Player A	Defection	Cooperation
Defection	(**P**: 1, **P**: 1)	(**T**: 5, **S**: 0)
Cooperation	(**S**: 0, **T**: 5)	(**R**: 3, **R**: 3)

This payoff matrix satisfies both conditions of T>R>P>S and 2R>T+S.

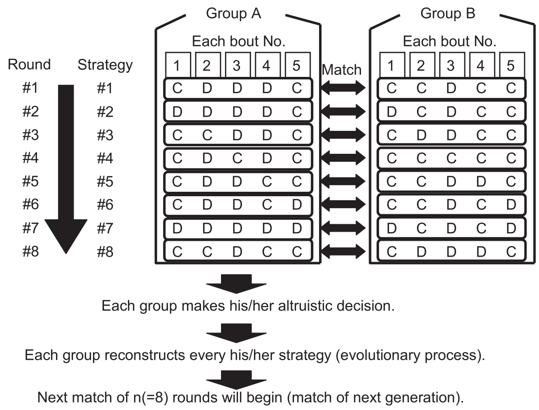

Fig. 1 Outline of the prisoner's dilemma game (PDG) with sequential strategy. There are two groups respectively having n (= 8, in the sample) sequences of strategy. Every group plays the PDG with sequential strategy against the opponent group in n rounds, utilizing each sequence of strategy of the same ID. At the end of the match of all n rounds, each group determines his/her one representative strategy in the decision. Based on his/her representative strategy, he/she generates his/her n sequences of strategy of the next generation, and plays the match again.

the altruistic decision in Ohdaira's previous works [45], [46]. The PDG with sequential strategy is the extension from standard PDG to parallel successive game. Sequential strategy regulates every action for finite bouts of PDG. In the PDG with sequential strategy, evolution of strategy does not start from a finite set of reaction patterns as in the IPDG studies. The initial state is a completely random sequence, and also has much variety. The PDG with sequential strategy does not explicitly have the information regarding the past move of the opponent which is different from IPDG. However, the information is implicitly informed as to the resulting payoff of each round. The basic framework of the PDG with sequential strategy is illustrated in **Fig. 1** and listed below.

- There are two different groups respectively having n sequences of strategy with ID u ($1 \leq u \leq n$). Every sequence of strategy describes the behavior of a group in each bout of PDG. All sequences of strategy are initialized as random sequences.
- Every sequence of strategy of the group i is an array whose length equals L shown as the following Eq. (1). Each character represents the strategy of one bout (D: Defection, C: Cooperation). Where D or C means the component vector, it is expressed as (0 1) or (1 0).

$$S_i(u) = \left\{ \alpha_i^L(u) \,\middle|\, \alpha_i(u) \in \{D, C\},\ 1 \leq u \leq n \right\} \quad (1)$$

The two groups play the PDG with sequential strategy utilizing their sequences of strategy of the identical ID. However, sequences of strategy of the two groups with the same ID are not similar because they independently evolve.

Table 1 determines the payoff of each bout, and follows the payoff matrix of standard PDG (also the same as Axelrod's IPDG tournament). The number of total sequences of strategy of each group is n, and the length of sequences of strategy is L. The payoff $p_i^g(u)$ of the u-th sequence of strategy of the group i at the generation g is numerically expressed in the following Eq. (2). A means the payoff matrix of Table 1, and where $\alpha_i^k(u)$ and $\alpha_j^k(u)$ each designate the element of the sequence of strategy in (1).

$$p_i^g(u) = \sum_k \alpha_i^k(u) A \alpha_j^k(u)^T \quad (1 \le k \le L,\ 1 \le u \le n),$$
$$A = \begin{pmatrix} 3 & 0 \\ 5 & 1 \end{pmatrix} \tag{2}$$

As also noted in the introduction, Ohdaira's previous works [42], [43], [44], [45], [46] including the altruistic decision [45], [46] do not mention the notion of the bounded rationality. Here, the author explains that the above design basically introduced by Ohdaira's previous works [42], [43], [44], [45], [46] is based on the bounded rational condition because this study intends to uncover the property of cooperation based on the altruistic decision in that condition. When we express the set of sequences of strategy of group A as S_1, an element of S_1 as $\{s_1^i\}$ and the payoff function as $f_1(s_1^i, s_2^j)$, and also the set of sequences of strategy of group B as S_2, an element of S_2 as $\{s_2^j\}$ and the payoff function as $f_2(s_1^i, s_2^j)$, the payoff matrix of the PDG with sequential strategy is given as the following bimatrix B_M. Note that each range regarding i, j is $1 \le i,\ j \le 2^L$. Now, as noted before, the obtainable information of the two groups is limited because they have only n sequences of strategy and cannot know all patterns (2^L) of sequences of strategy and B_M. Therefore, we can consider that the above design is surely based on the bounded rational condition.

$$B_M = \begin{pmatrix} \begin{pmatrix} f_1(s_1^1, s_2^1), \\ f_2(s_1^1, s_2^1) \end{pmatrix} & \begin{pmatrix} f_1(s_1^1, s_2^2), \\ f_2(s_1^1, s_2^2) \end{pmatrix} & \cdots & \begin{pmatrix} f_1(s_1^1, s_2^{2^L}), \\ f_2(s_1^1, s_2^{2^L}) \end{pmatrix} \\ \begin{pmatrix} f_1(s_1^2, s_2^1), \\ f_2(s_1^2, s_2^1) \end{pmatrix} & \begin{pmatrix} f_1(s_1^2, s_2^2), \\ f_2(s_1^2, s_2^2) \end{pmatrix} & \cdots & \begin{pmatrix} f_1(s_1^2, s_2^{2^L}), \\ f_2(s_1^2, s_2^{2^L}) \end{pmatrix} \\ \vdots & \vdots & \ddots & \vdots \\ \begin{pmatrix} f_1(s_1^{2^L}, s_2^1), \\ f_2(s_1^{2^L}, s_2^1) \end{pmatrix} & \begin{pmatrix} f_1(s_1^{2^L}, s_2^2), \\ f_2(s_1^{2^L}, s_2^2) \end{pmatrix} & \cdots & \begin{pmatrix} f_1(s_1^{2^L}, s_2^{2^L}), \\ f_2(s_1^{2^L}, s_2^{2^L}) \end{pmatrix} \end{pmatrix} \tag{3}$$

TFT strategy described before can be also regarded as bounded rational in that the range of memory which stores the past action(s) of the opponent is limited. Moreover, the folk theorem of game theory proves that there are many possible equilibria in the indefinitely repeated prisoner's dilemma game in addition to the particular TFT strategy [49]. However, the TFT strategy and other possible equilibria suppose that every player knows the whole payoff matrix. The design of this study does not have such explicit memory, and also every group does not know the whole payoff bimatrix B_M as noted before. Therefore, this design is more bounded rational than the TFT strategy and other possible equilibria in the indefinitely repeated prisoner's dilemma game.

After finishing the PDG with sequential strategy, each group separately makes his/her decision, i.e., he/she selects his/her representative strategy. As noted in the introduction, this study employs the altruistic decision in the bounded rational condition. In

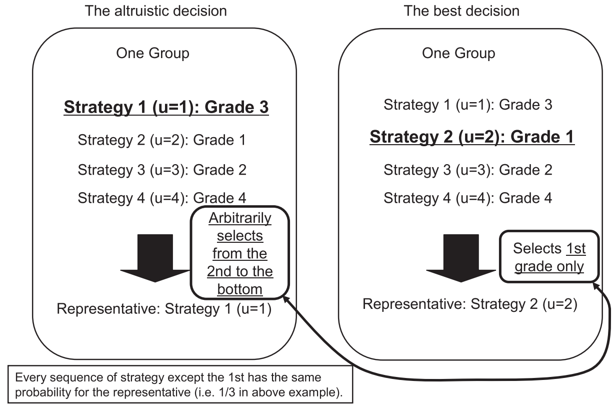

Fig. 2 Comparison between the altruistic decision and the best decision in the bounded rational condition. The altruistic decision is categorized as the decision with compromise, and has the concept that every group makes his/her decision without bias excluding the sequence of strategy of the highest payoff.

this decision, each group arbitrarily chooses his/her representative strategy from the one of the sequences of strategy except the highest. The sequence of strategy with the lowest payoff may be adopted as the representative strategy; however, this event does not occur in every process of decision. Payoff of the representative strategy of the group becomes eventually around the medium payoff when averaged. The following list gives the details of the altruistic decision. **Figure 2** also illustrates the distinction between the altruistic decision and the best decision in the bounded rational condition.

1. All sequences of strategy of each group are graded according to their payoff.
2. Each group arbitrarily selects one representative strategy from all sequences of strategy except the first grade. When all sequences of strategy have the same payoff, one sequence of strategy is randomly chosen as the representative strategy.

Through the above process, every group decides his/her representative strategy. After deciding the representative strategy, every group turns into the process of evolution (referred to as the evolutionary process in the following). In the process, as shown in **Fig. 3**, every group duplicates two fractions of his/her representative strategy to his/her other sequences of strategy in turn. Length of duplicated parts randomly changes from 1 to $L/2$. After finishing the evolutionary process, each character of created new sequences of strategy (including the representative strategy) is reversed according to the probability of mutation $m = 5.0\text{E-}04$. Through the evolutionary process, each group is ready for the following match of n rounds. This study defines the operation from the PDG with sequential strategy to the evolutionary process as one generation. One simulation lasts until the number of generations reaches 5,000 to investigate the fluctuation in a long period. Basically, the results described later are the average from 30 runs of simulation.

3. Results

3.1 The Altruistic Decision vs. the Second-best Decision

In the following **Figs. 4** and **5**, the altruistic decision is compared to the second-best decision [42] in the same bounded rational condition (limitation in the obtainable information as noted

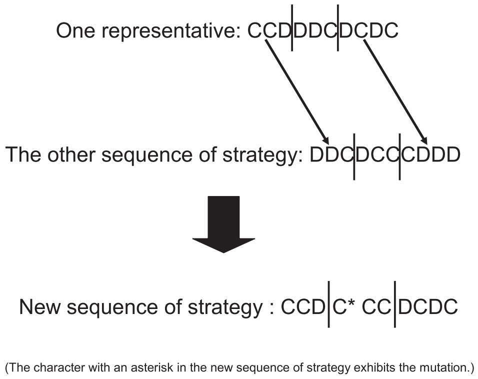

Fig. 3 Illustration of the evolutionary process. Every group updates all sequences of strategy according to the following manner. First, he/she chooses his/her one representative strategy. Second, he/she partly duplicates the representative strategy to his/her other sequences of strategy. The size of the duplicated fraction of the representative strategy randomly varies in every process of duplication regarding each sequence of strategy. Third, he/she mutates all generated new sequences of strategy by the reverse of every character with uniform probability.

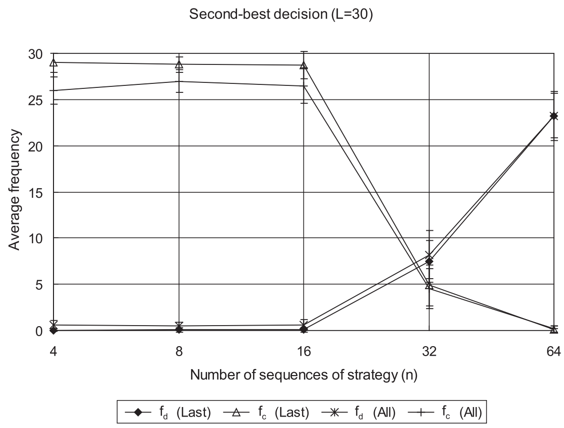

Fig. 5 This figure shows the dependence of the average frequency of mutual defection (f_d) and cooperation (f_c) on the number of sequences of strategy (n) at the last generation and in all generations for the second-best decision. The simulation setting of parameters regarding this figure is as follows; the length of sequences of strategy $L = 30$, the probability of mutation m = 5.0E-04. Each error bar in this figure shows the standard deviation of every frequency.

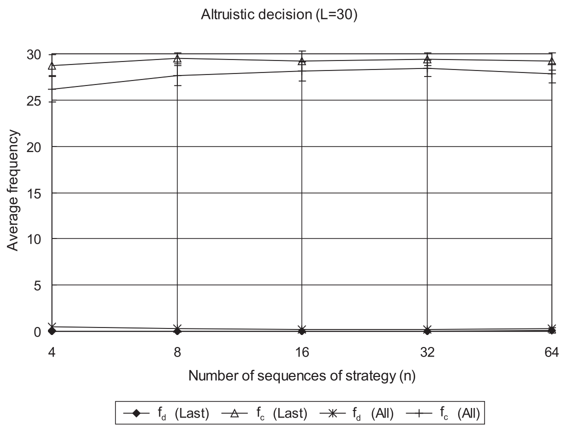

Fig. 4 This figure shows the dependence of the average frequency of mutual defection (f_d) and cooperation (f_c) on the number of sequences of strategy (n) at the last generation and in all generations for the altruistic decision. The simulation setting of parameters regarding this figure is as follows; the length of sequences of strategy $L = 30$, the probability of mutation m = 5.0E-04. Each error bar in this figure shows the standard deviation of every frequency.

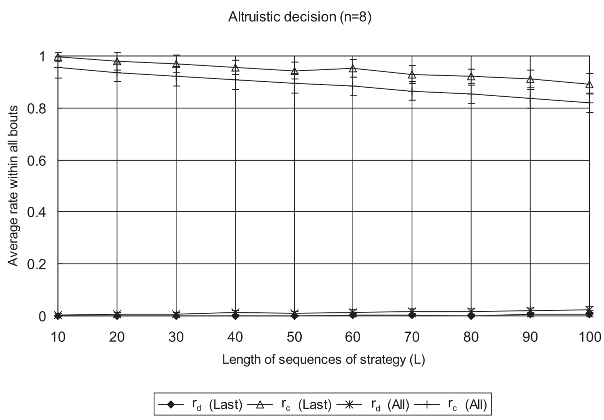

Fig. 6 This figure shows that the dependence of the average rate of mutual defection (r_d) and cooperation (r_c) on the length of sequences of strategy (L) at the last generation and in all generations for the altruistic decision. Note that r_d (r_c) can be obtained by the equation $r_d = f_d/L$ ($r_c = f_c/L$). The simulation setting of parameters regarding this figure is as follows; the number of total sequences of strategy of each group $n = 8$, the probability of mutation m = 5.0E-04. Each error bar in this figure shows the standard deviation of every rate.

before) to demonstrate the effect on the promotion of mutual cooperation and the suppression of mutual defection of the altruistic decision. The author also discusses the difference of outcome between each decision type. The simulation setting of parameters regarding Figs. 4 and 5 is as follows; the length of sequences of strategy $L = 30$, the probability of mutation m = 5.0E-04. The average frequency of mutual defection and cooperation from n rounds is given as f_d and f_c, and the length of sequences of strategy is L. In the result of the altruistic decision (Fig. 4), though the number of sequences of strategy in each group (n) increases, f_c does not change too much. Especially, employing the altruistic decision (see Fig. 4), f_c obviously increases in the case of $n = 32$, 64 while f_c does not grow in the same case of the second-best decision (see Fig. 5). In this case, f_c (Last and All) in the case of the altruistic decision is significantly larger than it in the case of the second-best decision. We have common knowledge that it is quite difficult in PDG to achieve mutual cooperation among many agents (= large number of sequences of strategy) [50], [51]. However, the model introducing the altruistic decision shows the different property and the high efficiency for the evolution of mutual cooperation.

Figure 6 illustrates another aspect of the altruistic decision. The simulation setting of parameters regarding Fig. 6 and **Fig. 7** is as follows; the number of total sequences of strategy of each group $n = 8$, the probability of mutation m = 5.0E-04. Note that the average rate of mutual defection (or cooperation) means the degree of mutual defection (cooperation) of groups in the PDG with sequential strategy. The average rate of mutual defection r_d (cooperation r_c) can be obtained by the equation $r_d = f_d/L$ ($r_c = f_c/L$). From those figures, it can be found that f_c decreases as the length of sequences of strategy (L) grows. This is because defection (D) is not easily replaced with cooperation (C) in every sequence of strategy when L becomes so long. However, contemplating Fig. 7 which illustrates the result of the second-best decision, it is shown that the altruistic decision works well on preserving mutual cooperation and suppressing the prevalence of

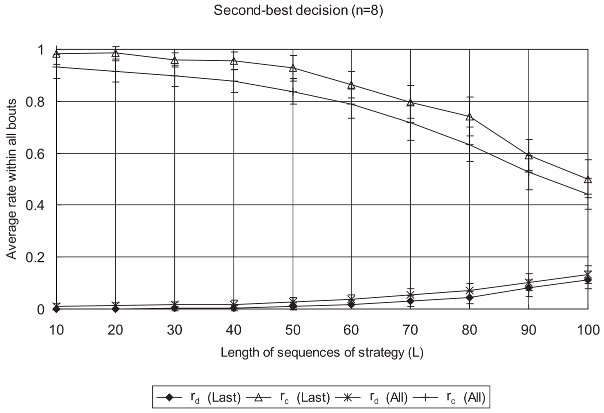

Fig. 7 This figure shows that the dependence of the average rate of mutual defection (r_d) and cooperation (r_c) on the length of sequences of strategy (L) at the last generation and in all generations for the second-best decision. Note that r_d (r_c) can be obtained by the equation $r_d = f_d/L$ ($r_c = f_c/L$). The simulation setting of parameters regarding this figure is as follows; the number of total sequences of strategy of each group $n = 8$, the probability of mutation m = 5.0E-04. Each error bar in this figure shows the standard deviation of every rate.

mutual defection especially in the case of $L \geq 60$. In particular, when $L = 100$, r_c (Last and All) in the case of the altruistic decision is significantly larger than it in the case of the second-best decision.

3.2 Further Experiment Regarding the Altruistic Decision

The previous results have revealed the effectiveness of the altruistic decision on the evolution of mutual cooperation in comparison with the second-best decision especially in large L. Someone may insist that it is not so surprising because each group employing the altruistic decision obeys the common knowledge for the evolution of mutual cooperation; that is, we should not stick to the temptation of recent future when we want to keep a cooperative relationship to the other [48]. However, the author considers it non-trivial and proposes that it cannot be predicted before executing simulation because in the model of the two groups, every group has a limited number of sequences of strategy and does not know the payoff matrix of the game. Therefore, in this subsection, based on the preceding results, the author probes further the traits of mutual cooperation based on the altruistic decision which the previous works [45], [46] do not mention.

Figures 8 and **9** (the number of total sequences of strategy of each group $n = 8$, the length of sequences of strategy $L = 30$, the probability of mutation m = 5.0E-04) exhibit the result which designates three categorized payoffs, i.e., the payoff of the representative strategy selected by the altruistic decision, the minimum payoff and the maximum payoff in each group. This result and following **Fig. 10** and **Table 2** is the one extraction from all 30 runs of simulation. The reason why the author does not show the averaged result is based on the fact that the difference of payoff cannot be distinguished when averaged because each development of f_c through 5,000 generations is so different between every simulation runs. From this investigation, the altruistic decision does not always select the sequence of strategy of the minimum payoff in the initial (from 1 to 10) generation. It can be found that the representative strategy has the payoff of the medium grade

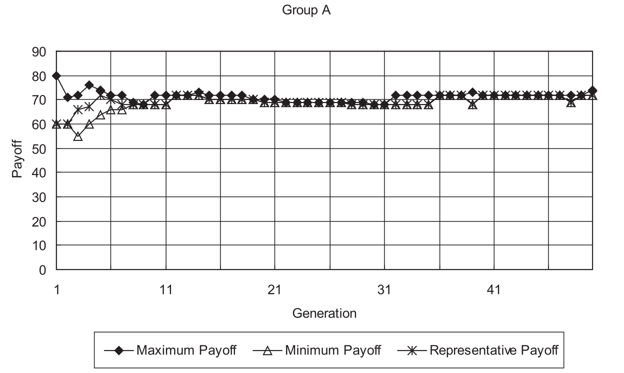

Fig. 8 Time series difference of the maximum payoff, the minimum payoff and the payoff of the representative strategy within 50 generations (Group A result). The simulation setting of parameters regarding this figure is as follows; the number of total sequences of strategy of each group $n = 8$, the length of sequences of strategy $L = 30$, the probability of mutation m = 5.0E-04.

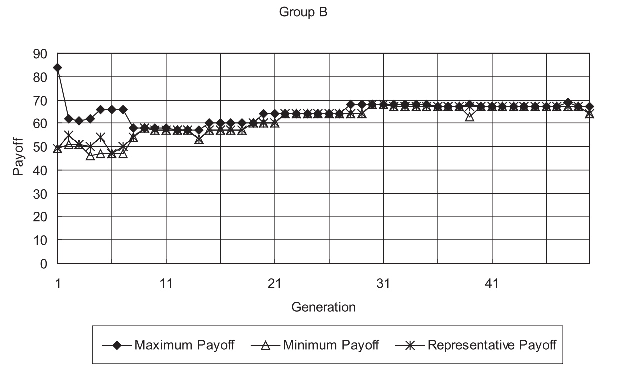

Fig. 9 Time series difference of the maximum payoff, the minimum payoff and the payoff of the representative strategy within 50 generations (Group B result). The simulation setting of parameters regarding this figure is as follows; the number of total sequences of strategy of each group $n = 8$, the length of sequences of strategy $L = 30$, the probability of mutation m = 5.0E-04.

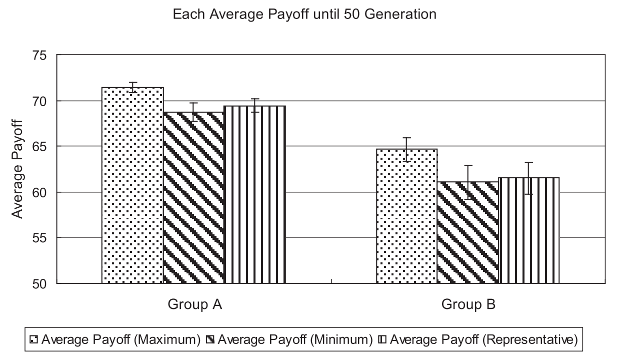

Fig. 10 This figure shows every average value of the maximum payoff, the minimum payoff and the payoff of the representative strategy within 50 generations. Each error bar illustrated in this figure is estimated with the significance level of 0.05. The simulation setting of parameters regarding this figure is the same as Figs. 8 and 9.

within each group during those periods. Figure 10 and Table 2 show that there is also a significant difference between each average payoff of the maximum and the representative within 50 generations with the significance level of 0.05. The simulation setting of parameters regarding Fig. 10 and Table 2 is the same as Figs. 8 and 9. As generation proceeds, function of the altruistic decision changes its role from the selection of the sequence of strategy of the medium grade to the exclusion of the sequence

Table 2 Average payoff within 50 generations with the significance level of 0.05. The simulation setting of parameters regarding this table is the same as Figs. 8 and 9.

Group A	Maximum payoff	71.44±0.582
	Minimum payoff	68.76±1.002
	Payoff of the representative strategy	69.4±0.73
Group B	Maximum payoff	64.68±1.29
	Minimum payoff	61.06±1.859
	Payoff of the representative strategy	61.5±1.695

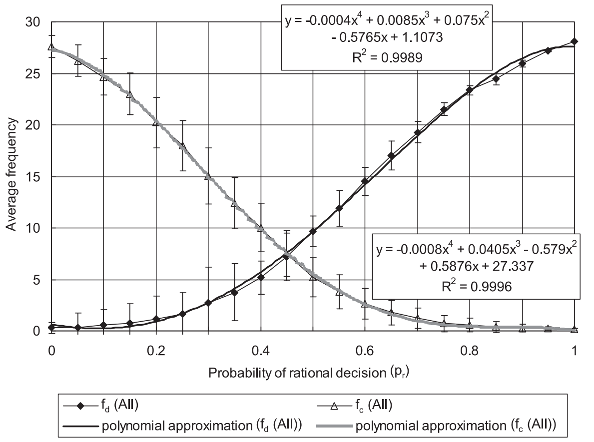

Fig. 11 Effect of the probabilistic rational decision on the average frequency of mutual defection (f_d) and cooperation (f_c) in all generations. Note that polynomial approximation is the fourth order and the value x can be found as follows with the probability of the rational decision (p_r); $x = 20p_r + 1$. R^2 means the coefficient of determination. The simulation setting of parameters regarding this figure is as follows; the number of total sequences of strategy of each group $n = 8$, the length of sequences of strategy $L = 30$. Those parameters are the same as the following Fig. 12. Each error bar in this figure shows the standard deviation of every frequency.

of strategy of the highest grade. The velocity of convergence regarding the variety of sequences of strategy within each group is so fast because the evolution of sequences of strategy is clearly directed to the growth of mutual cooperation.

The following **Figs. 11** and **12** are the detailed sensitivity analysis regarding the probability of the rational decision (p_r) and the probability of mutation (m) in the evolutionary process. Figure 11 is the outcome which illustrates the influence of the probabilistic rational decision on mutual cooperation employing the altruistic decision. The simulation setting of parameters regarding Fig. 11 is as follows; the number of total sequences of strategy of each group $n = 8$, the length of sequences of strategy $L = 30$. From the result, it can be found how the probability of the rational decision (p_r) affects the collapse of mutual cooperation. Now, the rational decision designates that a group selects the sequence of strategy of the maximum payoff as his/her representative strategy in the evolutionary process, and then p_r means how often each group makes the rational decision in every evolutionary process (generation). Therefore, when the value of p_r is 0.3, it indicates that each group makes the rational decision with that probability (and also the possibility of the altruistic decision should be 0.7). This probability is independent between groups. As shown in the result, the increase of p_r accelerates the growth

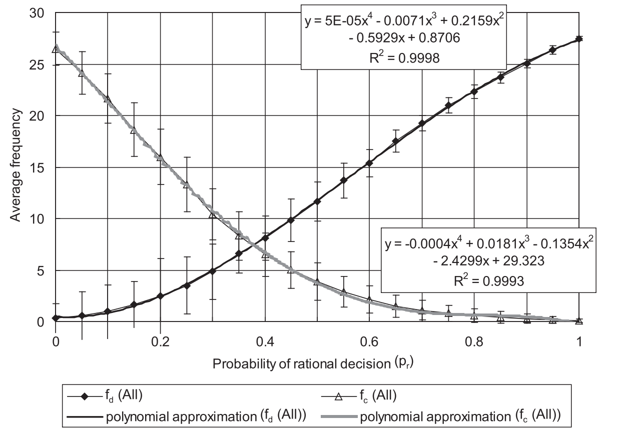

Fig. 12 Effect of the probabilistic rational decision on the average frequency of mutual defection (f_d) and cooperation (f_c) in all generations. Regarding this figure, the simulation setting of parameters except the probability of mutation (m) is the same as Fig. 11. The probability of mutation (m) is tripled from 5.0E-04 to 1.5E-03 in comparison with Fig. 11. Each error bar in this figure shows the standard deviation of every frequency.

of f_d, and also induces the decline of f_c. However, the acuteness for the increase of p_r is different between f_d and f_c. The variable f_c decreases more rapidly, whereas the growth of f_d is comparatively slow. The effect of the probabilistic rational decision offers the idea that mutual cooperation is strongly influenced by slightly rational behavior in the bounded rational condition.

In Fig. 11 of f_d and f_c for all generations, the author can approximately find the value of p_r on the point of intersection for two curves of f_d and f_c as 0.45602 with polynomial approximation of fourth order. Then, it is examined how this point of intersection alters with the change in the probability of mutation (m). This experiment also employs the same parameters as Fig. 11, however, only the probability of mutation (m) is tripled from 5.0E-04 to 1.5E-03. The result of this experiment is shown as Fig. 12 which also designates f_d and f_c for all generations like Fig. 11. The approximate value of p_r on the point of intersection can be found as 0.38349 with the same method of the previous approximation. The growth of the probability of mutation (m) certainly accelerates both the increase in f_d and the decrease in f_c; however, these changes are moderate in comparison with that growth.

4. Discussion

There are some similar frameworks that resemble the PDG with sequential strategy well, ex. centipede game [52] or Lindgren model [53] because the form of strategy is also a sequential array. However, this similarity is only apparent for the expression of strategy, and does not apply to the meaning of strategy. The centipede game discusses the length of cooperation from the beginning of the game. The Lindgren model exhibits the strategy of the player as bit strings describing his/her moves to each past action of the opponent, i.e., memory one strategy can be expressed as [s_1 s_2 s_3 s_4]. Note that s_1 corresponds to the case of previous match of (Player 1: C, Player 2: C), s_2: (C, D), s_3: (D, C) and s_4: (D, D) of each. Therefore, for example, TFT of memory one strategy is [1010]. The PDG with sequential strategy, unlike those models, discusses the frequency of cooperation in finite bouts without explicit reference to the past strategy of the opponent.

The works of IPDG define strategy as a reaction to the previous move(s) of the opponent. From this point of view, someone may criticize this study because sequences of strategy have no reactive action explicitly. However, when probing former studies of PDG, we can recognize that the case where a player cannot use his/her memory is not rare (such as Refs. [9], [54] or the case of no iteration and memory [19]). The PDG with sequential strategy is like PDG rather than IPDG in that a player has no chance to adjust his/her behavior once a game starts. However, due to the length of sequences of strategy, the PDG with sequential strategy has many various patterns of strategy (2^L) in comparison with PDG (only 2 patterns). That mechanism supports the operation like genetic algorithm in the evolutionary process. In addition, the model of the two groups in this paper is outstanding in comparison with former studies dealing with cooperation because all groups have only n sequences of strategy, and also cannot know all patterns (2^L) of sequences of strategy and the whole payoff matrix (i.e., in the bounded rational condition). That is, their obtainable information is quite limited. Because the elimination of memory and the limitation of information make it more difficult to construct cooperation, it is quite significant that mutual cooperation emerges within such restricted conditions.

5. Concluding Remarks

This paper enhances Ohdaira's previous discussion of the altruistic decision by newly introducing the notion of the bounded rationality, and then introduces the model of the two groups for discussing the altruistic decision in the bounded rational condition. The character of the model of the two groups can be summarized as follows. Firstly, the structure of strategy is a sequential array of fixed length, and strategy itself has no memory. Secondly, through the evolutionary process, each group makes the altruistic decision in the bounded rational condition, selects his/her representative strategy and overwrites his/her other sequences of strategy by the representative. This study shows that the altruistic decision can more effectively facilitate cooperation than the second-best decision in the case of large number of sequences of strategy n ($n = 32, 64$) and length of sequences of strategy L ($L \geq 60$) in the same bounded rational condition.

The additional outcome of this work is the detailed sensitivity analysis regarding the probability of the rational decision (p_r) and the probability of mutation (m) in the evolutionary process. The former is the influence of the probabilistic rational decision on mutual cooperation and mutual defection employing the altruistic decision (Fig. 11). Figure 11 shows that the acuteness for the increase of the probability of the rational decision (p_r) is different between the average frequency of mutual defection (f_d) and the average frequency of mutual cooperation (f_c). The variable f_c decreases more rapidly, while the growth of f_d is comparatively slow. The latter is the experiment of the tripled probability of mutation (m, from 5.0E-04 to 1.5E-03, see Fig. 12) which illustrates that the growth of the probability of mutation (m) certainly accelerates both the increase in f_d and the decrease in f_c; however, these changes are moderate in comparison with that growth. For further research based on this study, the author has already started to extend the model to the spatial game, utilizing various types of communication network. Some interesting properties have been obtained from the additional experiment. They have been already presented partly in Ref. [45], and also will appear as a new paper.

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References

[1] Nowak, M.A.: Five rules for the evolution of cooperation, *Science*, Vol.314, pp.1560–1563 (2006).
[2] Hamilton, W.D.: The genetical evolution of social behaviour, I and II, *J. Theor. Biol.*, Vol.7, pp.1–16 and pp.17–52 (1964).
[3] Trivers, R.L.: The evolution of reciprocal altruism, *The Quarterly Review of Biology*, Vol.46, pp.35–57 (1971).
[4] Nowak, M.A. and Sigmund, K.: The dynamics of indirect reciprocity, *J. Theor. Biol.*, Vol.194, pp.561–574 (1998).
[5] Nowak, M.A. and Sigmund, K.: Evolution of indirect reciprocity by image scoring, *Nature*, Vol.393, pp.573–577 (1998).
[6] Panchanathan, K. and Boyd, R.: A tale of two defectors: The importance of standing for the evolution of reciprocity, *J. Theor. Biol.*, Vol.224, pp.115–126 (2003).
[7] Panchanathan, K. and Boyd, R.: Indirect reciprocity can stabilize cooperation without the second-order free rider problem, *Nature*, Vol.432, pp.499–502 (2004).
[8] Nowak, M.A. and Sigmund, K.: Evolution of indirect reciprocity, *Nature*, Vol.437, pp.1291–1298 (2005).
[9] Nowak, M.A. and May, R.M.: Evolutionary games and spatial chaos, *Nature*, Vol.359, pp.826–829 (1992).
[10] Cao, Z. and Hwa, R.C.: Phase transition in evolutionary games, *Int. Jour. Mod. Phys. A*, Vol.14, pp.1551–1560 (1999).
[11] Ahmed, E., Hegazi, A.S. and Elgazzar, A.S.: On spatial asymmetric games, *Advances in Complex Systems*, Vol.5, pp.433–443 (2002).
[12] Kim, B.J., Trusina, A., Holme, P., Minnhagen, P., Chung, J.S. and Choi, M.Y.: Dynamic instabilities induced by asymmetric influence: Prisoner's dilemma game on small-world networks, *Phys. Rev. E*, Vol.66, 021907 (2002).
[13] Hauert, C. and Doebeli, M.: Spatial structure often inhibits the evolution of cooperation in the snowdrift game, *Nature*, Vol.428, pp.643–646 (2004).
[14] Nowak, M.A., Sasaki, A., Taylor, C. and Fudenberg, D.: Emergence of cooperation and evolutionary stability in finite populations, *Nature*, Vol.428, pp.646–650 (2004).
[15] Lieberman, E., Hauert, C. and Nowak, M.A.: Evolutionary dynamics on graphs, *Nature*, Vol.433, pp.312–316 (2005).
[16] Fort, H. and Sicardi, E.: Evolutionary Markovian strategies in 2×2 spatial games, *Physica A*, Vol.375, pp.323–335 (2007).
[17] Traulsen, A. and Nowak, M.A.: Evolution of cooperation by multilevel selection, *Proc. Natl. Acad. Sci. U.S.A.*, Vol.103, pp.10952–10955 (2006).
[18] Riolo, R.L.: The effects and evolution of tag-mediated selection of partners in populations playing the iterated prisoner's dilemma, *Proc. 7th Int. Conf. on Genetic Algorithms* (*ICGA97*), San Francisco, Morgan Kaufmann, pp.378–385 (1997).
[19] Hales, D.: Cooperation without space or memory: Tags, groups and the prisoner's dilemma, Multi-Agent-Based Simulation, *Lecture Notes in Artificial Intelligence*, Vol.1979, pp.157–166, Berlin, Springer-Verlag (2000).
[20] Riolo, R.L., Cohen, M.D.C. and Axelrod, R.: Evolution of cooperation without reciprocity, *Nature*, Vol.414, pp.441–443 (2001).
[21] Fowler, J.H.: Altruistic punishment and the origin of cooperation, *Proc. Natl. Acad. Sci. U.S.A.*, Vol.102, pp.7047–7049 (2005).
[22] Rankin, D.J., Santos M.D. and Wedekind, C.: The evolutionary significance of costly punishment is still to be demonstrated, *Proc. Natl. Acad. Sci. U.S.A.*, Vol.106, E135 (2009).
[23] Wang, Z., Szolnoki, A. and Perc, M.: Percolation threshold determines the optimal population density for public cooperation, *Phys. Rev. E*, Vol.85, 037101 (2012).
[24] Wang, Z., Szolnoki, A. and Perc, M.: If players are sparse social dilemmas are too: Importance of percolation for evolution of coop-

eration, *Scientific Reports*, Vol.2, 369 (2012).
[25] Ebel, H. and Bornholdt, S.: Coevolutionary games on networks, *Phys. Rev. E*, Vol.66, 056118 (2002).
[26] Zimmermann, M.G., Eguíluz, V.M. and Miguel, M.S.: Coevolution of dynamical states and interactions in dynamic networks, *Phys. Rev. E*, Vol.69, 065102(R) (2004).
[27] Zimmermann, M.G. and Eguíluz, V.M.: Cooperation, social networks, and the emergence of leadership in a prisoner's dilemma with adaptive local interactions, *Phys. Rev. E*, Vol.72, 056118 (2005).
[28] Eguíluz, V.M., Zimmermann, M.G., Cela-Conde, C.J. and Miguel, M.S.: Cooperation and the emergence of role differentiation in the dynamics of social networks, *American Journal of Sociology*, Vol.110, pp.977–1008 (2005).
[29] Pacheco, J.M., Traulsen, A. and Nowak, M.A.: Coevolution of strategy and structure in complex networks with dynamical linking, *Phys. Rev. Lett.*, Vol.97, 258103 (2006).
[30] Perc, M. and Szolnoki, A.: Coevolutionary games-A mini review, *Biosystems*, Vol.99, pp.109–125 (2010).
[31] Poncela, J., Gómez-Gardeñes, J., Floría, L.M., Sánchez, A. and Moreno, Y.: Complex cooperative networks from evolutionary preferential attachment, *PLoS ONE*, Vol.3, No.6, e2449 (2008).
[32] Szolnoki, A. and Perc, M.: Coevolution of teaching activity promotes cooperation, *New J. Phys.*, Vol.10, 043036 (2008).
[33] Szolnoki, A., Perc, M. and Danku, Z.: Making new connections towards cooperation in the prisoner's dilemma game, *Europhys. Lett.*, Vol.84, 50007 (2008).
[34] Szolnoki, A. and Perc, M.: Emergence of multilevel selection in the prisoner's dilemma game on coevolving random networks, *New J. Phys.*, Vol.11, 093033 (2009).
[35] Szolnoki, A. and Perc, M.: Resolving social dilemmas on evolving random networks, *Europhys. Lett.*, Vol.86, 30007 (2009).
[36] Van Segbroeck, S., Santos, F.C., Nowé, A., Pacheco, J.M. and Lenaerts, T.: The evolution of prompt reaction to adverse ties, *BMC Evol. Biol.*, Vol.8, 287 (2008).
[37] Van Segbroeck, S., Santos, F.C., Lenaerts, T. and Pacheco, J.M.: Reacting differently to adverse ties promotes cooperation in social networks, *Phys. Rev. Lett.*, Vol.102, 058105 (2009).
[38] Dreber, A., Rand, D.G., Fudenberg, D. and Nowak, M.A.: Winners don't punish, *Nature*, Vol.452, pp.348–351 (2008).
[39] Wu, J.-J., Zhang, B.-Y., Zhou, Z.-X., He, Q.-Q., Zheng, X.-D., Cressman, R. and Tao, Y.: Costly punishment does not always increase cooperation, *Proc. Natl. Acad. Sci. U.S.A.*, Vol.106, pp.17448–17451 (2009).
[40] Szolnoki, A., Szabó, G. and Czakó, L.: Competition of individual and institutional punishments in spatial public goods games, *Phys. Rev. E*, Vol.84, 046106 (2011).
[41] Perc, M. and Szolnoki, A.: Self-organization of punishment in structured populations, *New J. Phys.*, Vol.14, 043013 (2012).
[42] Ohdaira, T. and Terano, T.: Cooperation in the prisoner's dilemma game based on the second-best decision, *Journal of Artificial Societies and Social Simulation*, Vol.12, No.4 (2009).
[43] Ohdaira, T. and Terano, T.: Scale-free relationships facilitate cooperation in spatial games with sequential strategy, *Journal of Artificial Societies and Social Simulation*, Vol.14, No.3 (2011).
[44] Ohdaira, T. and Terano, T.: The diversity in the decision facilitates cooperation in the sequential prisoner's dilemma game, *Advances in Complex Systems*, Vol.14, pp.377–401 (2011).
[45] Ohdaira, T., Ohashi, H., Chen, Y. and Hashimoto, Y.: Partiality causes unhappiness: Randomness of network induces difficulty in establishing cooperative relationship, *Proc. 1st World Congress on Social Simulation* (*WCSS06*), Kyoto, Japan (2006).
[46] Ohdaira, T. and Ohashi, H.: The effect of occasional rational decision on the cooperative relationship between groups, *Proc. 12th International Symposium on Artificial Life and Robotics* (*AROB 12th '07*), Beppu, Oita, Japan (2007).
[47] Maynard Smith, J.: *Evolution and the Theory of Games*, Cambridge University Press, Cambridge (1982).
[48] Axelrod, R.: The Evolution of Cooperation, Basic Books, New York (1984).
[49] Binmore, K.: Review of the complexity of cooperation: Agent-based models of competition and collaboration by Robert Axelrod, *Journal of Artificial Societies and Social Simulation*, Vol.1, No.1 (1998).
[50] Duran, O. and Mulet, R.: Evolutionary prisoner's dilemma in random graphs, *Physica D*, Vol.208, pp.257–265 (2005).
[51] Chen, X., Fu, F. and Wang, L.: Prisoner's dilemma on community networks, *Physica A*, Vol.378, pp.512–518 (2007).
[52] McNamara, J.M., Barta, Z. and Houston, A.I.: Variation in behaviour promotes cooperation in the prisoner's dilemma game, *Nature*, Vol.428, pp.745–748 (2004).
[53] Lindgren, K. and Nordahl, M.G.: Evolutionary dynamics of spatial games, *Physica D*, Vol.75, pp.292–309 (1994).
[54] Fort, H.: Cooperation with random interaction and without memory or "tags", *Journal of Artificial Societies and Social Simulation*, Vol.6, No.2 (2003).

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